Cornell University

PHYS1205

The Physics of Sailing

Join us to learn physics on the water and in the classroom! Have fun while receiving science credits.

Contact: Prof. Hoffstaetter de Torquat at gh77@cornell.edu

PHYS 1205 Physics of Sailing

Goals

1. Have fun sailing experiences and understand what happens to the boat and the water/air environment.
2. Introduce elementary physics with the applied and tangible topic of sailing.
3. Perform hands-on experiments and understand basic concepts of experimentation.

Instructor: Georg Hoffstaetter de Torquat gh77@cornell.edu

Class Times and Participation

Classes:

Tuesday 2:55 – 5:15pm at the Cornell Sailing center until October 17 (fall break)

Tuesday 2:55 – 4:10pm in Rockefeller Hall B15 after fall break

Thursday 2:55 - 4:10 in Rockefeller Hall B15 throughout the semester

TAs – experienced sailors – one per 6-people boats

George Patte gp397@cornell.edu

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Christian Imre Varallyay civ2@cornell.edu

Jason Mannka jmannka@gmail.com

Director of the Cornell Sailing center: Ivan Sagel ivan.s@cornell.edu

Attendance will be taken at all classes

Class structure and safety

Structure:

- ½ of the classes will be on boats before fall break.
- Boat experiences will be a basis for physics explorations in on-campus classes.
- Physics phenomena from on-campus classes will lead to onboard experiments.
- Grades are based on: Boating agility (10%), onboard experiments (10%), Experiment reports (20%), Homework (30%), Final (30%)
- Homework will be posted on canvas Thursday night and is due before class on the following Thursday.

Safety:

- All need to have taken the Cornell swim test
- All bring life jackets on board
- There will be one experienced sailor TA per 6-person boat
- There will be man overboard training in the first boat class

Physics of Sailing Literature

Physics of Sailing

- Physics of Sailing (John Kimball)
- The Physics of Sailing Explained (Bryan D. Anderson)
- Physik des Segelns (Wolfgang Püschl)

Practial Sailing instruction with superficial physics explanations

- Basi Cruising (U.S. Sailing Association)
- Basic Keelboat (U.S. Sailing Association)
- Sailing Made Easy (American Sailing Association)
- Bareboat Cruising Made Easy (American Sailing Association)
- Coastal Cruising Made Easy (American Sailing Association)
- Advance Cruising and Seamanship (American Sailing Association)
- The Complete Sailing Manual (Steve Sleight)
- The Sailing Bible (Jeremy Evans)

Sailing Adventure Literature

- Sailing Alone Around the World (Joshua Slocum)
- The Long Way (Bernard Moitessier)
- Memories of the Open Sea (Eric Tabarly)
- Left for Dead: Surviving the Deadliest Storm in Modern Sailing History (Nick Ward)

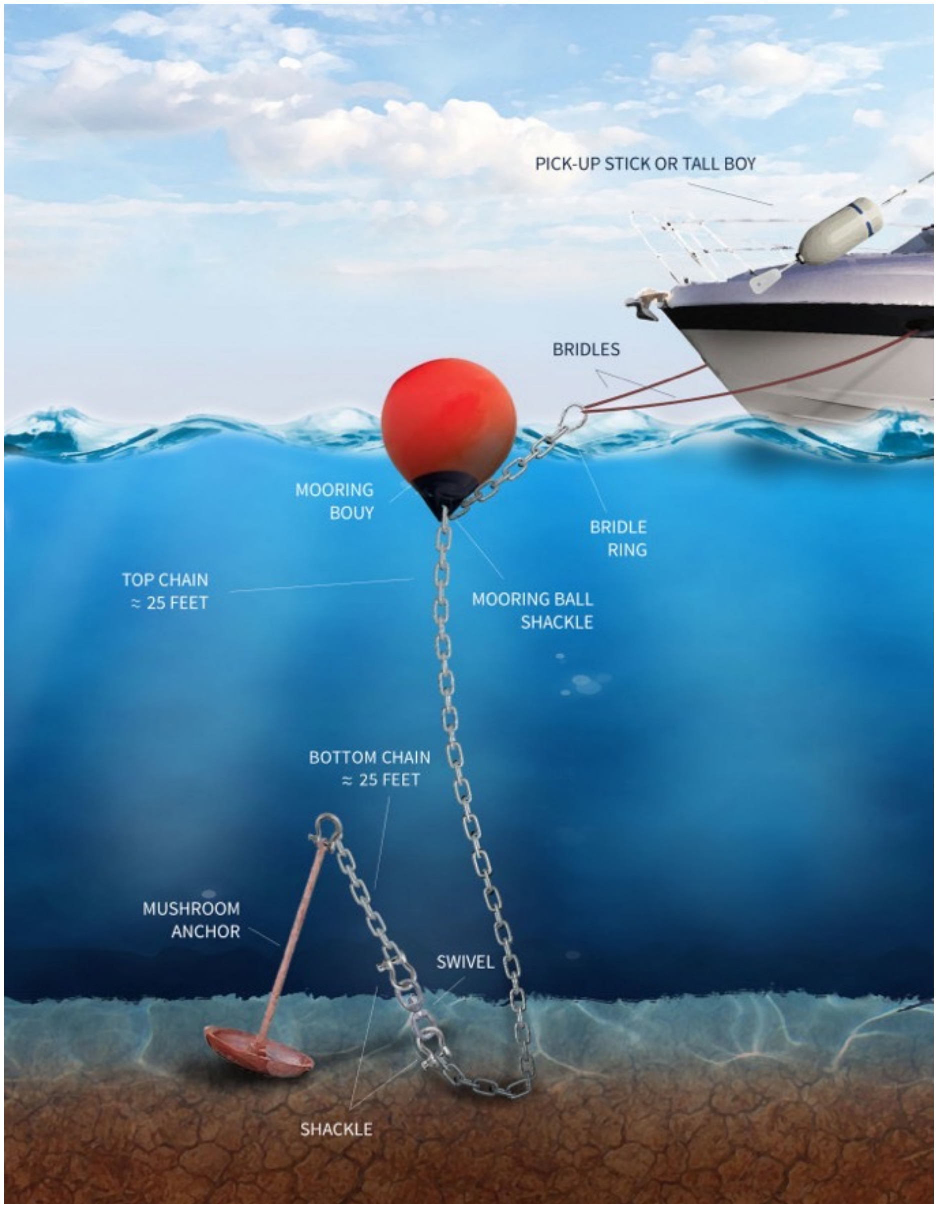
1st Week - Introduction – August 27 & 29

Goal:

1) Safety instruction
2) Parts of the boat
3) Sailing observations

1) Safety instruction (Summary)
   a) Make sure you have the Cornell swim test.
   b) Have your Personal Floatation Device (PFD) close by and wear it when windy.
   c) Man overboard training
      - Throw a PFD
      - Gybe (turn by steering away from the wind)
      - Approach against the wind
   d) Safe mooring
      - Approach against the wind
      - Hold on to the mooring line (not the mooring ball)
   e) Safe docking
      - Approach against the wind
      - Tie up so the boat does not touch the dock, if possible, otherwise use plenty of fenders.
      - Use the Cleat Hitch.

Vocabulary



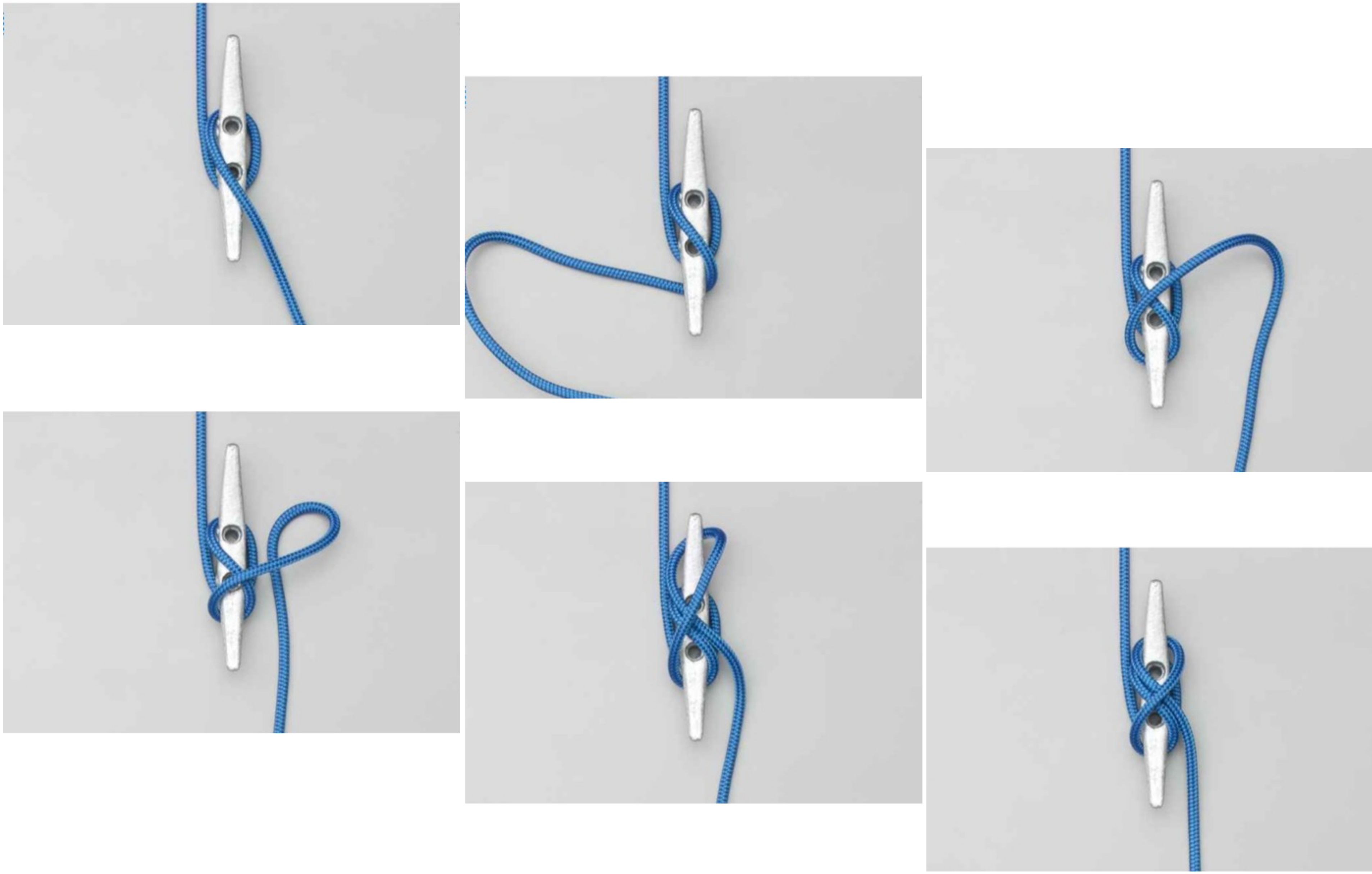
PFD

auto inflatable PFD

Mooring

Fenders

The Cleat Hitch

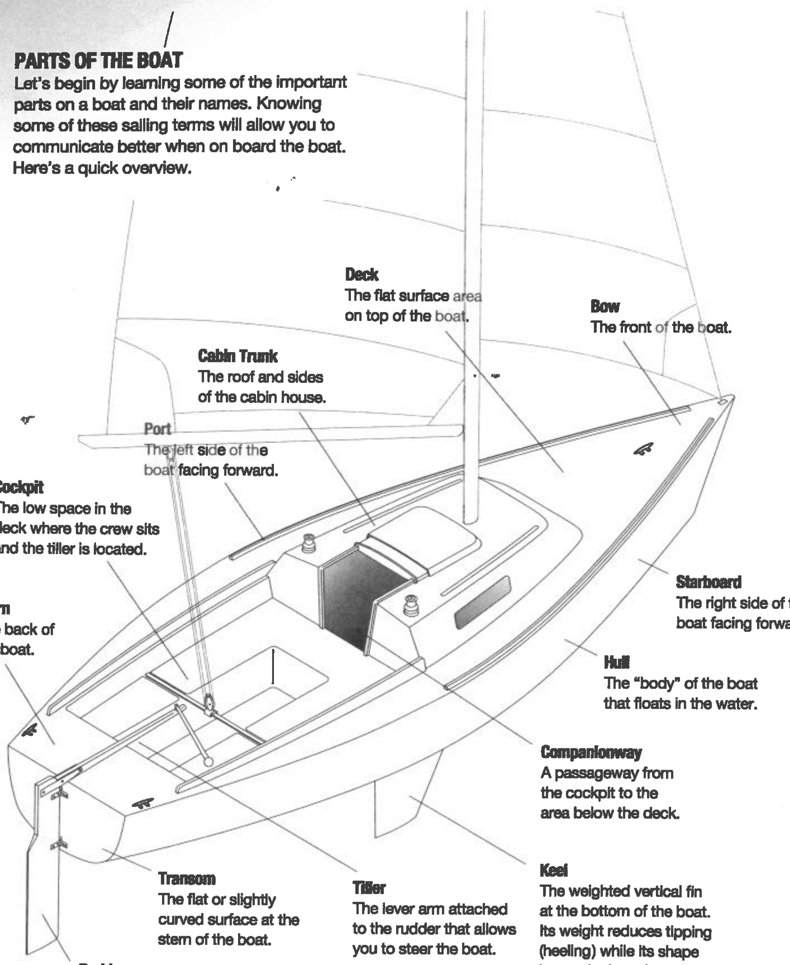


Parts of the Boat

PART 1

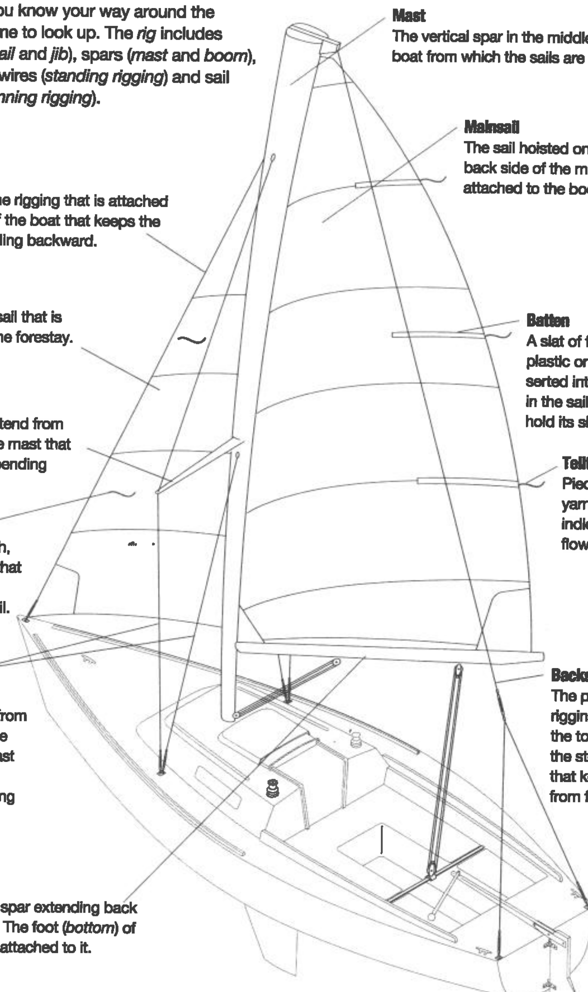
PARTS OF THE BOAT

Let's begin by learning some of the important parts on a boat and their names. Knowing some of these sailing terms will allow you to communicate better when on board the boat. Here's a quick overview.



PARTS OF THE RIG

Now that you know your way around the deck, it's time to look up. The *rig* includes sails (*mainsail* and *jib*), spars (*mast* and *boom*), supporting wires (*standing rigging*) and sail controls (*running rigging*).

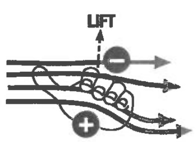


from Basi Cruising (U.S. Sailing Association)

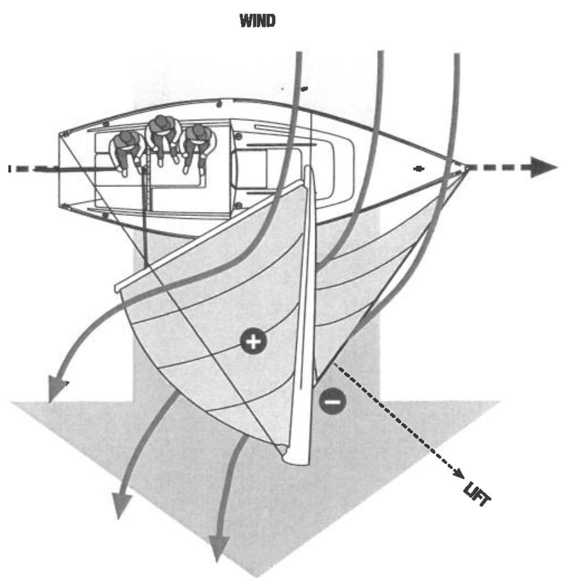
Parts of the Boat

HOW A SAIL WORKS

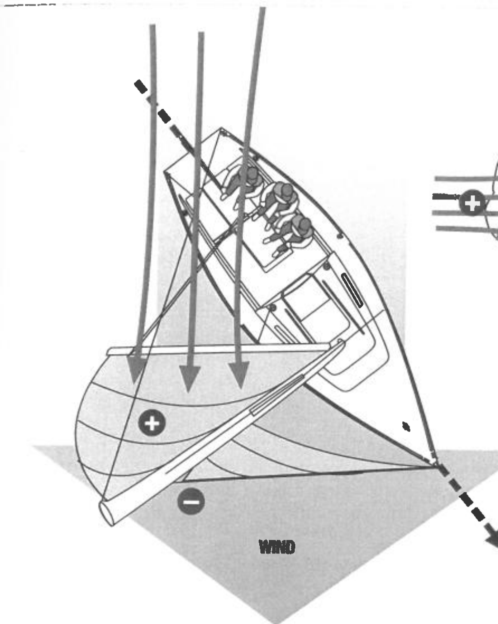
Sails are a boat's engine, and they produce power in one of two ways. When the wind is coming from the side of the boat, it flows around both sides of the sail (like an airplane wing), creating lift which "pulls" the boat forward. When the wind is coming from behind the boat, it "pushes" against the sail and simply shoves the boat forward.

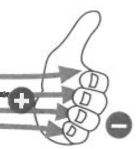


If you hold your hand out the window of a moving car, you can feel the force of the wind lifting your hand. This is the same force that "pulls" a sailboat forward when the wind comes over the side of the boat.



PULL MODE
Your sail is much more efficient at using the wind than your hand. It is shaped to bend the wind as it flows by, creating higher pressure on the inside of the sail ⊕ and lower pressure on the outside ⊖, thus creating lift. The lift the sail creates "pulls" the boat forward and sideways. The boat's keel keeps the boat from being pulled sideways through the water.

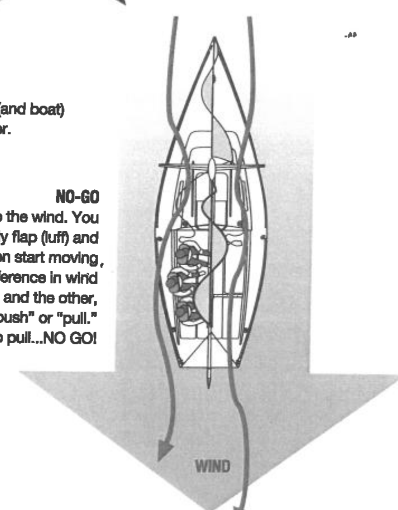




If you hold your hand out the window of a moving car with your palm facing the wind, you can feel the wind "push" your hand back. This is how a sail works when the wind is coming from behind.

PUSH MODE
With the wind coming from behind, the sail (and boat) are simply pushed forward through the water.

NO-GO
A sailboat cannot sail directly into the wind. You can try it, but your sails will only flap (luff) and you'll be dead in the water...or even start moving backward. Because there is no difference in wind pressure between one side of the sail and the other, the sail cannot generate either "push" or "pull." No push...no pull...NO GO!



from Basi Cruising (U.S. Sailing Association)

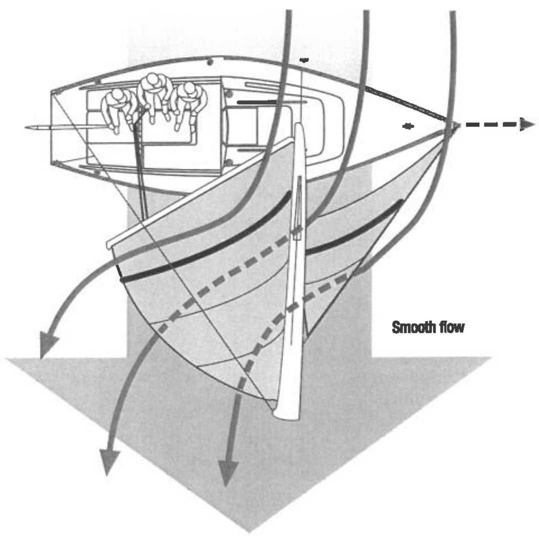
Parts of the Boat

TRIMMING A SAIL

Many sailors view a boat's mainsheet and jib sheet as they would the accelerator on a car...the sails are sheeted (pulled) in until the sails stop luffing to make the boat go. As we described on the previous spread, a sail creates lift by redirecting wind flow. If the wind flows smoothly around (not past) the sails, maximum power is achieved, resulting in maximum boatspeed. If the sails are sheeted in or out too much, turbulent flow will result, reducing flow and slowing the boat.

Trimmed Just Right

- Smooth flow around sail
- Optimum power
- Optimum boat speed
- Easy steering
- Well-balanced

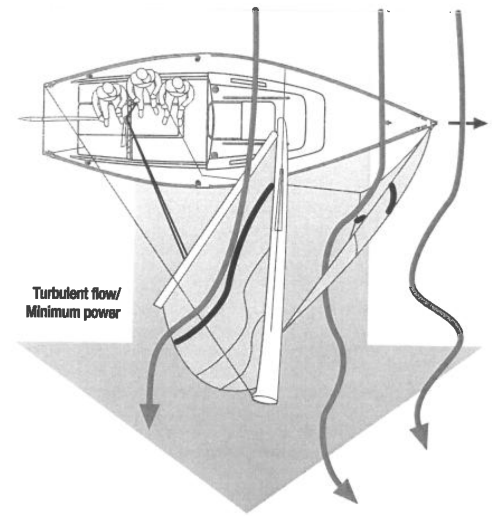


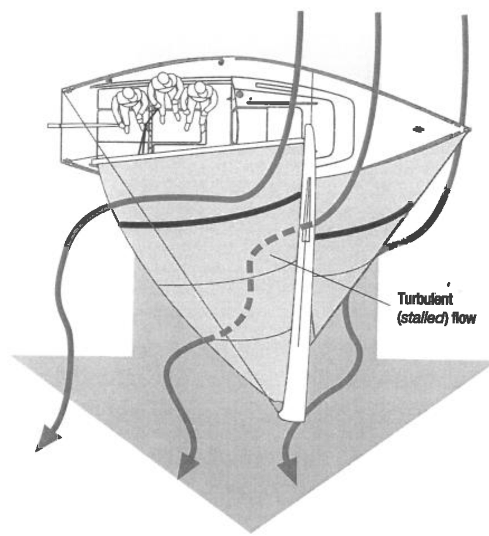
Trimmed Too Loose

- Turbulent flow around sail
- Minimum power
- Reduced boat speed

Sailing with your sails trimmed too far out is not necessarily bad. There are times when you will want to sail along slowly (at less than maximum speed) and will trim in your sails only part way.

Sheet in sails to the point where they stop luffing to trim them correctly.





Trimmed Too Tight

- Turbulent flow around sail
- Reduced power
- Boat slows down
- Difficult steering
- Excessive heeling

If you lose wind flow around your sails, your boat will begin to feel sluggish. To get smooth flow going again, let out the sails until they luff, then bring them back in until they stop luffing and feel the boat pick up speed.

from Basi Cruising (U.S. Sailing Association)

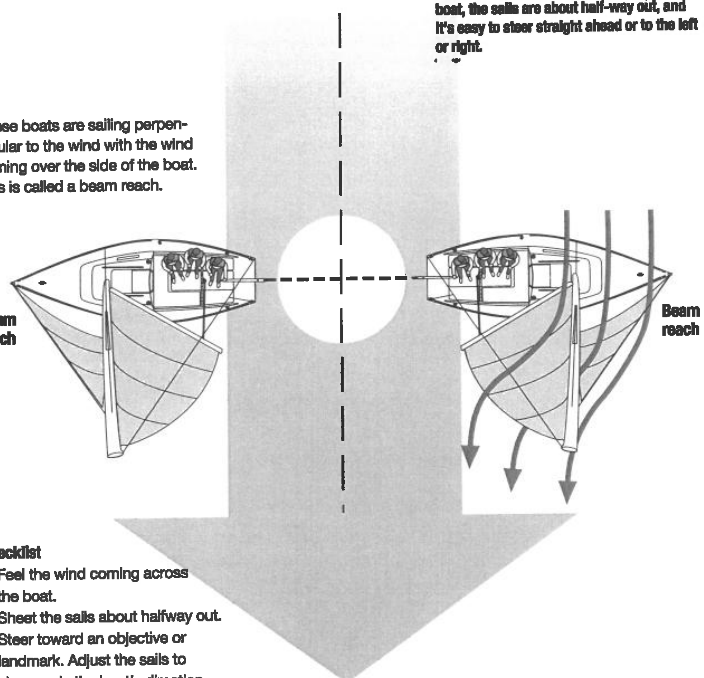
Parts of the Boat

SAILING ACROSS THE WIND

Sailing across the wind, with the wind perpendicular to the side of the boat, is a fast and easy way to sail — certainly easier than sailing upwind. In your first lesson, you will spend a lot of time sailing across the wind, learning how to steer and trim the sails.

Reaching (sailing across the wind) is easy, fun, and lively. There's a slight heel to the boat, the sails are about half-way out, and it's easy to steer straight ahead or to the left or right.

These boats are sailing perpendicular to the wind with the wind coming over the side of the boat. This is called a beam reach.



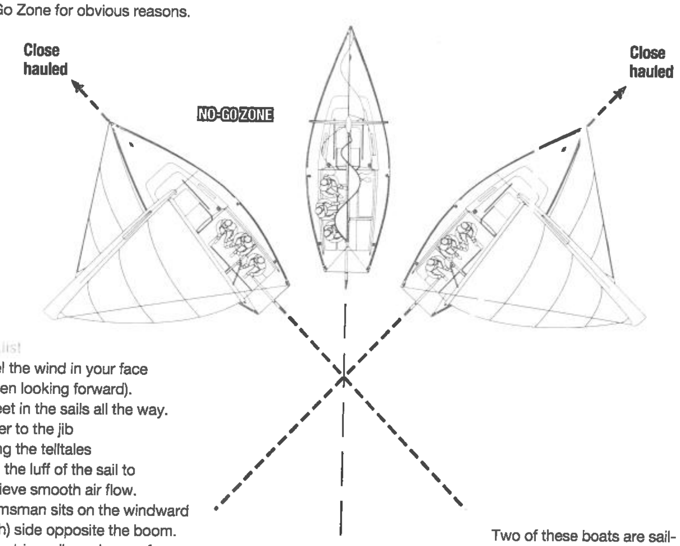
Checklist
- Feel the wind coming across the boat.
- Sheet the sails about halfway out.
- Steer toward an objective or landmark. Adjust the sails to changes in the boat's direction or changes in wind direction.

SAILING UPWIND

Although a boat cannot sail directly into the wind, it can sail upwind, or close to where the wind is coming from. Sailing about 45 degrees from the direction of the wind is about the closest a boat can sail upwind (although some high performance boats can sail as close as 30 to 35 degrees).

Sailing upwind is fun and exhilarating. You can feel waves passing under the hull, wind and spray in your face; knowing it is these natural elements that power the boat.

If you try to sail too close to the wind, your sails will luff and lose power, and the boat will come to a stop. This 90 degree area is called the No-Go Zone for obvious reasons.



Checklist
- Feel the wind in your face (when looking forward).
- Sheet in the sails all the way.
- Steer to the jib using the telltales and the luff of the sail to achieve smooth air flow.
- Helmsman sits on the windward (high) side opposite the boom.
- Crew trim sails and move from one side to the other to help keep the boat from heeling too much or too little.

Two of these boats are sailing as close to the direction of the wind as possible without entering the no-go zone. This is called sailing close-hauled.

from Basi Cruising (U.S. Sailing Association)

Parts of the Boat

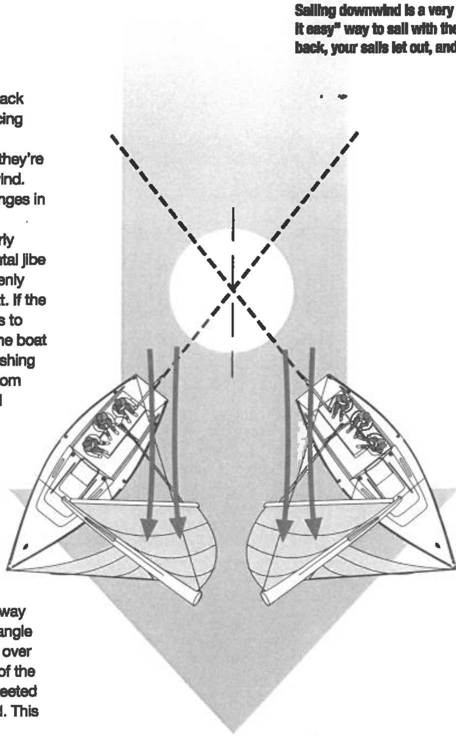
SAILING DOWNWIND

Sailing downwind, with the wind coming over the stern, is the most comfortable and relaxing point of sail. The wind and the waves are following you, the ride is smooth, and the boat stays upright. Sailing at a slight angle downwind (broad reach) is faster, safer, and easier than sailing directly downwind (with the wind coming directly behind the boat) because there is less chance for the boom to accidentally swing across the boat.

Sailing downwind is a very relaxing, "take it easy" way to sail with the wind at your back, your sails let out, and no spray.

Checklist

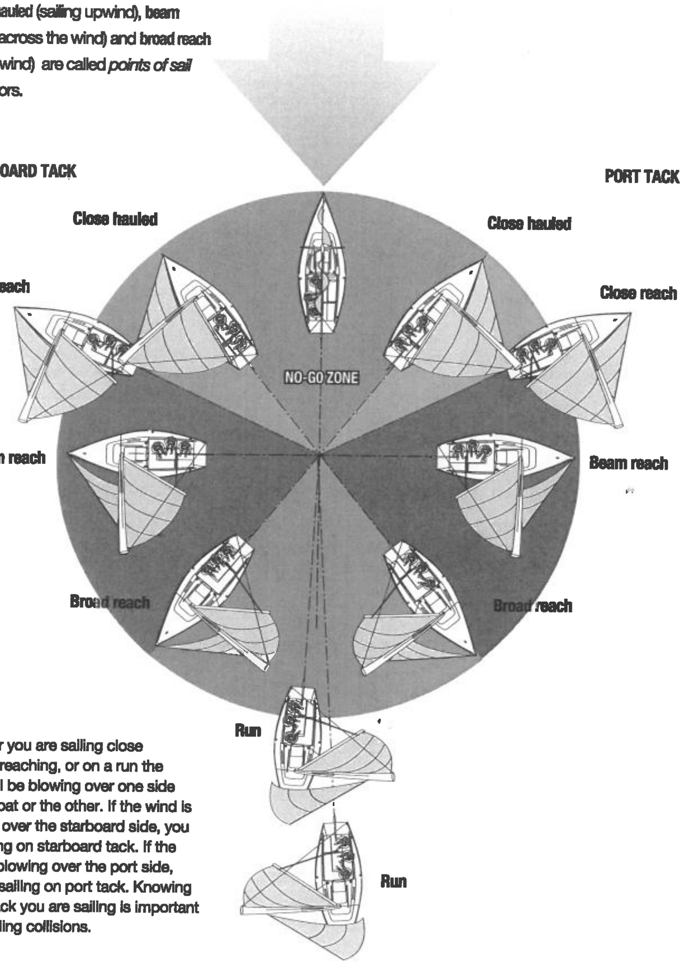
- Feel the wind on the back of your neck (when facing forward).
- Sheet out the sails so they're perpendicular to the wind.
- Adjust the sails to changes in boat or wind direction.
- Watch the jib as an early warning for an accidental jibe where the boom suddenly swings across the boat. If the jib goes limp and starts to cross the boat, head the boat toward the wind by pushing the tiller toward the boom until the jib returns and fills with wind again.



These boats are sailing away from the wind, but at an angle to it. The wind is blowing over the quarter (back corner of the boat) and the sails are sheeted perpendicular to the wind. This is called a broad reach.

POINTS OF SAIL

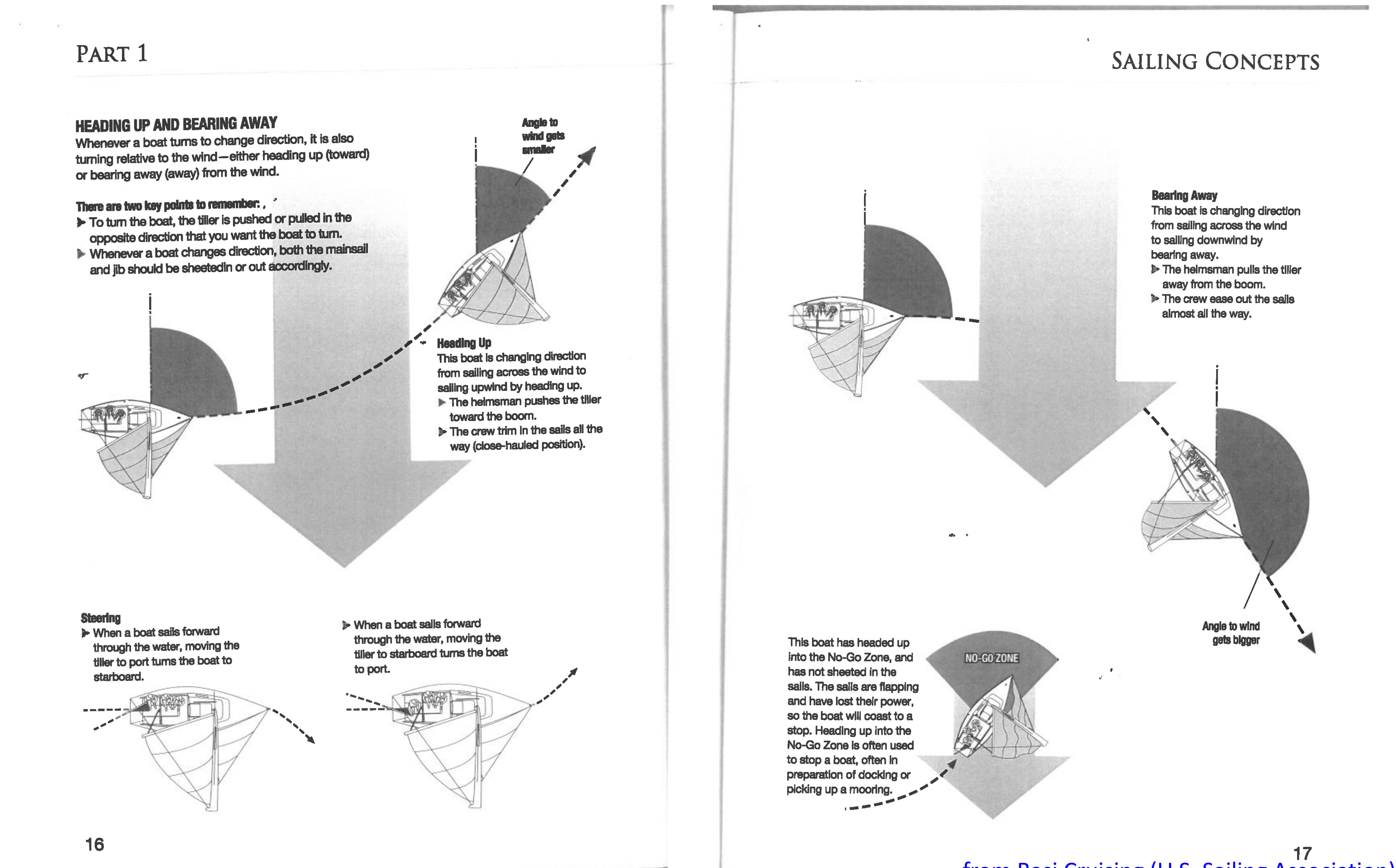
Close hauled (sailing upwind), beam reach (across the wind) and broad reach (downwind) are called *points of sail* by sailors.



Whether you are sailing close hauled, reaching, or on a run the wind will be blowing over one side of the boat or the other. If the wind is blowing over the starboard side, you are sailing on starboard tack. If the wind is blowing over the port side, you are sailing on port tack. Knowing which tack you are sailing is important for avoiding collisions.

from Basi Cruising (U.S. Sailing Association)

Parts of the Boat



PART 1

SAILING CONCEPTS

HEADING UP AND BEARING AWAY

Whenever a boat turns to change direction, it is also turning relative to the wind—either heading up (toward) or bearing away (away) from the wind.

There are two key points to remember:

- To turn the boat, the tiller is pushed or pulled in the opposite direction that you want the boat to turn.
- Whenever a boat changes direction, both the mainsail and jib should be sheetedin or out accordingly.

Heading Up
This boat is changing direction from sailing across the wind to sailing upwind by heading up.

- The helmsman pushes the tiller toward the boom.
- The crew trim in the sails all the way (close-hauled position).

Steering

- When a boat sails forward through the water, moving the tiller to port turns the boat to starboard.
- When a boat sails forward through the water, moving the tiller to starboard turns the boat to port.

Bearing Away
This boat is changing direction from sailing across the wind to sailing downwind by bearing away.

- The helmsman pulls the tiller away from the boom.
- The crew ease out the sails almost all the way.

This boat has headed up into the No-Go Zone, and has not sheeted in the sails. The sails are flapping and have lost their power, so the boat will coast to a stop. Heading up into the No-Go Zone is often used to stop a boat, often in preparation of docking or picking up a mooring.

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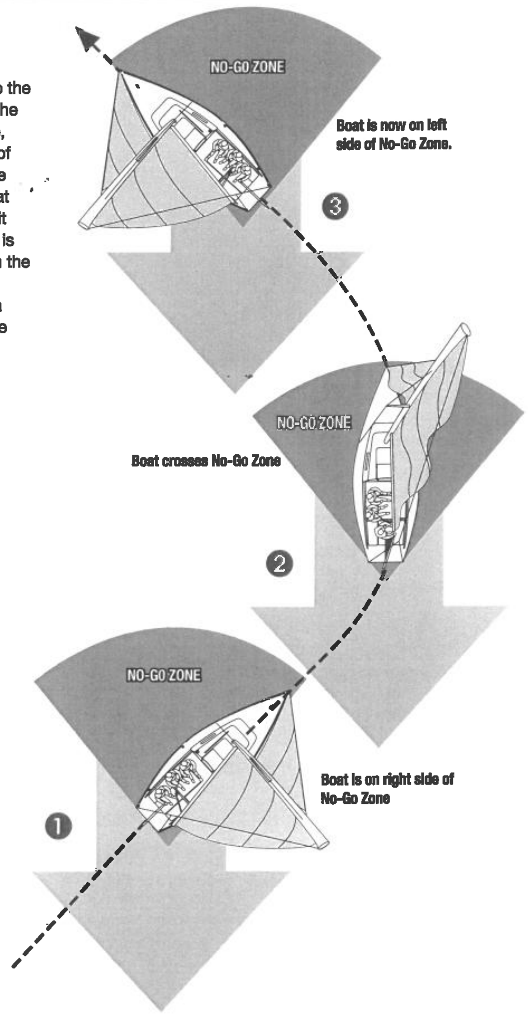
from Basi Cruising (U.S. Sailing Association)

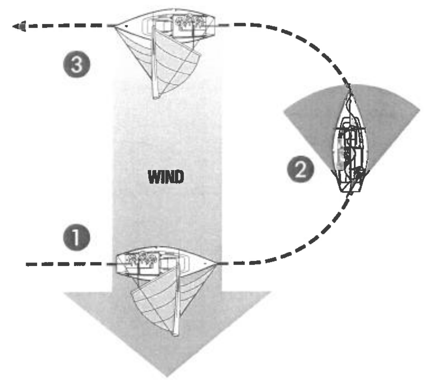
Parts of the Boat

TACKING

A sailboat cannot sail directly into the wind. To make progress toward the wind it must sail a zig-zag course, much as you would use a series of angular switch-backs to reach the top of a steep hill. When a sailboat switches from a "zig" to a "zag," it is called a tack. A tack or tacking is turning the bow of a boat through the wind from one side of the No-Go Zone to the other. When a boat crosses the No-Go Zone, the sails will also cross the boat.



At the beginning of the tack ①, the sailors are sailing close-hauled with the wind coming over the port side of the boat. In the middle of the tack ②, the boat crosses the wind and No-Go Zone, and the sails lose all their power. In the final part of the tack ③, their boat is again picking up speed, this time with the wind coming over starboard side of the boat. The boat's direction changed about 90 degrees.



TACKING FROM REACH TO REACH

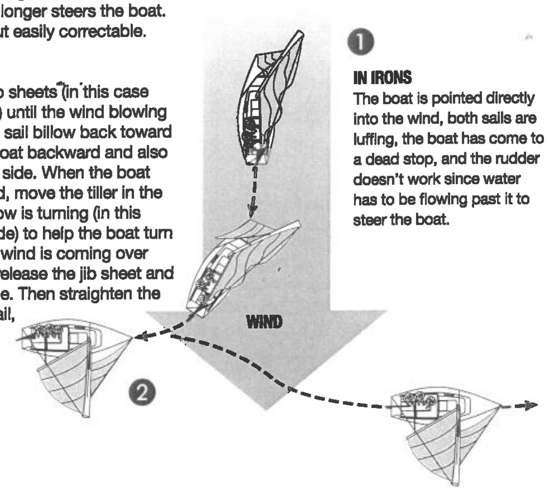
Tacking doesn't only happen when you are trying to sail toward the wind. Any time you switch the wind from one side of the boat to the other by sailing through the No-Go Zone, you are performing a tack. In the sequence to the left, the boat is reaching with the wind coming over the port side ①, then sailing through the No-Go Zone ②, and finally reaching with the wind coming over the starboard side ③.

GETTING OUT OF IRONS

At some point while you are learning to sail, you will tack the boat too slowly through the wind and get stuck in the No-Go Zone. You are now in irons. The sails are luffing, the boat slows to a stop, and the rudder no longer steers the boat. It's a helpless feeling, but easily correctable.

Here's how:

① Sheet in one of the jib sheets (in this case the one on the port side) until the wind blowing over the bow makes the sail billow back toward you. This will push the boat backward and also push the bow off to one side. When the boat starts to move backward, move the tiller in the same direction as the bow is turning (in this case to the starboard side) to help the boat turn more quickly. When the wind is coming over the side of the boat ②, release the jib sheet and trim it in on the other side. Then straighten the tiller, sheet in the mainsail, and off you go!



from Basi Cruising (U.S. Sailing Association)

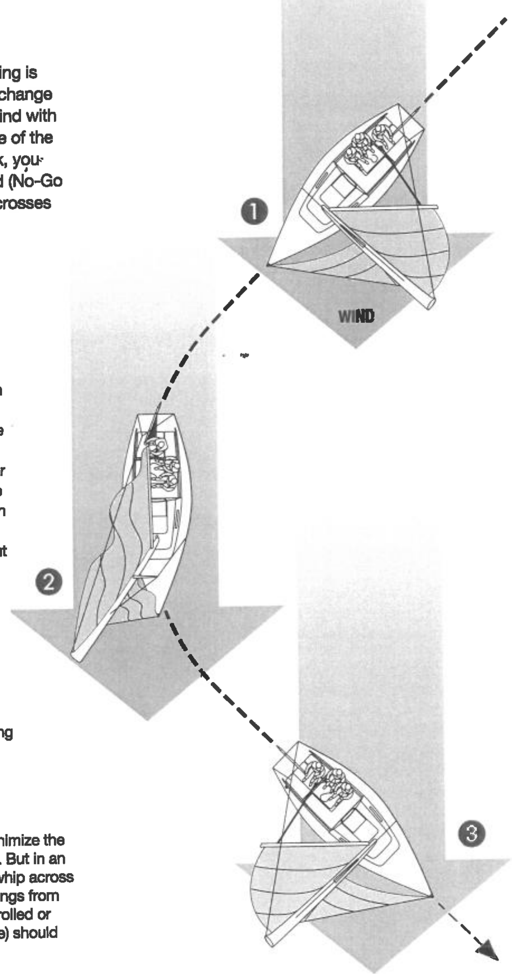
Parts of the Boat

JIBING

Another basic maneuver in sailing is the jibe. Like a tack, a jibe is a change in boat direction through the wind with the sails crossing from one side of the boat to the other. During a tack, you steer the bow through the wind (No-Go Zone). During a jibe, the wind crosses over the back of the stern.

At the beginning of the jibe ①, the sails are let out almost all the way with the wind coming over the starboard side of the boat. In the middle of the jibe ②, the stern of the boat crosses the wind and the sail swings over from one side to the other. A key to controlling the mainsail before it crosses over during a jibe is to sheet it in to the center of the boat before the stern crosses the wind. After the boom flops over, the mainsheet is let out quickly. Remember: *KEEP YOUR HEAD LOW AS THE BOOM SWINGS OVER!* In the final part of the jibe ③, the tiller is straightened and the mainsail is let back out almost all the way. The boat continues on with the wind coming over the port side of the boat.

NOTE: A "controlled" jibe helps minimize the speed of the boom crossing over. But in an uncontrolled jibe, the boom can whip across the cockpit quickly as the sail swings from one side to the other. An uncontrolled or *accidental jibe* (see opposite page) should be avoided.



DANGEROUS UNCONTROLLED JIBES!

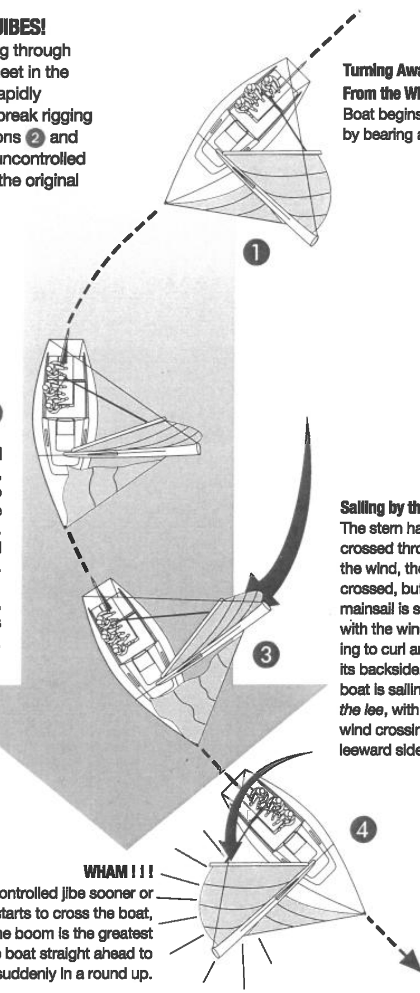
The boat in this illustration is going through an uncontrolled jibe, forgetting sheet in the mainsail. The force of the boom rapidly swinging across the cockpit can break rigging or hit a crew member. In illustrations ② and ③ it is still possible to avoid the uncontrolled jibe if the helmsman heads up to the original course in illustration ①. The key thing to do if the uncontrolled jibe occurs ④ is to quickly duck under the boom's path. *The alert sailor should shout out a warning, "Duck!"*

Turning Away From the Wind
Boat begins the jibe by bearing away.

Dead Downwind
Wind is directly behind the stern, and the jib is limp and starts to cross the boat. These are the warning signs of an impending jibe. The mainsail has not been sheeted in to the middle of the boat.

To prevent the jibe from happening, head up, moving the tiller towards the boom until the jib fills again.

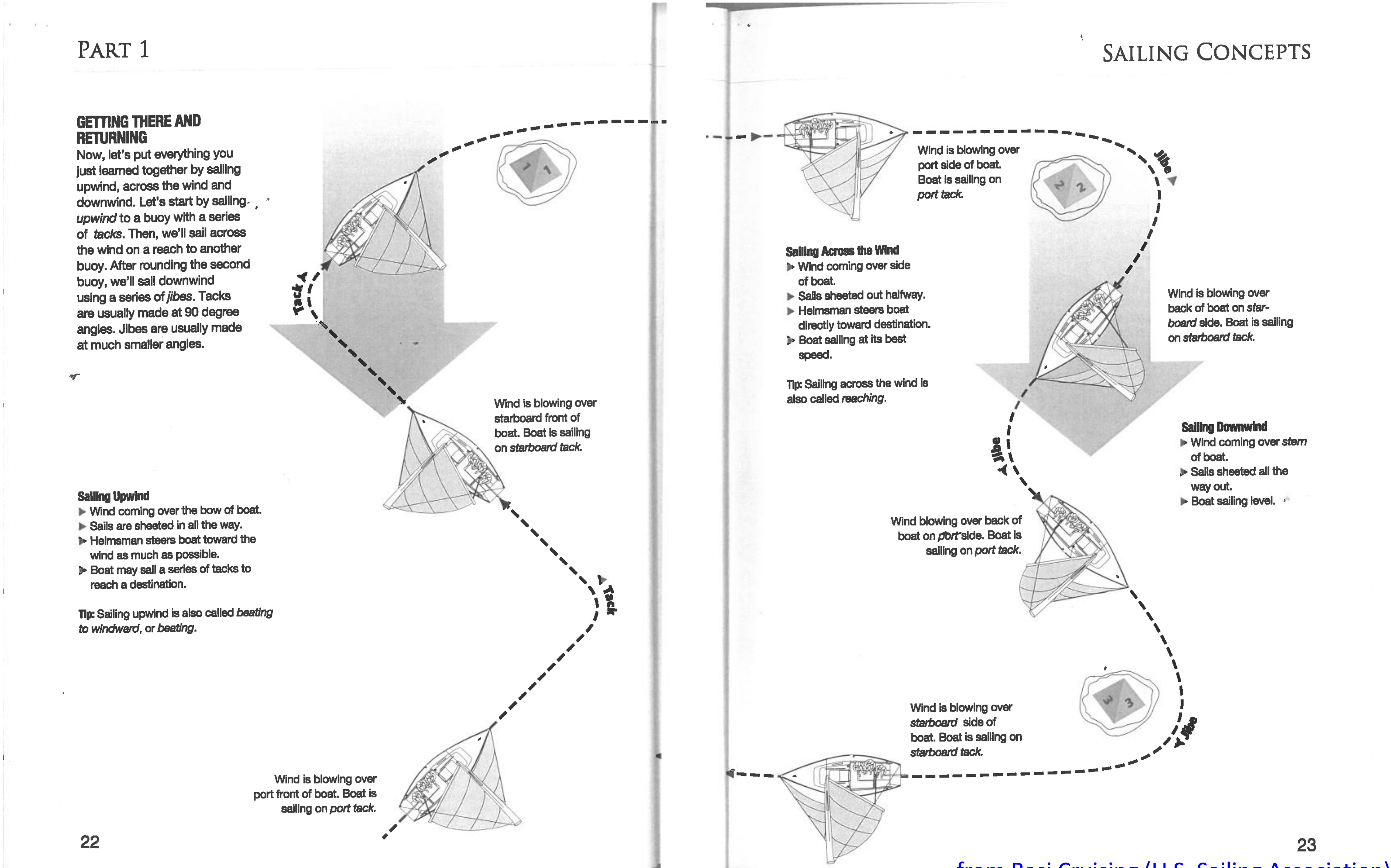
Sailing by the Lee
The stern has crossed through the wind, the jib has crossed, but the mainsail is still out with the wind starting to curl around its backside. The boat is sailing *by the lee*, with the wind crossing the leeward side first.



WHAM ! ! !
Everyone has an uncontrolled jibe sooner or later. Once the sail starts to cross the boat, ducking under the boom is the greatest priority. Next, steer the boat straight ahead to prevent it from turning suddenly in a round up.

from Basi Cruising (U.S. Sailing Association)

Parts of the Boat



PART 1

SAILING CONCEPTS

GETTING THERE AND RETURNING

Now, let's put everything you just learned together by sailing upwind, across the wind and downwind. Let's start by sailing *upwind* to a buoy with a series of *tacks*. Then, we'll sail across the wind on a reach to another buoy. After rounding the second buoy, we'll sail downwind using a series of *jibes*. Tacks are usually made at 90 degree angles. Jibes are usually made at much smaller angles.

Sailing Upwind

- Wind coming over the bow of boat.
- Sails are sheeted in all the way.
- Helmsman steers boat toward the wind as much as possible.
- Boat may sail a series of tacks to reach a destination.

Tip: Sailing upwind is also called *beating to windward*, or *beating*.

Wind is blowing over port front of boat. Boat is sailing on *port tack*.

Tack

Wind is blowing over starboard front of boat. Boat is sailing on *starboard tack*.

Tack

Wind is blowing over port side of boat. Boat is sailing on *port tack*.

Sailing Across the Wind

- Wind coming over side of boat.
- Sails sheeted out halfway.
- Helmsman steers boat directly toward destination.
- Boat sailing at its best speed.

Tip: Sailing across the wind is also called *reaching*.

Jibe

Wind is blowing over back of boat on *starboard* side. Boat is sailing on *starboard tack*.

Sailing Downwind

- Wind coming over *stern* of boat.
- Sails sheeted all the way out.
- Boat sailing level.

Jibe

Wind blowing over back of boat on *port* side. Boat is sailing on *port tack*.

Jibe

Wind is blowing over *starboard* side of boat. Boat is sailing on *starboard tack*.

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from Basi Cruising (U.S. Sailing Association)

The Apparent Wind

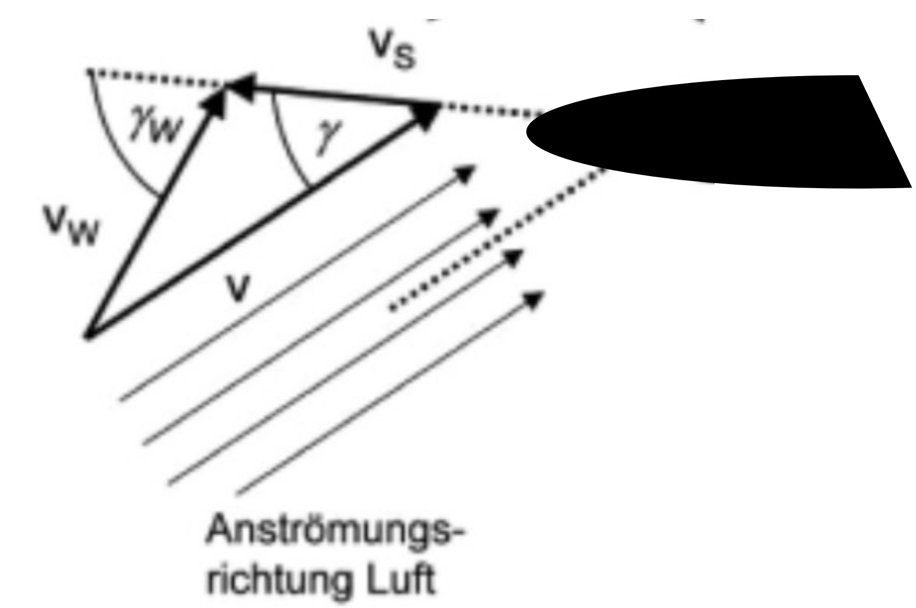
On a close reach, the angle of the apparent wind v to the boat velocity v_s is less than the angle of the true wind v_w to the boat velocity.

➔ After a turn, the direction of the apparent wind therefore changes, even if the true wind does not change.

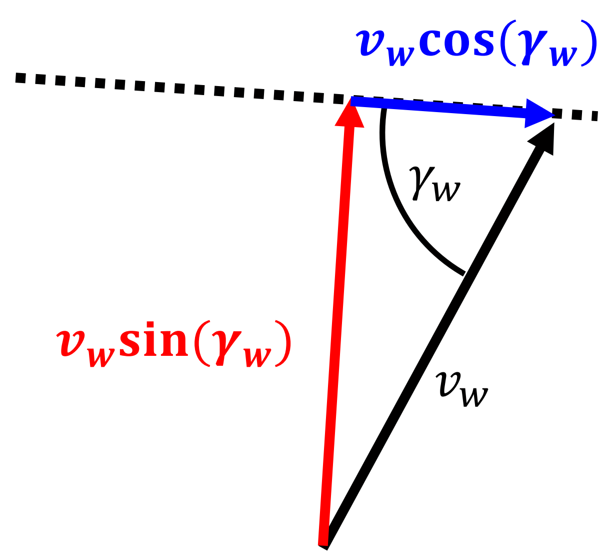
True wind: $v_w cos(\gamma_w)$ in direction of the boat, $v_w sin(\gamma_w)$ perpendicular to it.

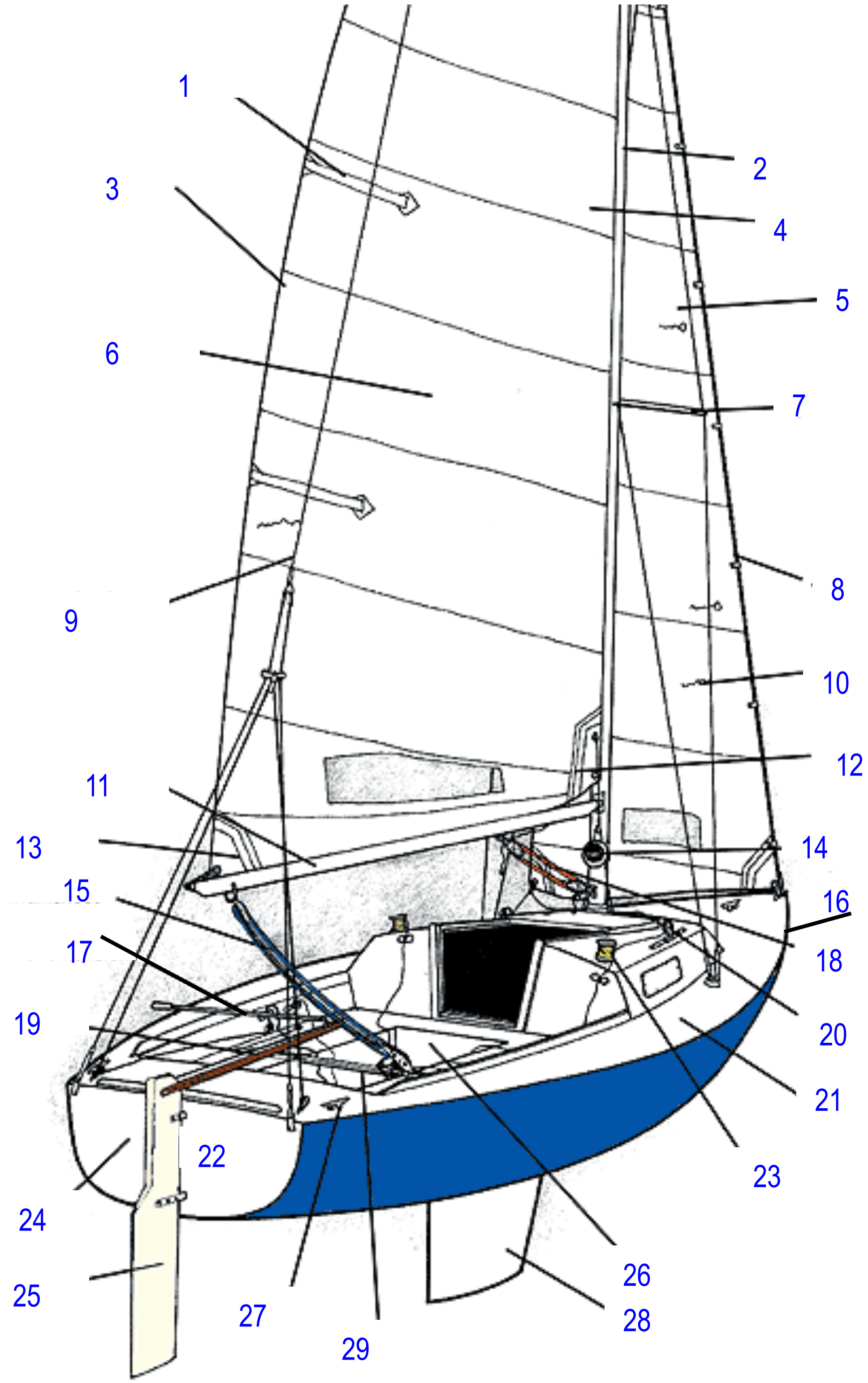
Apparent wind: $v_s + v_w cos(\gamma_w)$ in direction of the boat, $v_w sin(\gamma_w)$ perpendicular.

Boat velocity



Components of the wind velocity w.r.t. the boat direction:





Homework 1

1)	________	16)	________
2)	________	17)	Tiller extension
3)	Leech	18)	Boomvang
4)	Luff	19)	________
5)	________	20)	Block
6)	________	21)	________
7)	________	22)	________
8)	________	23)	Winch
9)	________	24)	________
10)	________	25)	________
11)	________	26)	________
12)	________	27)	________
13)	Clew	28)	________
14)	Compass	29)	________
15)	________		

Fill out the lines and explain

In a $v_w = 10kn$ north wind you sail to the north-north-west ($\gamma_w = 45°$) from the north with $v_s = 7kn.$

a) What is your apparent speed and from which direction does it come? (a diagram may help)

b) Are you on starboard or port tack?

c) If you turn to the other tack (also 45^0) from north, how does the apparent wind change?

Sailing on a river, against the wind

Two friends have a competition of sailing down a swift river.

Friend *A* records their time after struggling against a strong wind against them.

By the time friend *B* sails down the river, the wind has completely died down and they can just let themselves drift to the finish line.

Surprised, *B* realizes that they lost the race. How come?

History of the Sailboat

Around 4000BC:

1st image of a sail, Egyptian vase in Luxor

Around 3000-2000BC:

Images of auxiliary sails, Egyptian

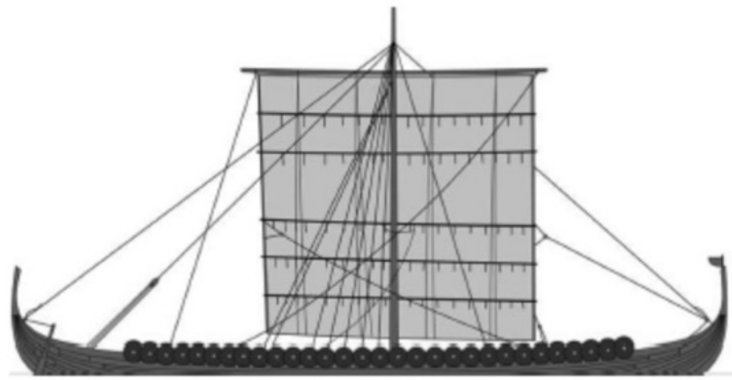
Around 2000-1000BC: Phoenician open sea sailboats

600BC: Phoenicians are reported to have sailed around Africa under Pharaoh Necho II (about 2000year before Vasco da Gama)

1000AC: Vikings sail to North America (about 500 years before Columbus)

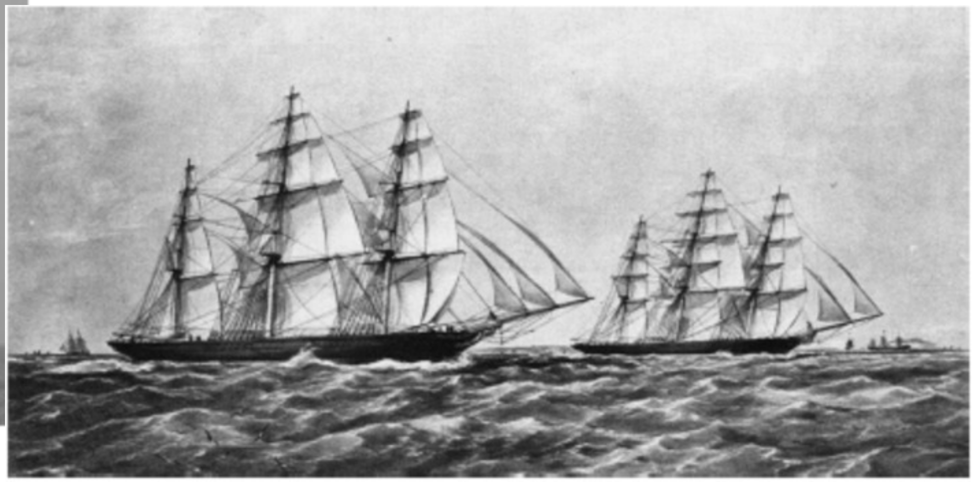
History of Different Sails: square sails

Early Egyptian and north European used square sails: good for broad reach, e.g. in the trait winds.



Viking Longboat

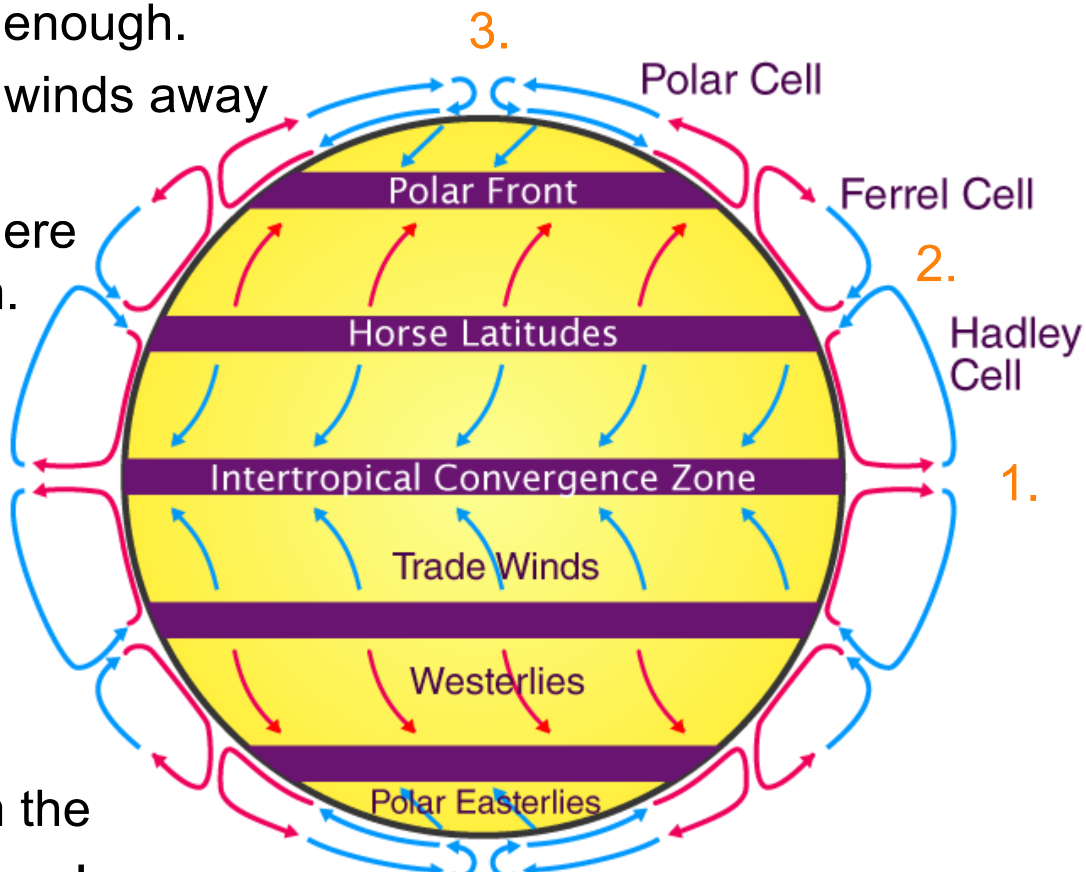
1600s AC, Atlantic and India trait



Tea Clipper, late 1800s AC

Trait winds

1. Hot air at the equator rises and pulls wind in from the north and south. ➔ Wind toward the equator
2. The risen wind pushes north and south and sinks when it has cooled enough.
3. Cold air at the poles sinks ➔ winds away from the pole
4. The thickness of the atmosphere produces 6 cells of circulation.



This would produce winds in north-south directions, if it were not for the Coriolis Force.

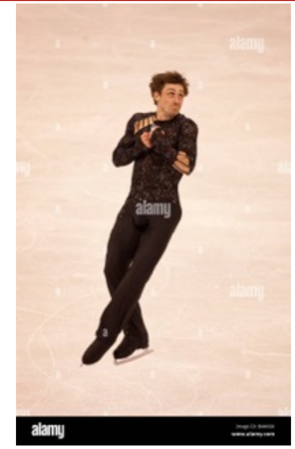
Note: The east-west direction of global winds does not come from the earth rotating under the atmosphere!

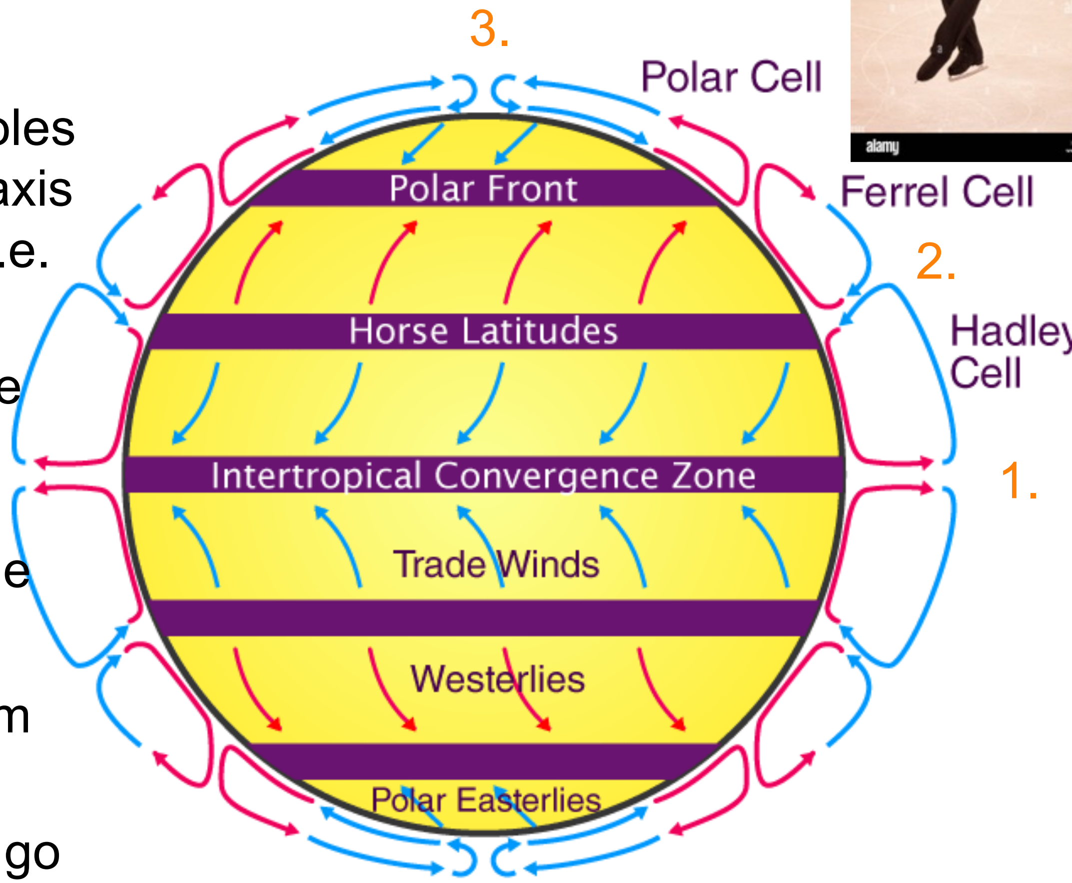
The Coriolis Force producing Trait Winds

1. In a pirouette an ice skater rotates faster when pulling the arms into the axis of rotation.
2. They rotate slower when lifting the arms sideways, away from the rotation axis.
3. The Earth rotates to the east.
4. Wind that flows toward the poles moves closer to the rotation axis and therefore rotates faster, i.e. toward the east.
5. Wind that flows away from the poles moves away from the rotation axis and therefore rotates slower, i.e. towards the west.

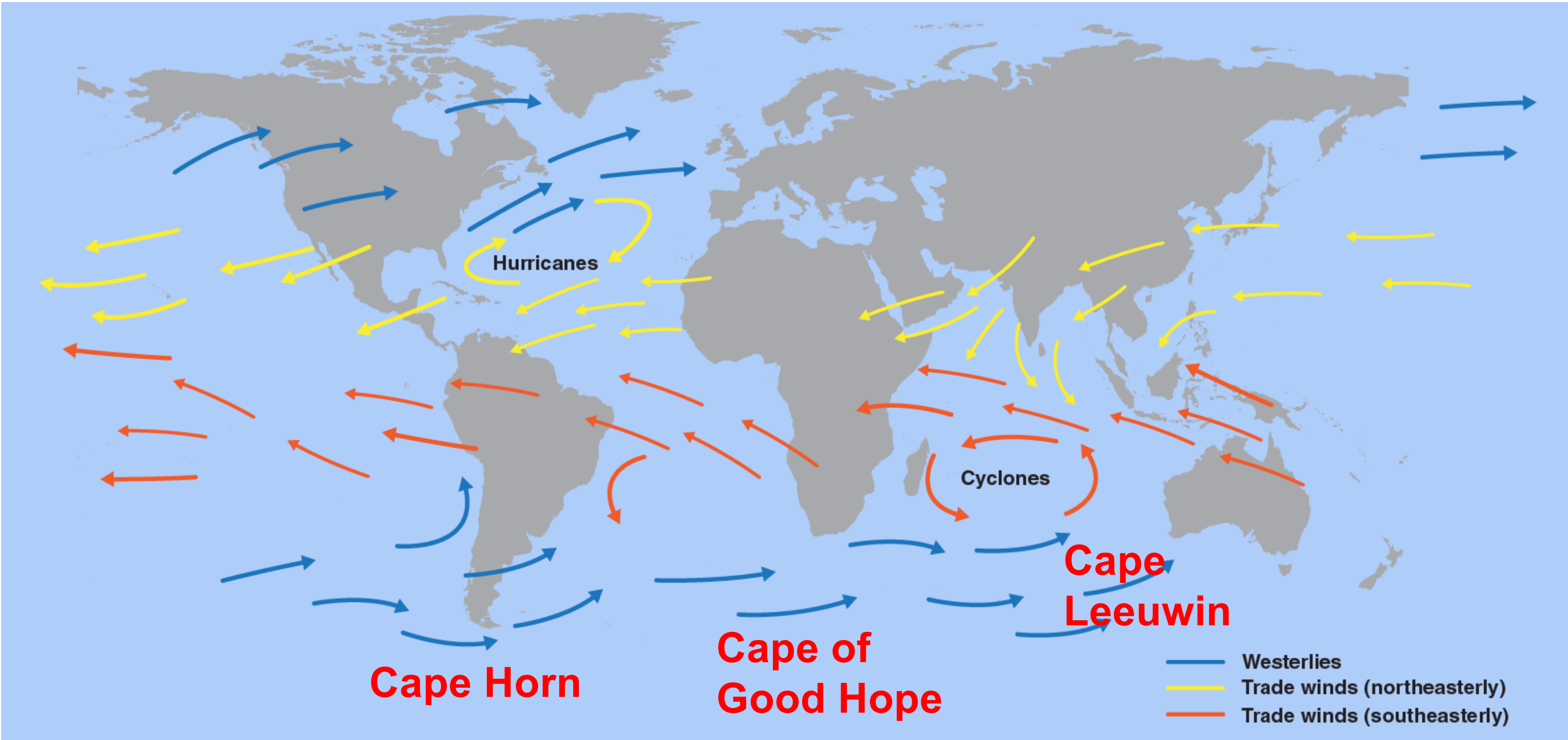
➔ NY latitude is good to sail from America to Europe.

➔ Caribbean latitude is good to go from Europe to America.





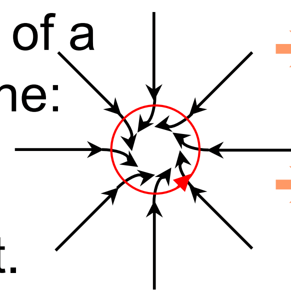
The hard direction at the great capes



- Cape Horn
- Cape of Good Hope
- Cape Leeuwin

All have preferred winds from the west.

Bottom of a hurricane:



➔ Hurricanes rotate counterclockwise

➔ Cyclones rotate clockwise.

Why do pirouettes turn faster with closed arms?

Explanation 1: Energy conservation

Pulling arms in against their centrifugal force needs energy.

This energy must go somewhere and the only place it can go is into faster rotation.

Remaining question: Where does the centrifugal force come from, and why is energy conserved?

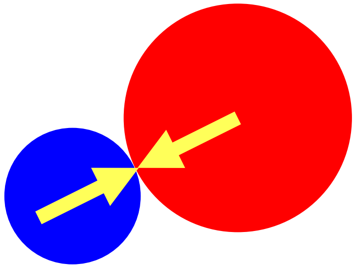
Newton's Laws:

1. Masses move in a straight line except if forces act on them.
2. A force is equal to the mass times the acceleration it produces.

$$F = m\,a = m\frac{\Delta v}{\Delta t}$$

3. When to objects interact, they experience forces that are equal, but opposite in reaction.

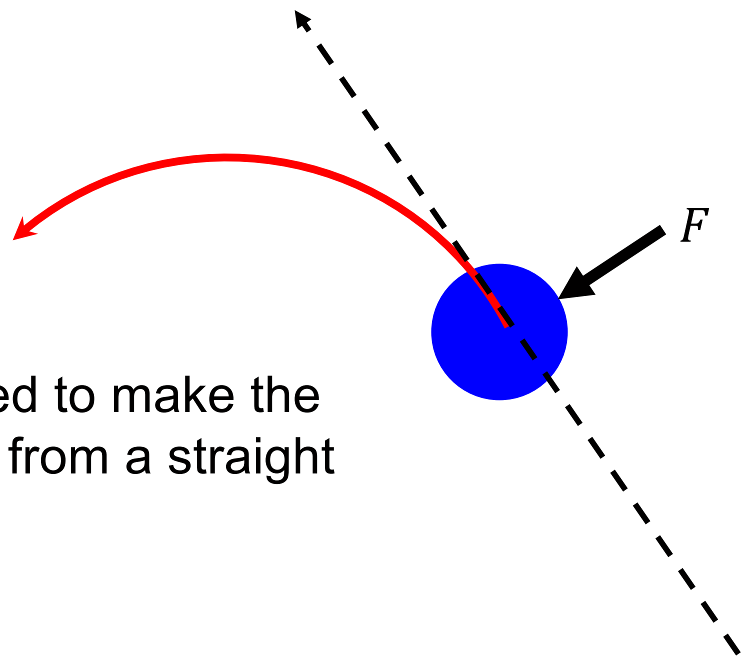
$$F_1 = -F_2$$



Centrifugal force?

Newton's Law:

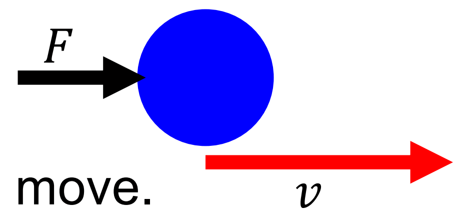
1. Masses move in a straight line except if forces act on them.



A force F is needed to make the mass curve away from a straight line.

Energy conservation

A force moving an object by a small distance Δs changes its velocity by a a small amount Δv.



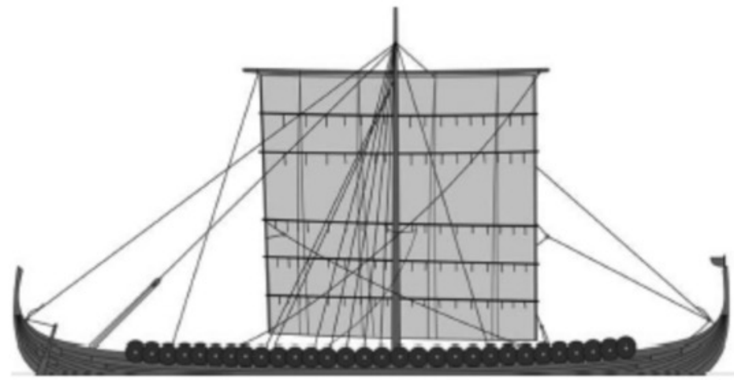
1. Acceleration $a = \frac{\Delta v}{\Delta t}$ and velocity $v = \frac{\Delta s}{\Delta t}$.
2. $F = m \cdot \frac{\Delta v}{\Delta t}$ ➔ Objects with more mass m are harder to move.
3. This can be used to compute how much the velocity is change by the force:
$$\Delta v = \frac{F}{m} \cdot \Delta t = \frac{F}{m} \cdot \frac{\Delta t}{\Delta s} \cdot \Delta s = \frac{F}{m \cdot v} \cdot \Delta s$$
4. There is a quantity that does not change, the energy
$$\Delta \mathrm{E} = \Delta\left(\frac{1}{2} m \cdot v^2 - F \cdot s\right) = 0$$
   a) The energy part due to velocity is the kinetic energy: $\mathrm{E}_{\mathrm{kin}} = \frac{1}{2} m \cdot v^2$
   b) Distance times force pushed is the potential energy: $\Delta \mathrm{E}_{\mathrm{pot}} = -\mathrm{F} \cdot \Delta s$

$$\Delta \mathrm{E} = \Delta E_{kin} + \Delta E_{pot} = \frac{1}{2} m \cdot v_2^2 - \frac{1}{2} m \cdot v_1^2 - F \cdot \Delta s = \frac{1}{2} m \cdot (v_2^2 - v_1^2) - F \cdot \Delta s$$

$$\Delta \mathrm{E} = m \frac{v_2 + v_1}{2} \cdot (v_2 - v_1) - F \cdot \Delta s = m \cdot v \cdot \Delta v - F \cdot \Delta s = 0$$

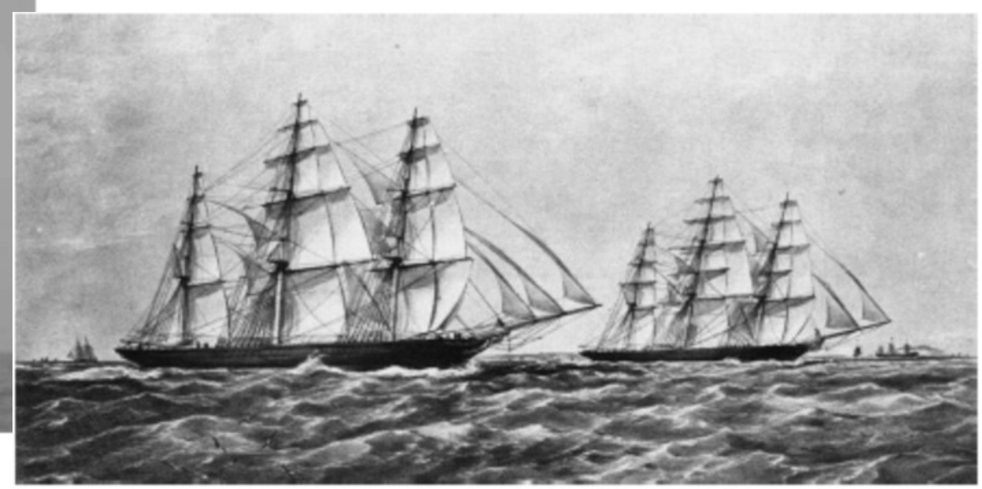
So much physics from talking about square sails !

Early Egyptian and north European used square sails: good for broad reach, e.g. in the trait winds.



Viking Longboat, 1000s

1600s AC, Atlantic and India trait



Tea Clipper, late 1800s AC

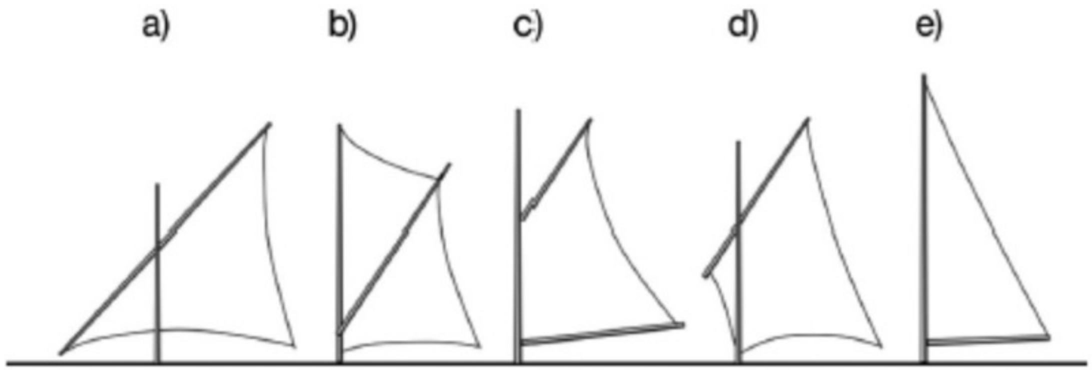
Cornell University

Homework 2

1) Which knot should you use to tighten a boat to a dock?
2) If a sink were perfectly round, filled with water at rest.
   a) In which direction would the draining water rotate in the northern hemisphere?
   b) What is the name of the force that would be responsible for this rotation?
   c) Does water from your sink always rotate in that direction?
   d) The draining water builds a funnel. What is the name of the force that presses water to the sloped wall of that funnel?
   e) The funnel gets narrower at the bottom. Does the water rotate faster or slower there, explain.
3) The Suez Canal was finished in 1869, the Panama Canal in 1914. If you had sailed around the world before the WW1, would you have preferred a westward or eastward direction. Today, would you prefer an eastward or westward direction. Explain your choices.
4) Explain why tall ship trading vessels often had square sails, but today's yachts rarely do.
5) If a sailboat crosses a straight river of 6mi width with a velocity of 10mi/h at an angle of 30^0 to a perpendicular crossing, how long will it take? How long will it take if you tack in the middle and continue with an angle of 15^0. Make a sketch and show where you use trigonometric functions.

History of Different Sails: fore-aft-sails

Arab region, later dominant in the Mediterranean: Lateen sails, i.e. fore-and-aft sails.



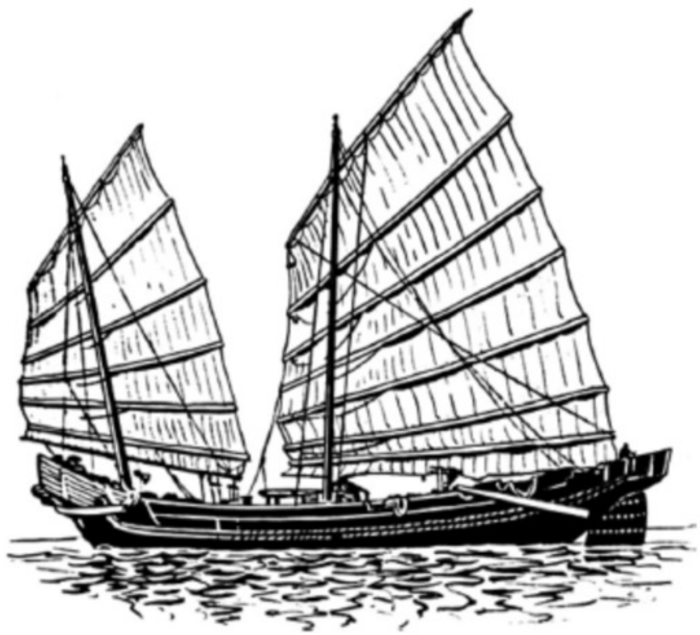
Other fore-and-aft sails:

(a) Lateen sail, (b) Sprit sail, (c) Gaff sail, (d) Lug sail, (e) Bermuda sail

History of Different Sails: Junk sails

Early 1400s AC: Ming Dynasty reaches the Red Sea with 100s of sailboats.

Sail type: Junk sail



Advantages: keeps shape well, easy to reef (reduce size in strong winds), no standing rigging (shrouds, fore or backstay) as mast is supported by the sail.

Pacific (Indonesian) migration: 3500BC to 1000BC

Type sailing canoes, often with two hulls, bidirectional with 180^0 sail rotation. Reported by Roman writers to have reach the Red Sea.

History of the Rudder

Early Rudders:

Mediterranean, rudders on both sides.

Norther European, one rudder on the right, the starboard side, leaving the port side for docking at port.

Around 1000AC: Invention of central rudder with firm connection to the bow

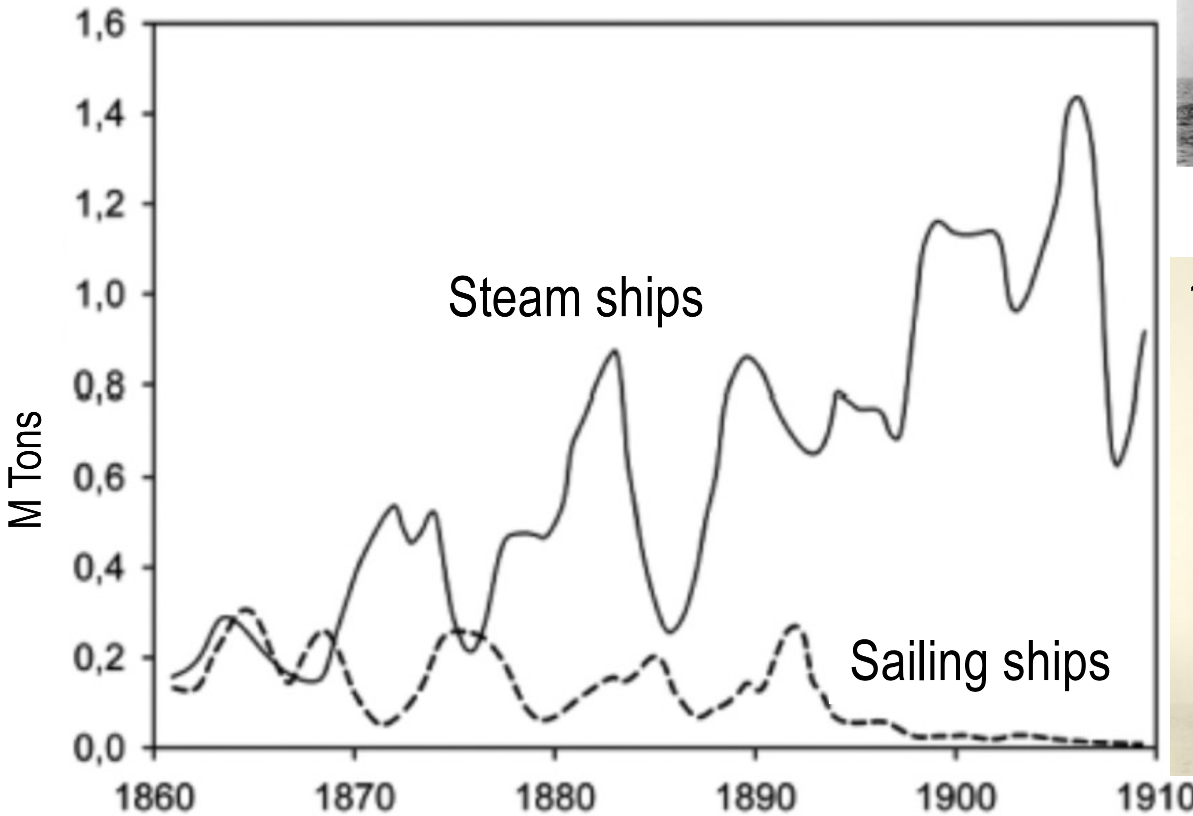
The end of commercial sailing

Commercial long-distance sailing diminished in the late 1800s AC:

1890: 41% of tons were transported by sail.

1914: 7.5% of tons were transported by sail.

1937: 1.5% of tons were transported by sail.



Newy built ships in Great Brittain

1902-1910

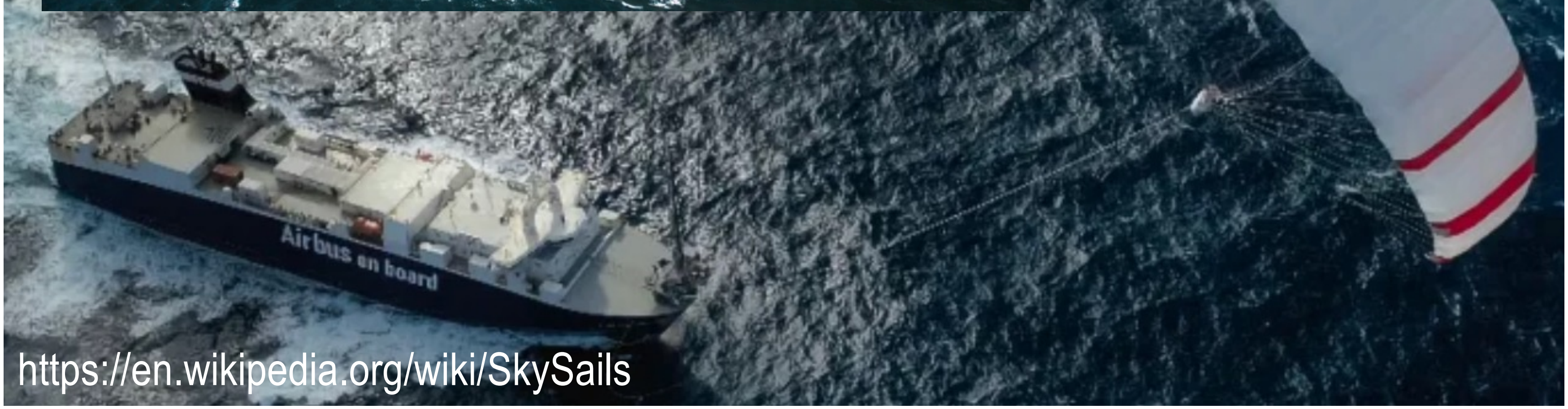
Preußen, only 5-mast full ship.

1902-1907

Thomas W. Larson, largest pure sailing ship, only 7-mast schooner.

Moden commercial sailing ideas

Sporadically new ideas get tested for commercial sailing, e.g. the large glider support up to 1200ft height, providing up to 2000kW or Flettner rotots using the Magnus effect.



Transition to pleasure and sport sailing

For sport and pleasure sailing, smaller boats are used, one or two masts, focusing on speed and comfort, rather than volume.

Gaff rigged Schooner, wooden hull, 1900

The term, yacht, originates from the Dutch word jacht, which means "hunt", and originally referred to light, fast sailing vessels that the Dutch Republic navy used to pursue pirates and other transgressors around and into the shallow waters of the Low Countries.

Transition to pleasure and sport sailing

For sport and pleasure sailing, smaller boats are used, one or two masts, focusing on speed and comfort, rather than volume. Original strategy: low weight, deep keel**, large sail**!

Herreshoff Westward, 1910

Toward Higher Speeds by Aerodynamics

Speed records are now held by multi hulls and foiling boats, to above to 60 knots.

(Commercial displacement boat usually about 10 knots, record was 22knots)

Aerodynamics and Hydrodynamics

America cup 2024 – more than 50 knots

American Magic

https://youtu.be/n0JFEL1rVBs

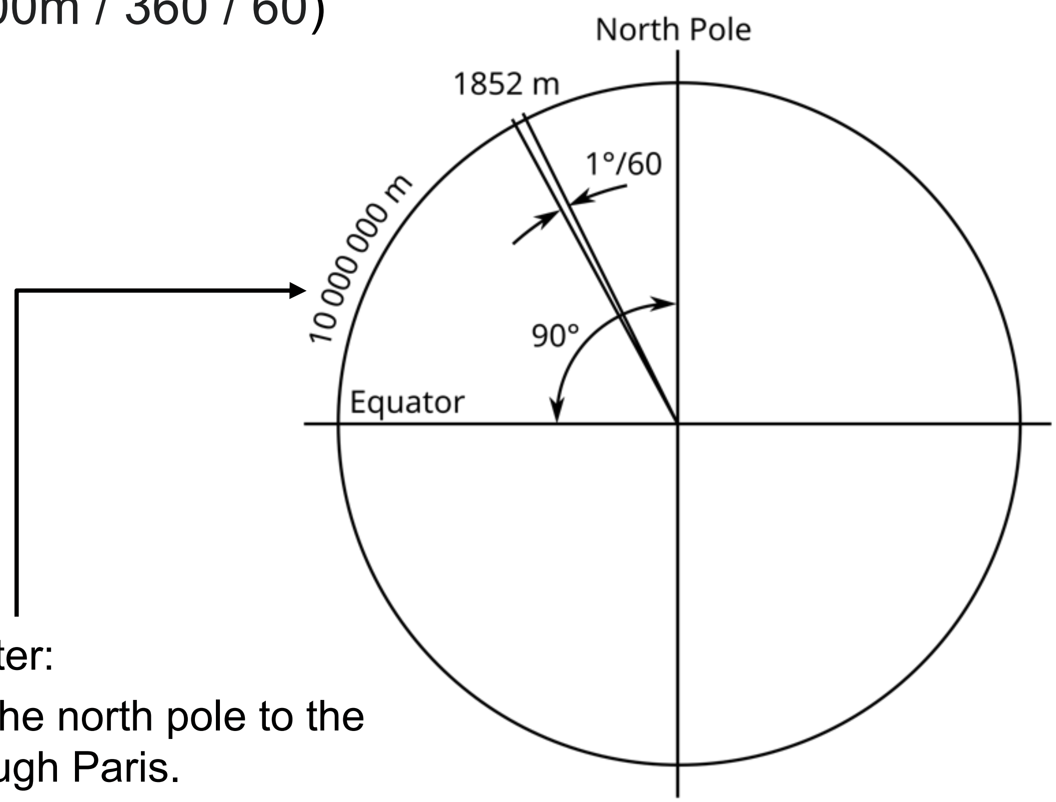
Note: Ice sailing can be even (much) faster, record 124kn ! (already 1938)

Speed on a sailboat – the Knot

Boat speeds are measured in knots.

1 knot ($1kn$) = 1 nautical mile / hour [$1\frac{nmi}{h}$]

1 nmi is the length of one arc-minute on the equator = 1852m = 1.151mi
(1852m = 40,003,200m / 360 / 60)



Old definition of one meter:
10,000,000th part from the north pole to the equator, on a circle through Paris.

Distances Sailed

24h records: For a Tea Clipper a good “Day’s run” was 200 nautical miles.

The record day’s run was 465 nautical miles (1854).

Today’s 24h distance record = 908 nautical miles (trimaran, average 38kn) (https://www.sailspeedrecords.com/24-hour-distance)

Trans Atlantic records

19th century trans Atlantic trait: 3-4 weeks.

Record 12 days (1905, 3-mast Atlantik) held until 1980.

Today: 3day 15 h (2009)

Note: the Steam ship record (the blue ribbon) is 3days 12h (1952)

Around the world records

Tea Clipper: around 130 days (subtracting times for loading)

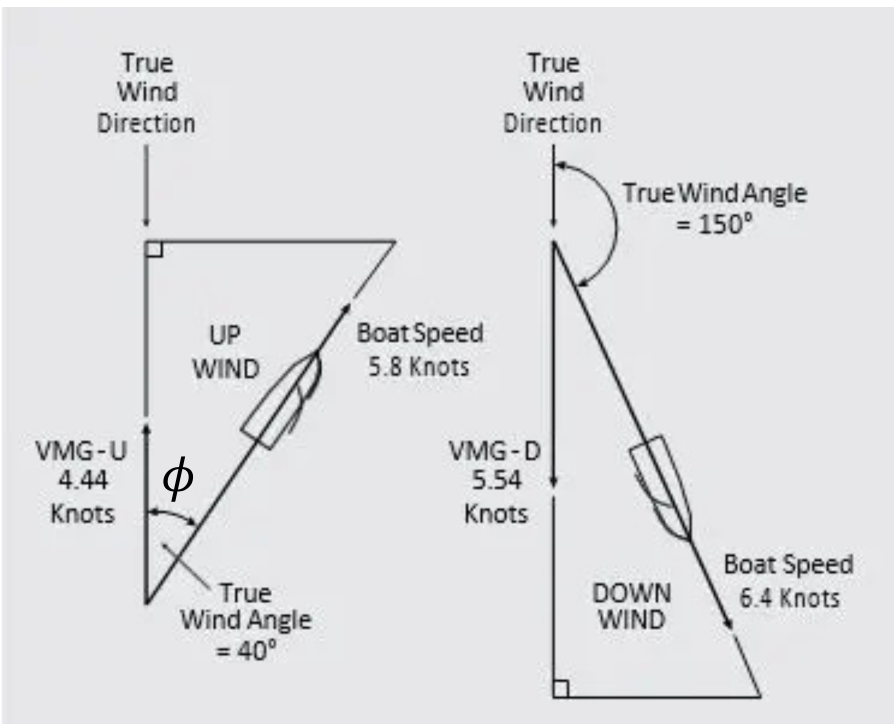
Today: 40days 23h (trimaran, 2017)

Velocity Made Good (VMG)

Global trait ships were optimized for the trait winds (from stern).

For races, the speed against the wind is important.

Velocity Made Good is the component of the velocity against the wind.



$$VMG = velocity \cdot cos\phi$$

For a square-sail rigged clipper, a good upwind angle was 67.5^0.

→ VMG = 38% of boat velocity

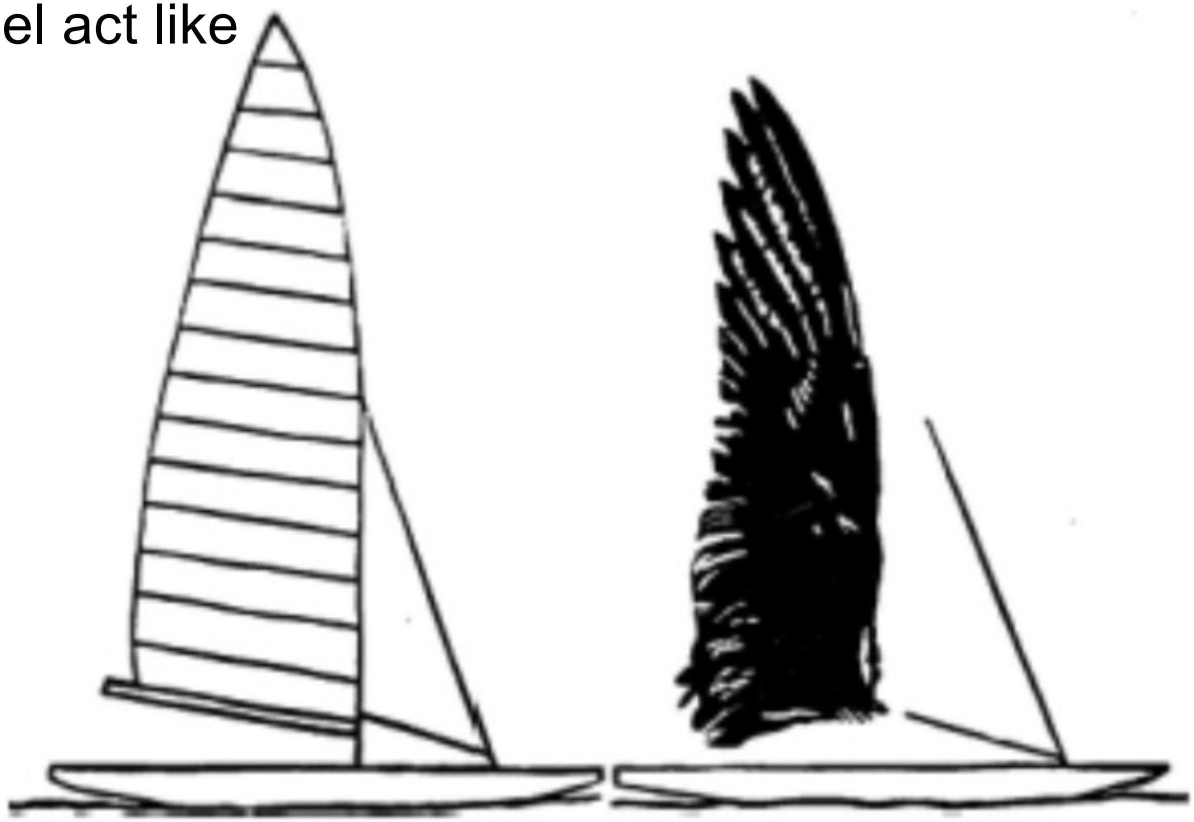
Modern yacht reaches about 40^0.

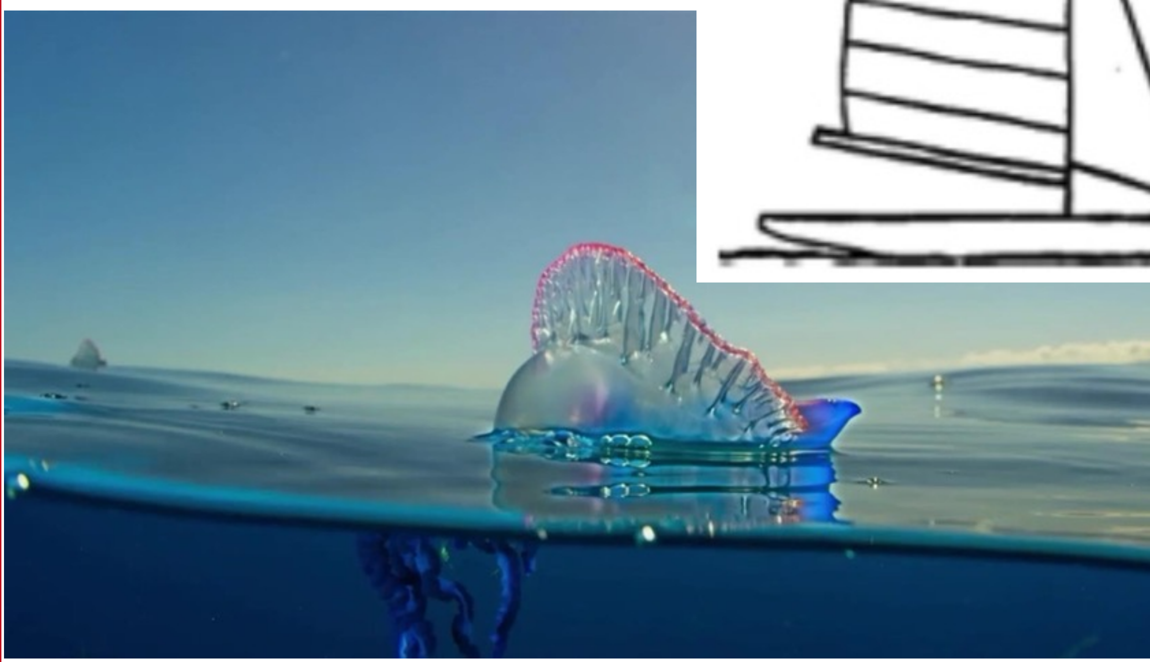
→ VMG = 77% of boat velocity

Today's top performers have VMG of nearly 2 times the wind speed *(against the wind!)*.

The Portuguese man'o war

The sail, and the keel act like bird wings !





A Portuguese man'o war has a sail as body part.

Cornell University

Homework 3

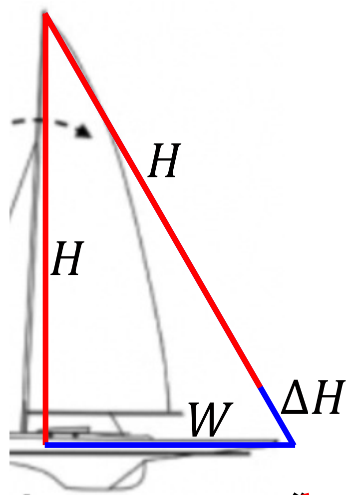
1) Which knot should you use to attach a rope that you might need to release after it has been tightened by large tension?
2) What sail type does this historic boat from Brittany have?
3) What is its boat type?
4) What are advantages of Junk sails? Check web to find advantages beyond those mentioned in class.
5) For what purpose did square-rigged ships have a fore-aft spanker sail?
6) How many nautical miles is the equator long?
7) If you sail with 10kn close howled with 45^0 to the wind, what is you VMG? If you angle is ϕ, what is your VMG?
8) If you sail along the equator for 2700nmi and then make a right angle to the north and sail 5400nmi. What is the shortest distance on the earth's surface to your starting point?

Cornell University

Homework 4

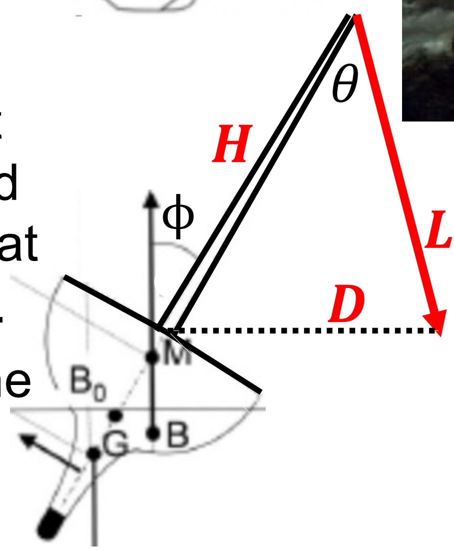
1) What sail types can you identify in this mid 17th century painting from Simon de Vlieger?

2) If H is the mast height, equal to the length of your main halyard and you measure ΔH some distance W behind the mast, how can you calculate the mast height?

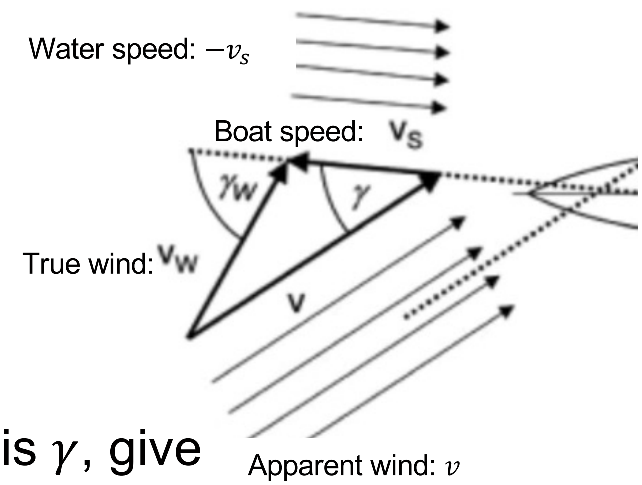


3) You pull over the mast and measure H, L, and D. Derive a formula that calculates the angle θ.

4) Derive a formula for the angle ϕ.



5) If your speed is v_s, the angle to the apparent wind is γ, give a formula for the true wind speed v_w.



Measuring the Metacentric Height times Mass

1) Measure the torque as a function of angle

$$T = F \cdot \sin(\theta) = m \cdot g \cdot \overline{GM} \cdot \sin(\phi)$$

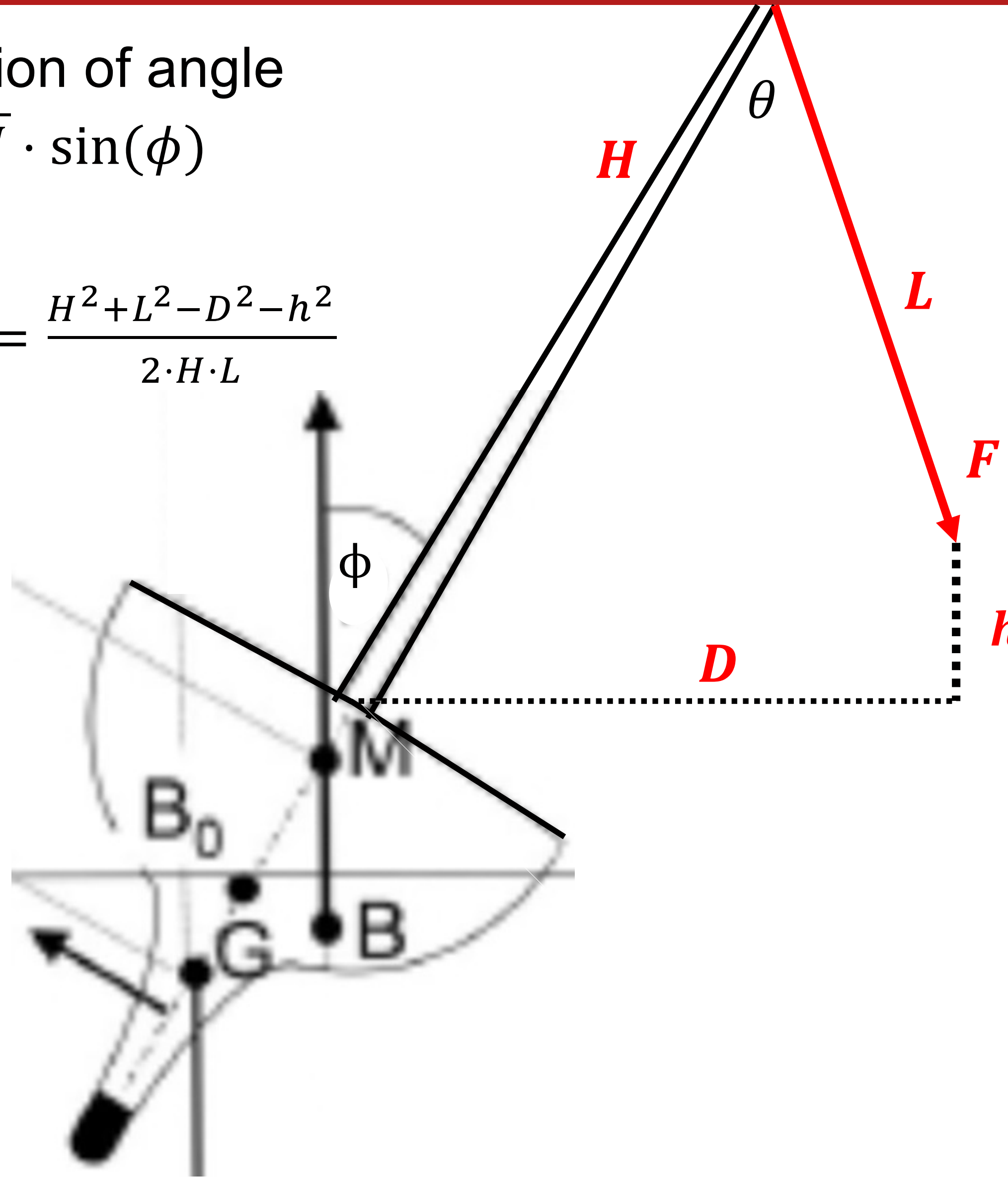
2) Determine the angle θ.

$$\vec{L} - \vec{H} = D\vec{e}_x + h\vec{e}_y \Rightarrow \cos(\theta) = \frac{H^2+L^2-D^2-h^2}{2 \cdot H \cdot L}$$

3) Determine the angle ϕ.

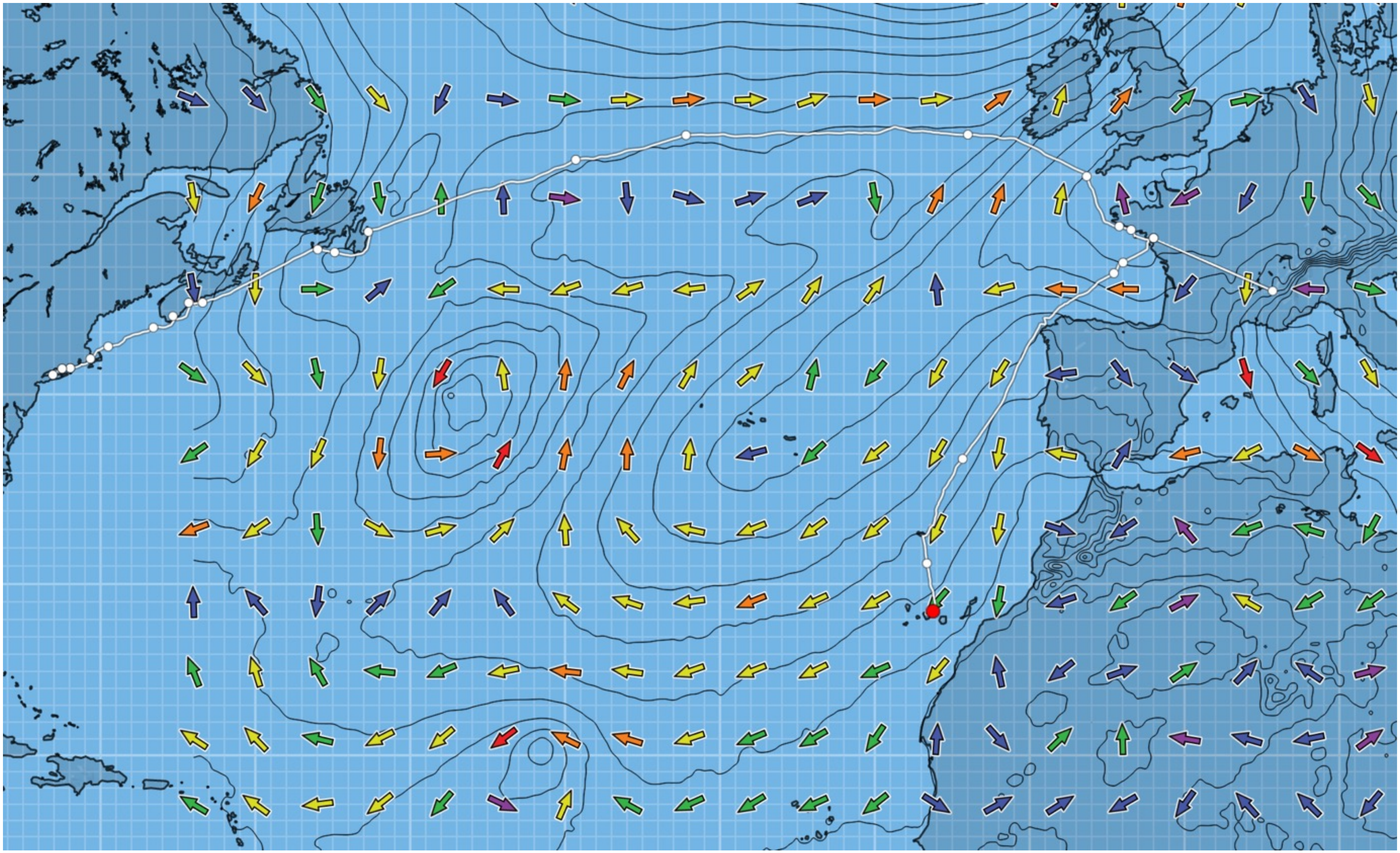
$$\Rightarrow \sin(\phi) = \frac{D\,H - D\,L\,\cos(\theta) - h\,L\,\sin(\theta)}{D^2+h^2}$$

$\Rightarrow m \cdot \overline{GM}$ and ϕ.



Measure the force for many different L and compute $m \cdot \overline{GM}$ and ϕ each time. Plot $m \cdot \overline{GM}$ against ϕ.

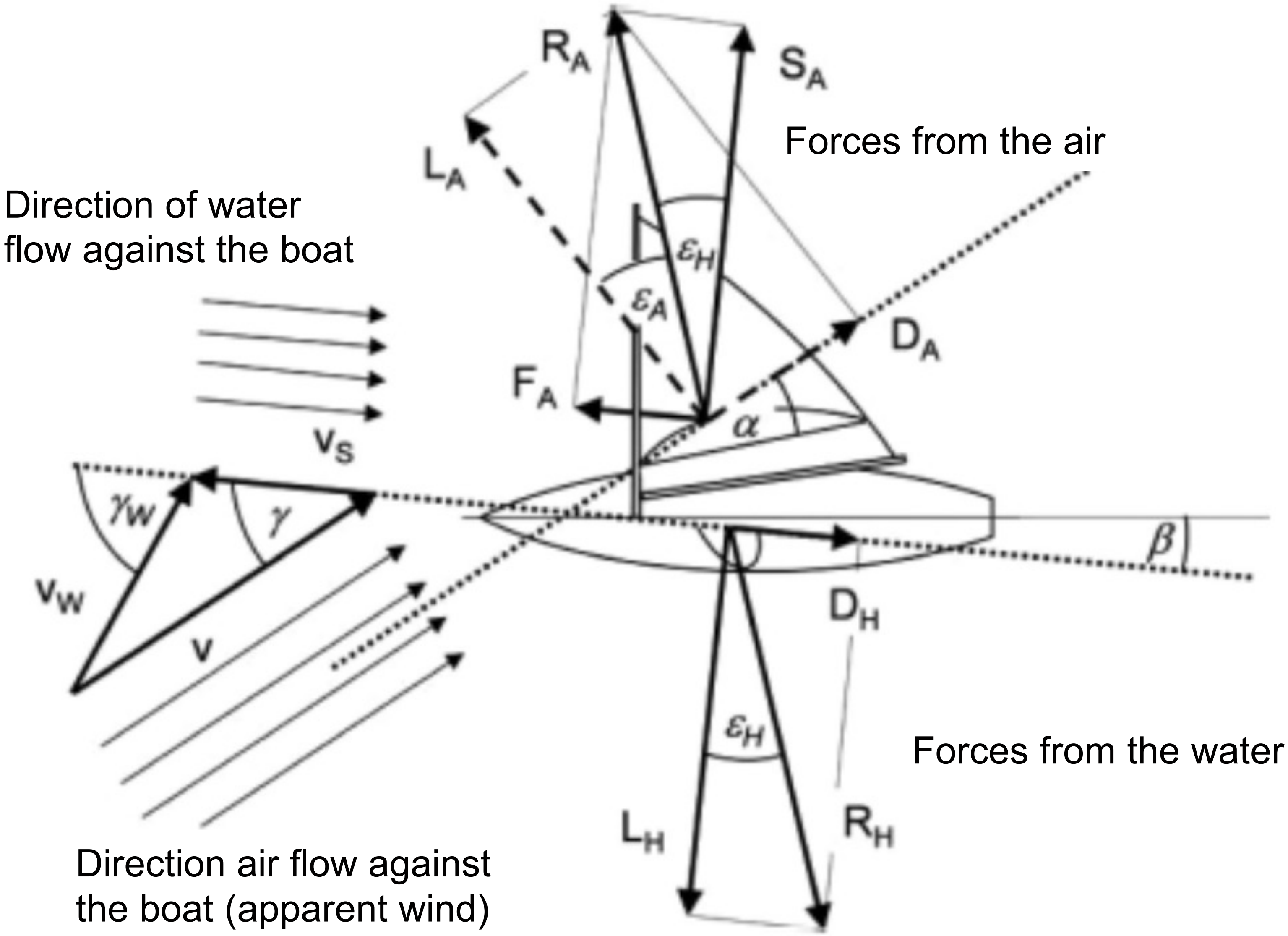
Travels of Physicist Dr. Francois Meot



https://forecast.predictwind.com/tracking/display/Dolcenea/

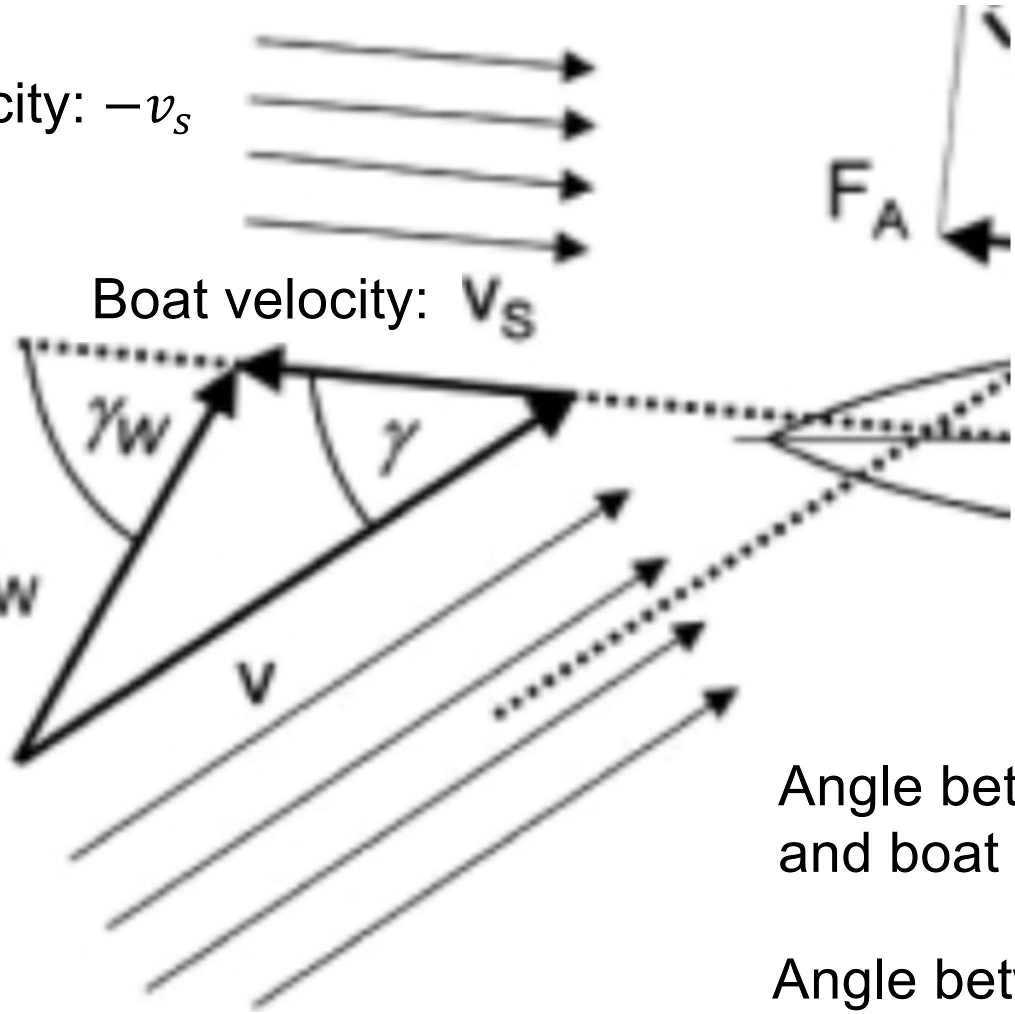
Equilibrium of Forces in Uniform Motion

Newton's 3rd law: All forces are balanced; each force has a counter force.



a) Force diagram: Velocities relative to the boat

Water velocity: $-v_s$

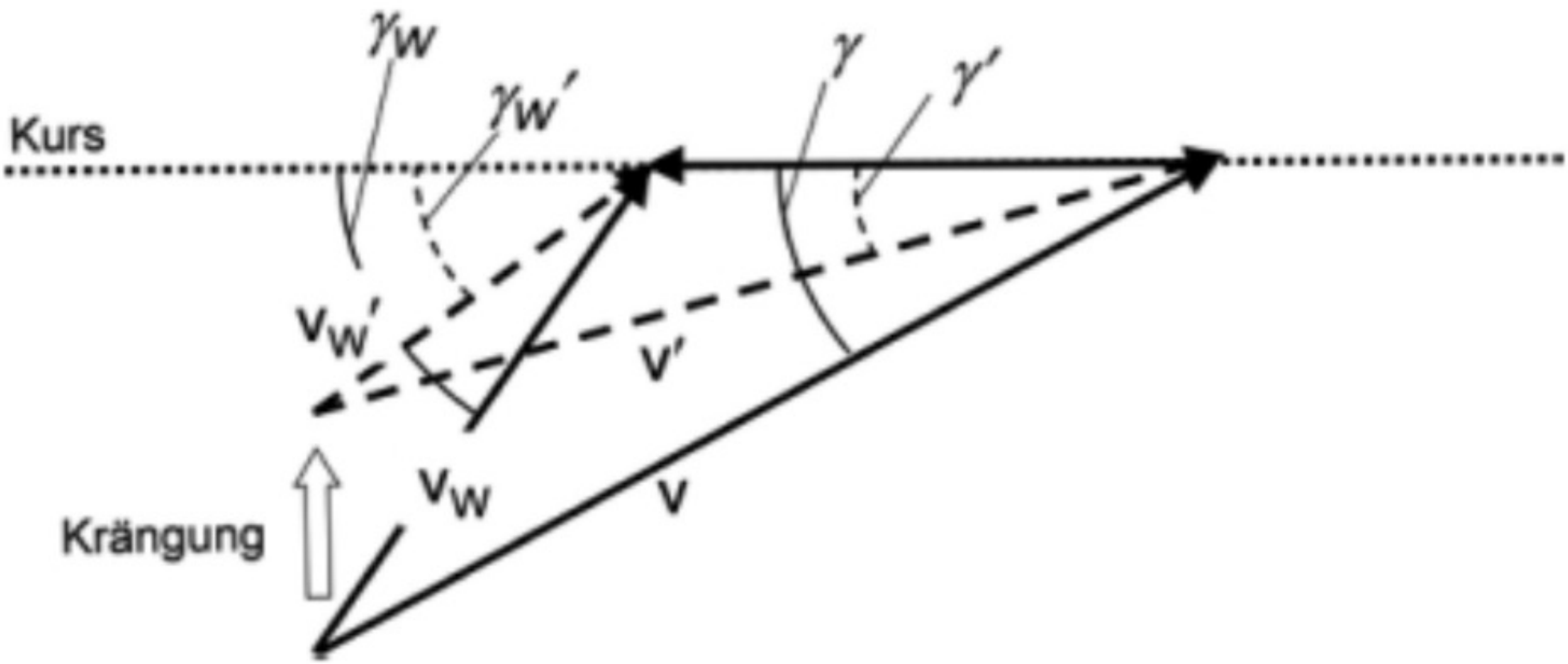


Apparent wind: v

Angle between true wind and boat velocity: γ_w

Angle between apparent wind and boat velocity: γ

One Disadvantage of Heeling: More Upwind

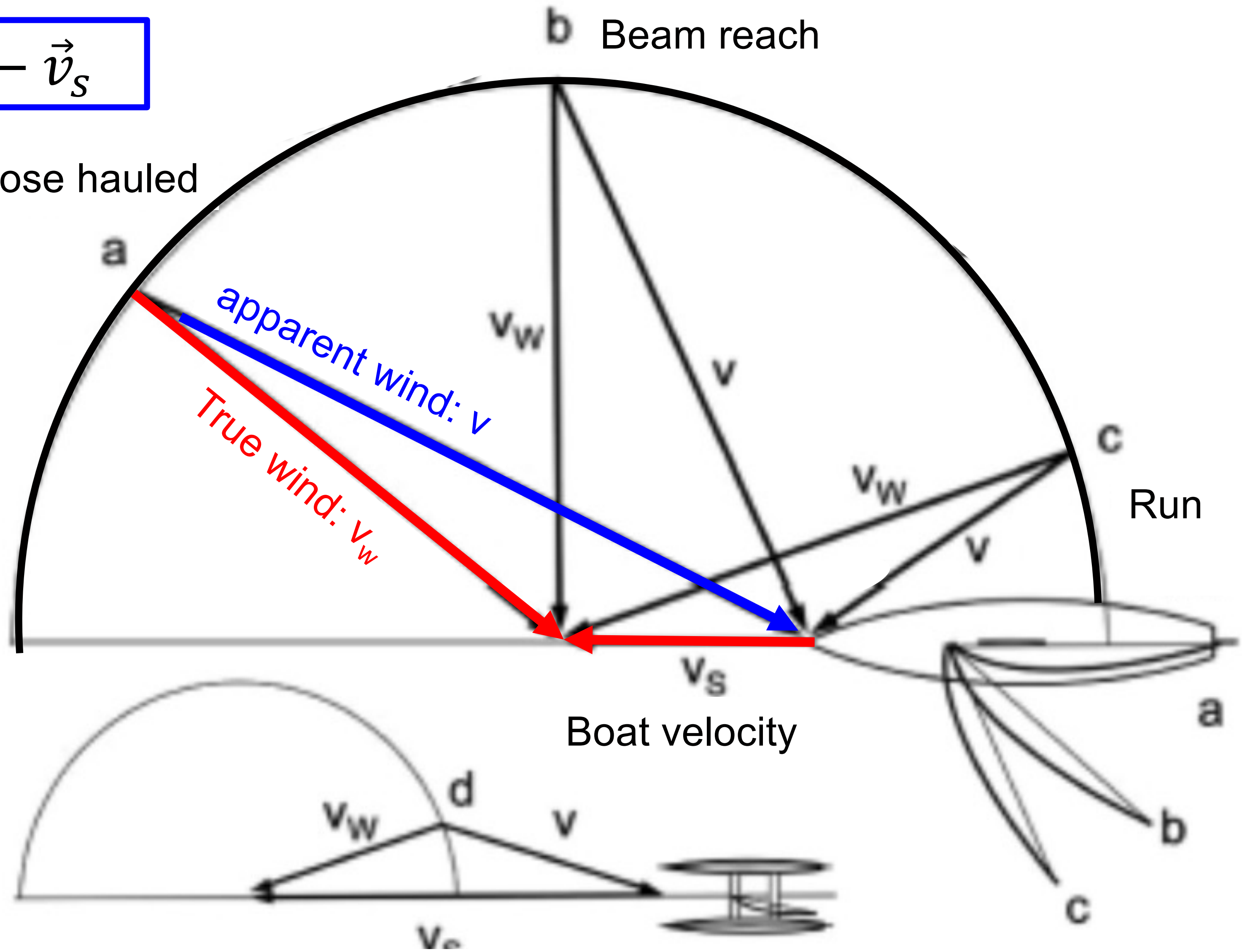


The wind and apparent wind projected onto the heeling surface of the boat have a smaller angle to the boat direction, as if the boat was closer hauled.

➔ Disadvantage: A heeling boat can go less close to the wind.

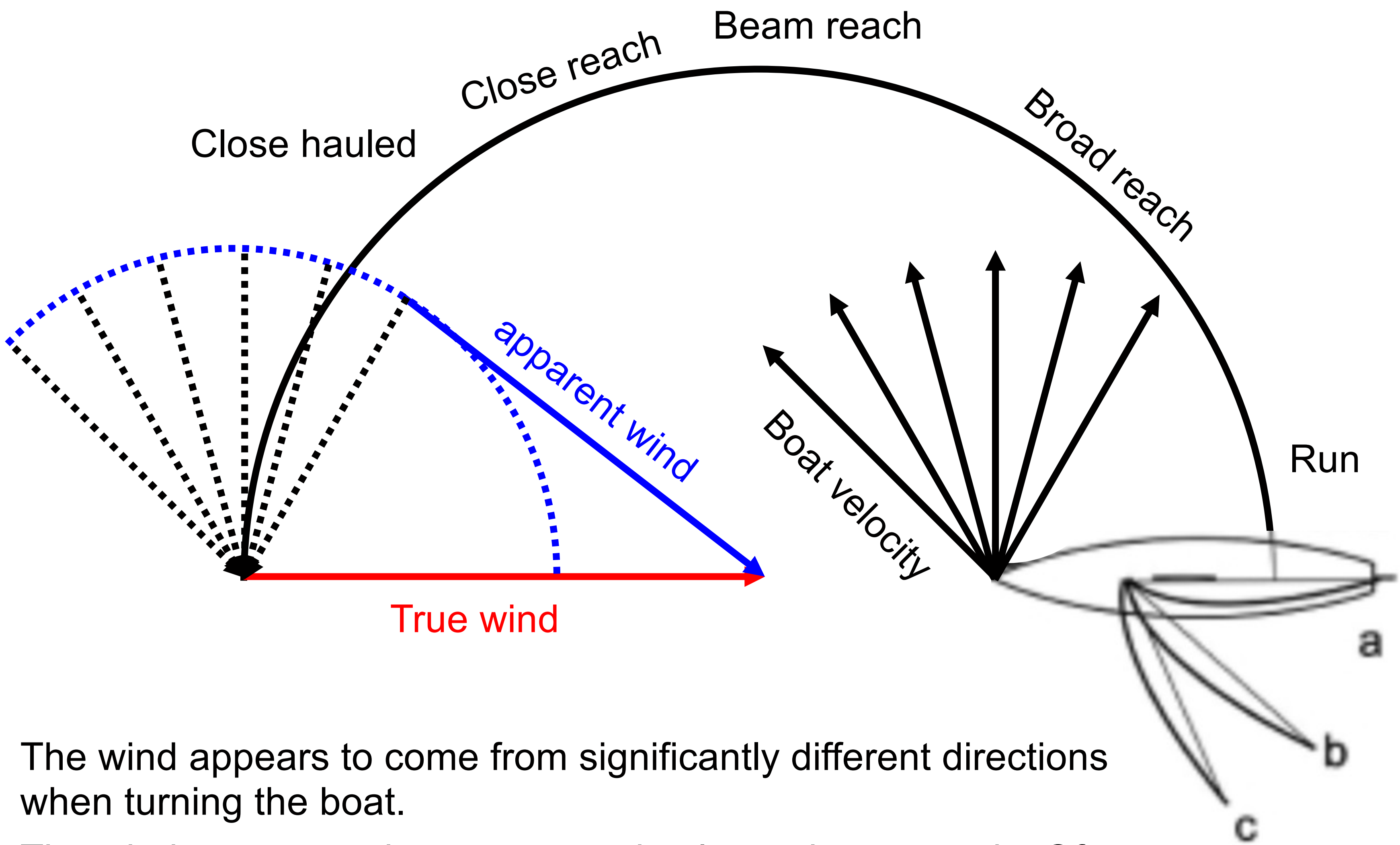
Changing apparent wind for different directions

$$\vec{v} = \vec{v}_w - \vec{v}_s$$



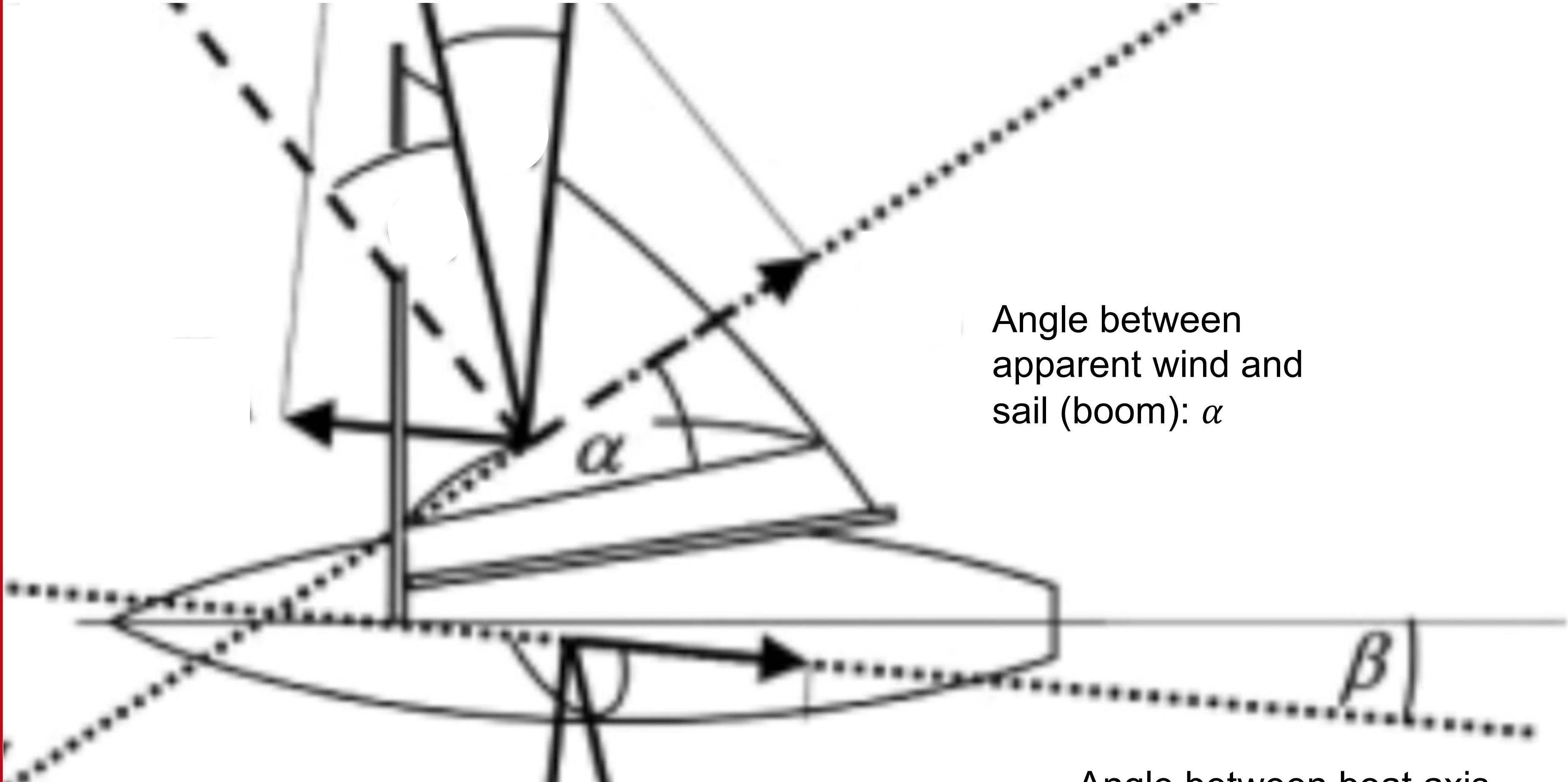
Especially in ice sailing, the wind can appear from the front, even it truly is from behind.

Changing apparent wind for different directions



1) The wind appears to come from significantly different directions when turning the boat.
2) The wind appears to increase, causing larger boat speeds. Often the increased boat speed only increased the apparent wind,

b) Force diagram: Orientation of the boat

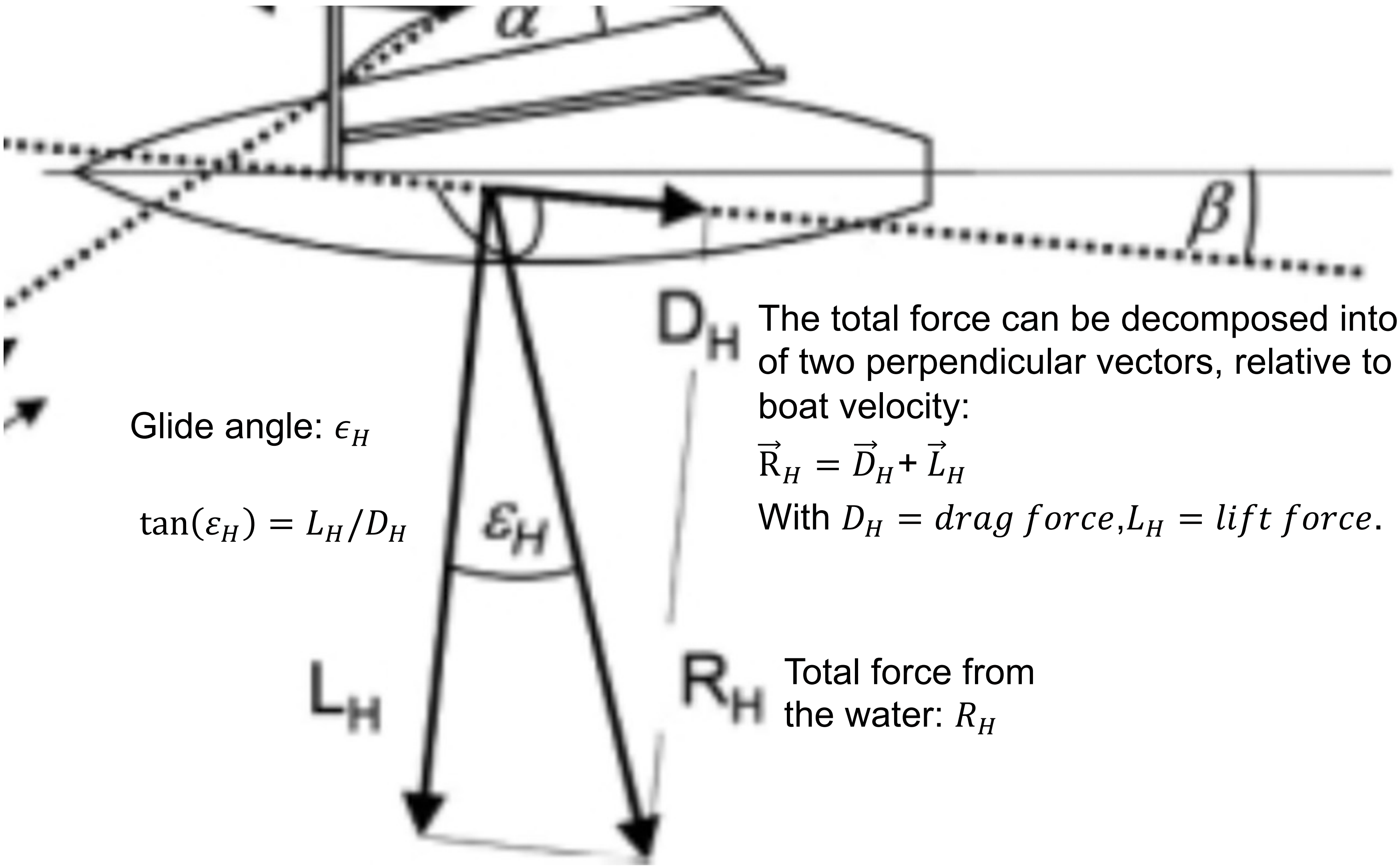


Angle between apparent wind and sail (boom): α

Angle between boat axis and boat velocity: β

These are the angles when looking perpendicular down on the deck, which is not necessarily parallel to the water surface, because the boat heels by an angle δ.

c) Force diagram: Forces from the water



Glide angle: ϵ_H

$\tan(\varepsilon_H) = L_H/D_H$

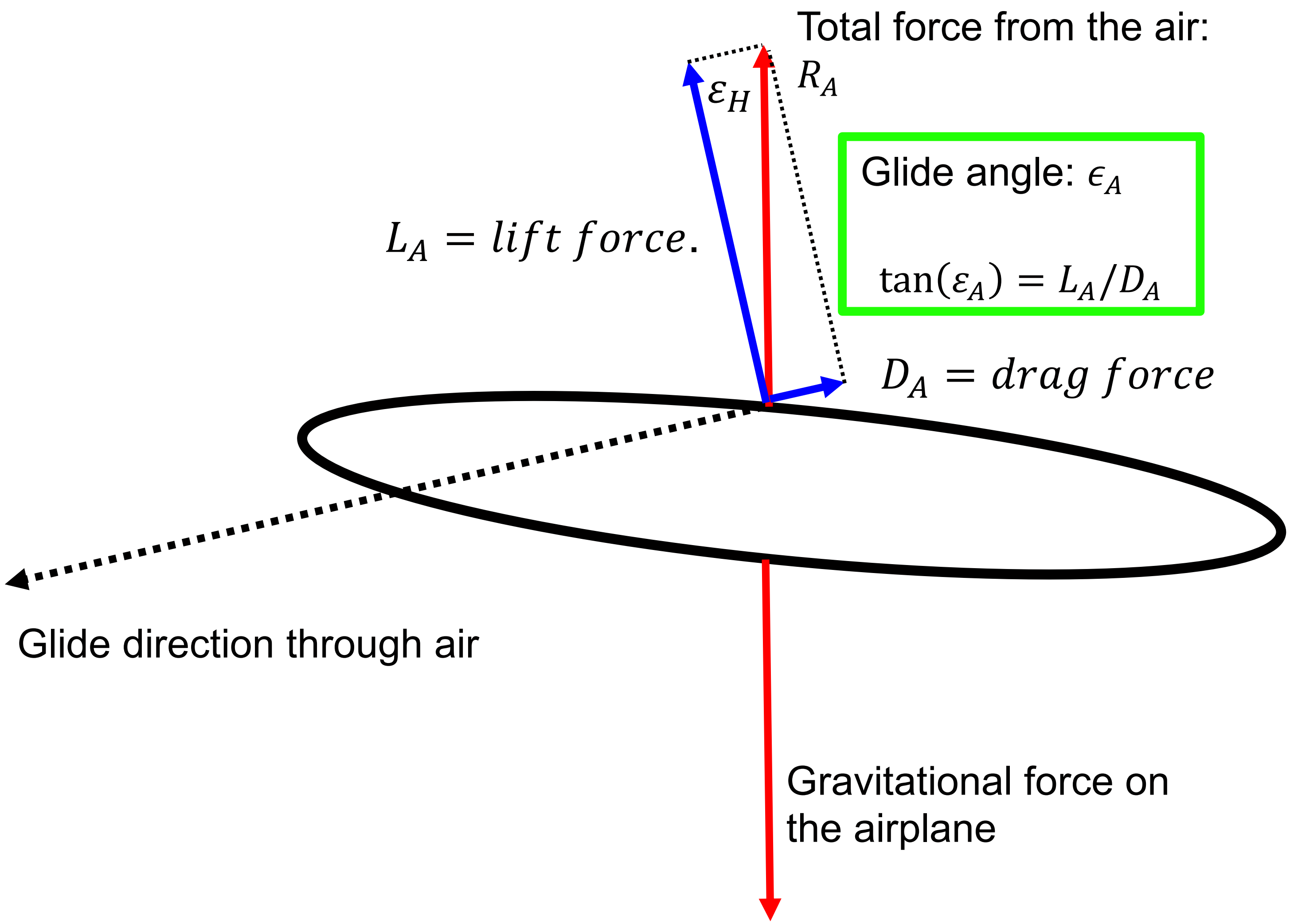
The total force can be decomposed into sums of two perpendicular vectors, relative to the boat velocity:

$\vec{R}_H = \vec{D}_H + \vec{L}_H$

With $D_H = drag\ force$, $L_H = lift\ force$.

Total force from the water: R_H

Glide angle for airplanes

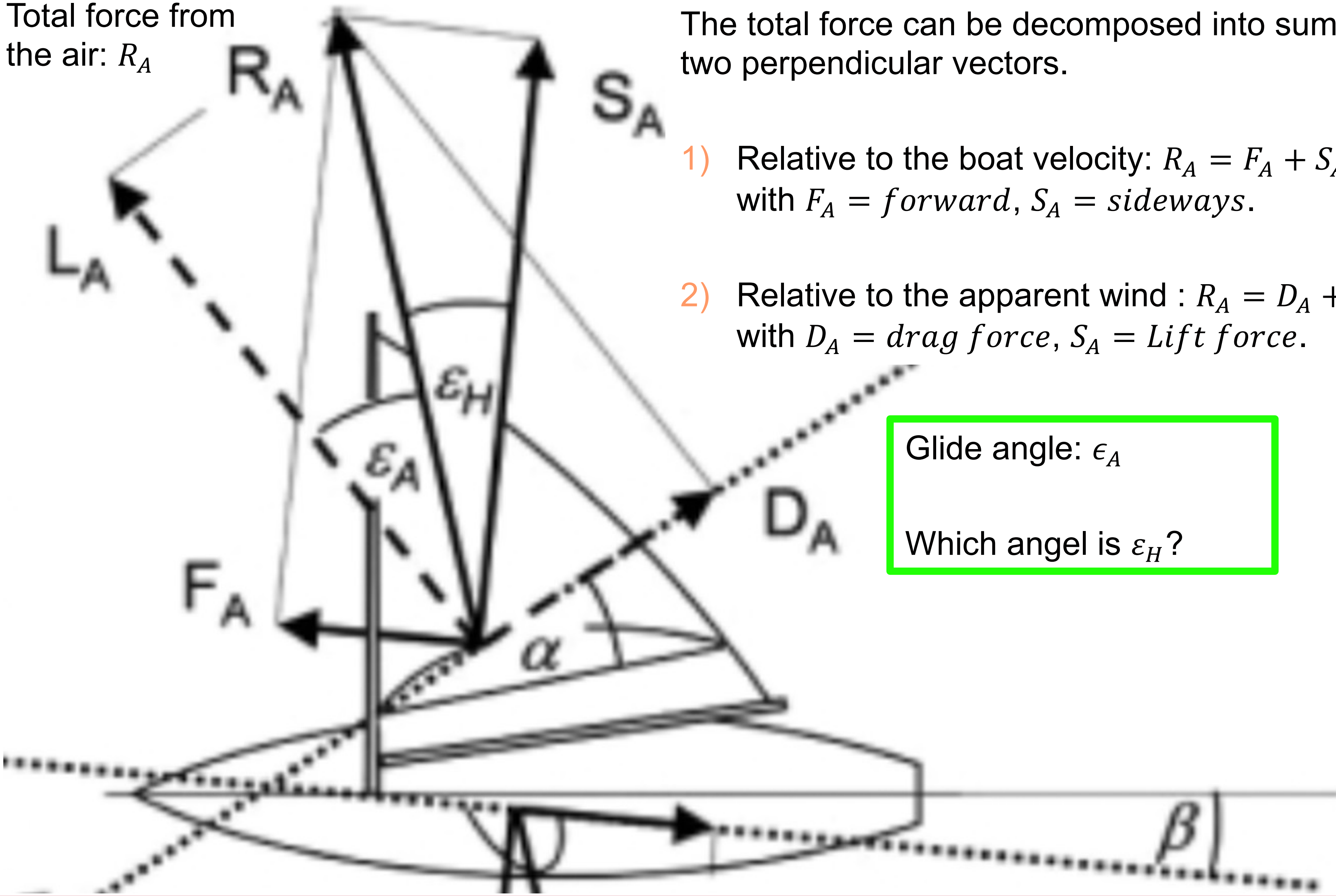


Glide angle: ϵ_A

$\tan(\varepsilon_A) = L_A/D_A$

d) Force diagram: Forces from the air

Total force from the air: R_A



The total force can be decomposed into sums of two perpendicular vectors.

1) Relative to the boat velocity: $R_A = F_A + S_A$ with $F_A = forward, S_A = sideways.$

2) Relative to the apparent wind : $R_A = D_A + L_A$ with $D_A = drag\ force, S_A = Lift\ force.$

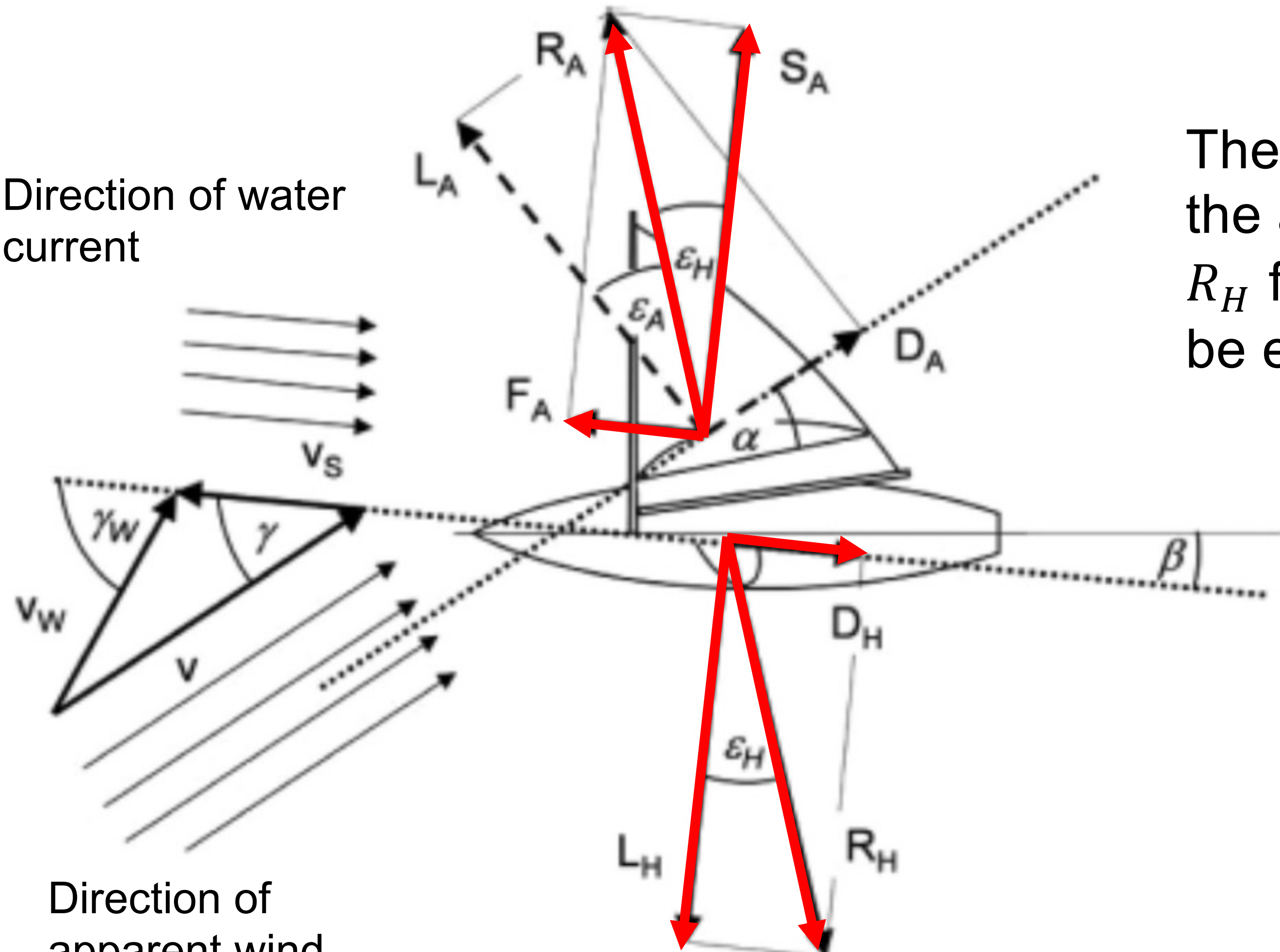
Glide angle: ϵ_A

Which angel is ε_H?

Balance of air and water forces

Newton’s 3rd law: All forces are balanced; each force has a counter force.

Direction of water current



Direction of apparent wind

The total force R_A from the air and the total force R_H from the water must be exactly opposite:

$$R_A + R_H = 0$$

➔

$$F_A = -D_H$$
$$S_A = -L_H$$

Glide angle in ice sailing and gliding planes

Starting to gain speed:

After acceleration in beam reach:

Ice sailing:

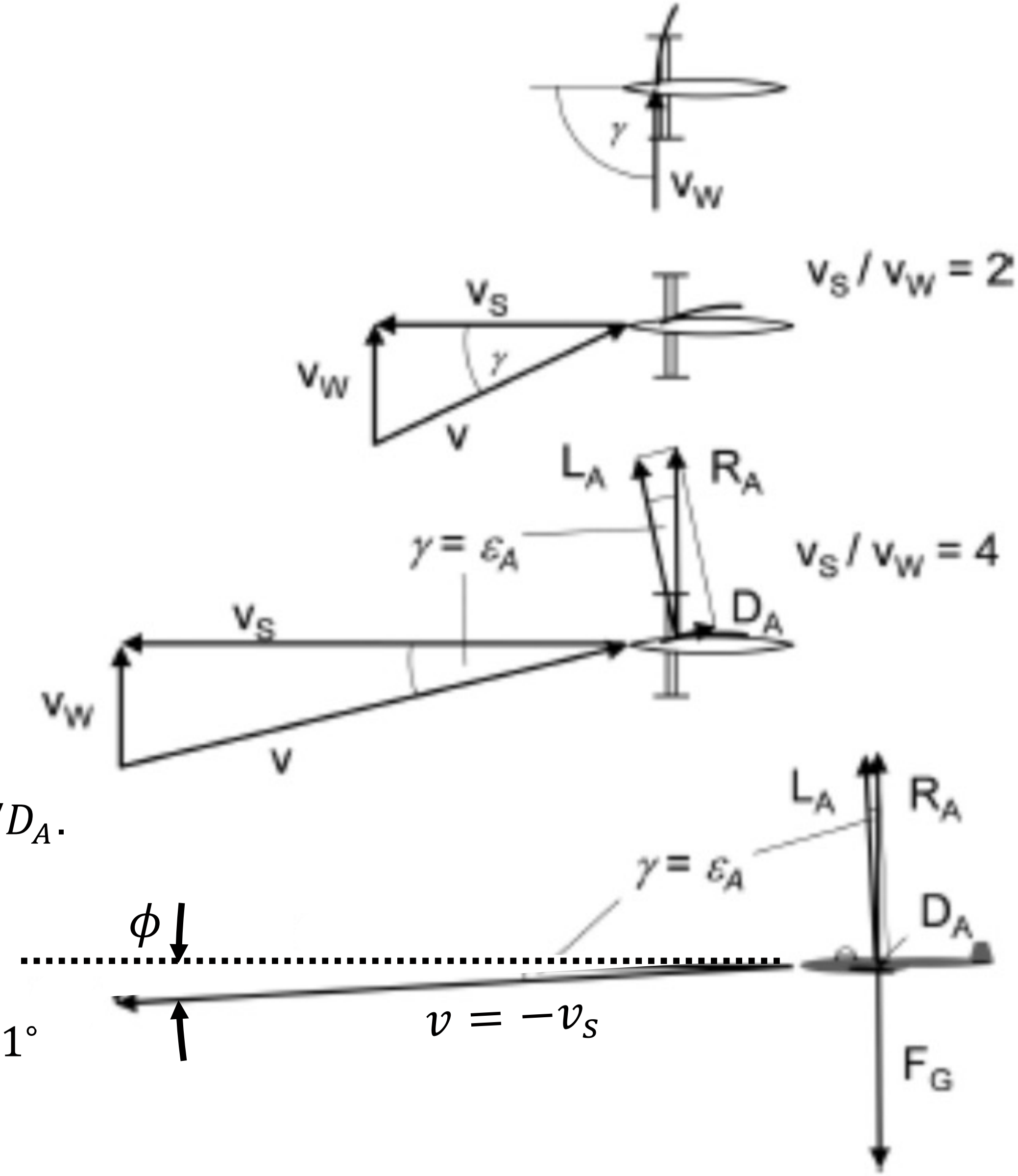
Ice friction $D_H = 0$.

➔ Total hull force $R_H = L_H \perp$ to skis

➔ Apparent wind direction from L_A / D_A.

Airplane gliding:

In optimized gliders $\frac{L_A}{D_A} \approx 50 \rightarrow \phi \approx 1^\circ$



Sailing strategy

1. Chose a course ➔ γ_w
2. Choose an angle of the sail to the apparent wind ➔ α
3. It will take a while until equilibrium is reached.
4. At equilibrium ➔ $L_A \& D_A$ of the sail ➔ $S_A \& F_A$ ➔ $v \& \beta$

After equilibrium has been established, the velocity in the chosen direction is only a function of the sail angle α. One usually wants to fined the α that makes v the largest.

Several challenges remain:

a) Choose the best course to reach the final goal.
b) Establish the equilibrium for maximum v as quickly as possible.
c) Change other available sail properties that increase the maximum v.
    a) Leech tension (via Boom Vang)
    b) Luff tension (via Cunningham)
    c) Mast bending (via backstay tension)
    d) Changing sails

Different Sails, Different Riggings

Square rigged Lateen rigged Sprit rigged Gaff rigged Lug rigged Bermuda rigged

Different Types of Boats: The Hull

Different Types of Yachts

Cat: one foreword mast, no jib

Sloop: one center mast, one jib

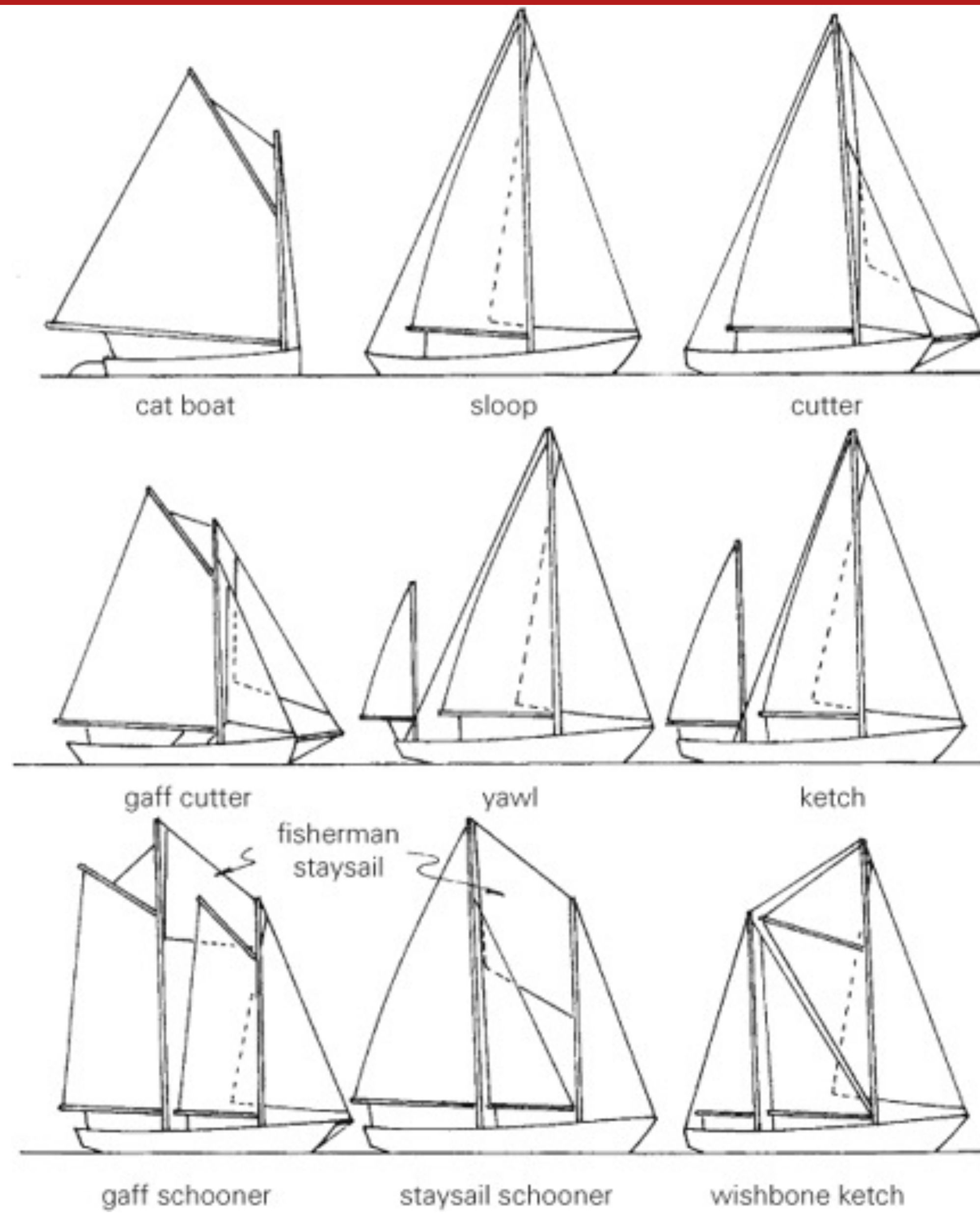
Cutter: one center mast, two jibs

Two masts:

Yawl: Smaller, aftmost mast behind rudder post

Ketch: Smaller, aftmost mast before rudder post

Schooner: Largest mast aftmost



Different Types of Tall Ships

Ship = Full-rigged ship, all square sails + a fore-aft spanker

Brig = 2-mast full-rigged ship

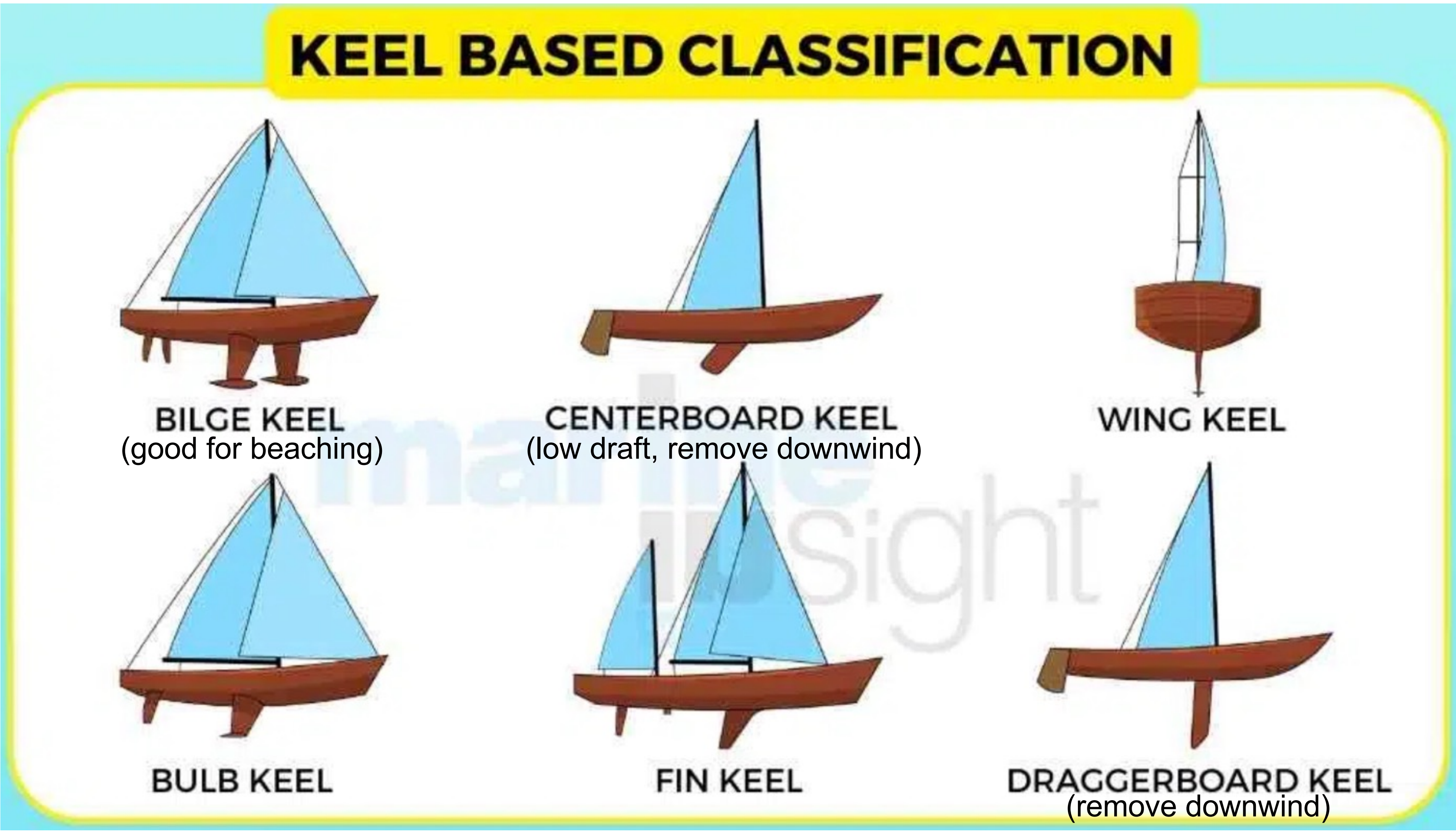
Barque: only aftmost mast (mizzen) is fore-aft rigged

… antine: only foremast is square rigged

Schooner: all fore-aft rigged, no mast higher than the second

Freedom: uncommon, without standing rigging

Types of Sailboat – The Keel



Homework 5

1) What hull type is shown?
2) What boat type is shown, what type of rigging does it have?
3) What boat type is shown?
4) What kind of keel do these boats have?

(1)

(2)

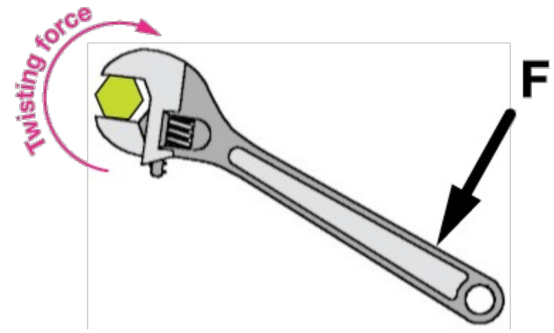
(4)

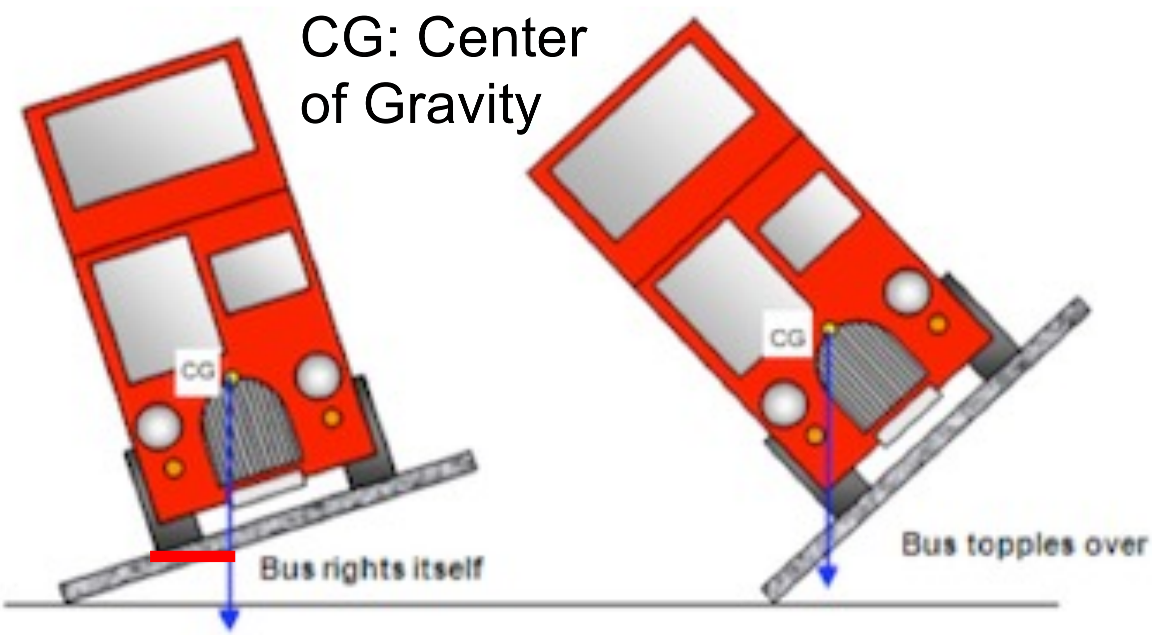
(3)

5) How large is the forward force on the sail of an ice sailor at constant velocity.
6) If the drag force on a hull is $D_H = 10 Newton$, and the lift in the water is $L_H = 100 Newton$. What is the total force on the hull? How large is the forward force F_A of the sail? If the apparent wind angle is $\gamma = 45°$, what is the lift L_A of the sail?

Equilibrium of Torques in Uniform Motion

Righting Torques: $Torque = \boldsymbol{Arm} \cdot \boldsymbol{Force}$





Righting torque

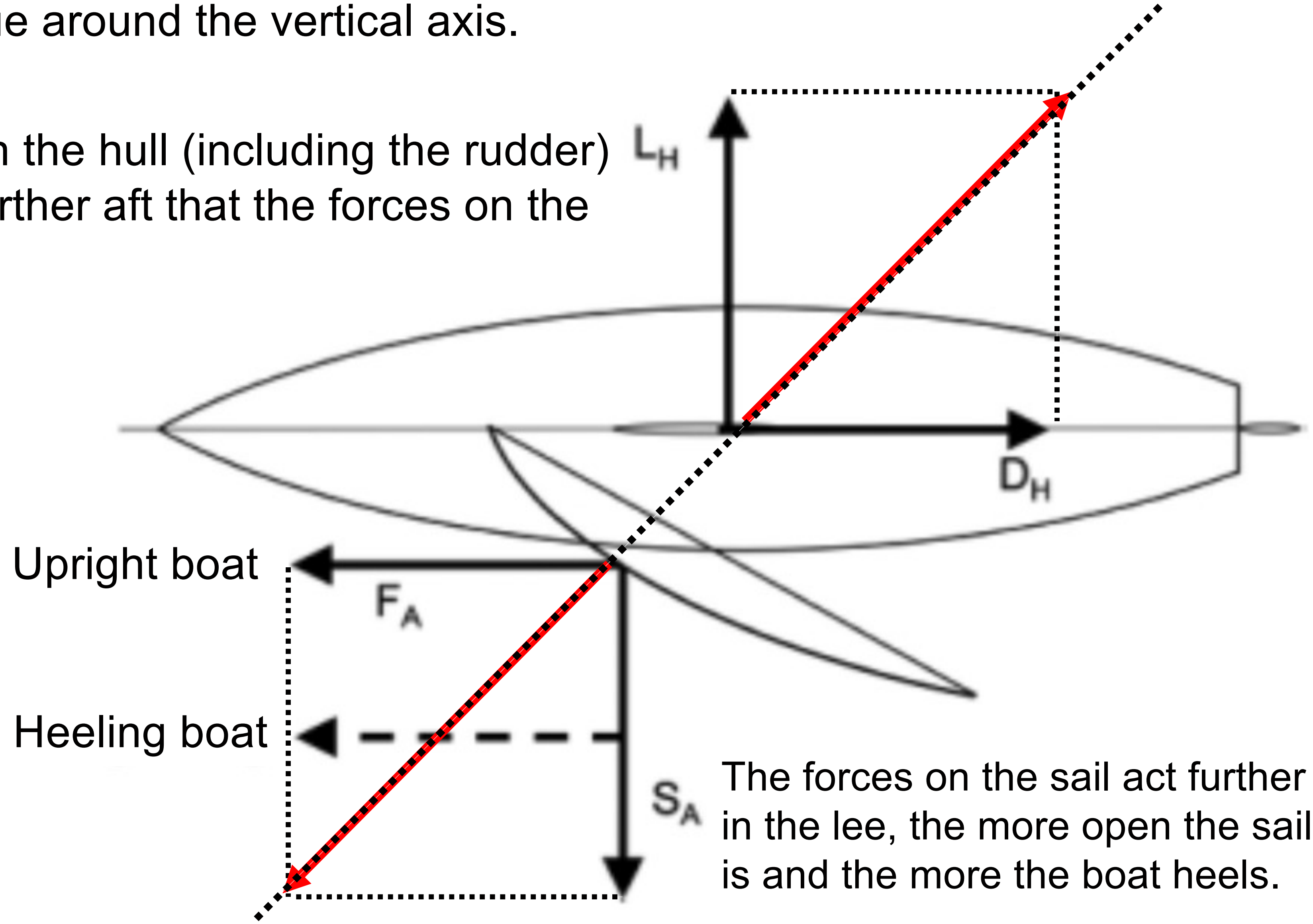
No righting torque

Celtic torque

Equilibrium of Torques in Uniform Motion

1) Torque around the vertical axis.

Forces on the hull (including the rudder) are act further aft that the forces on the sail.

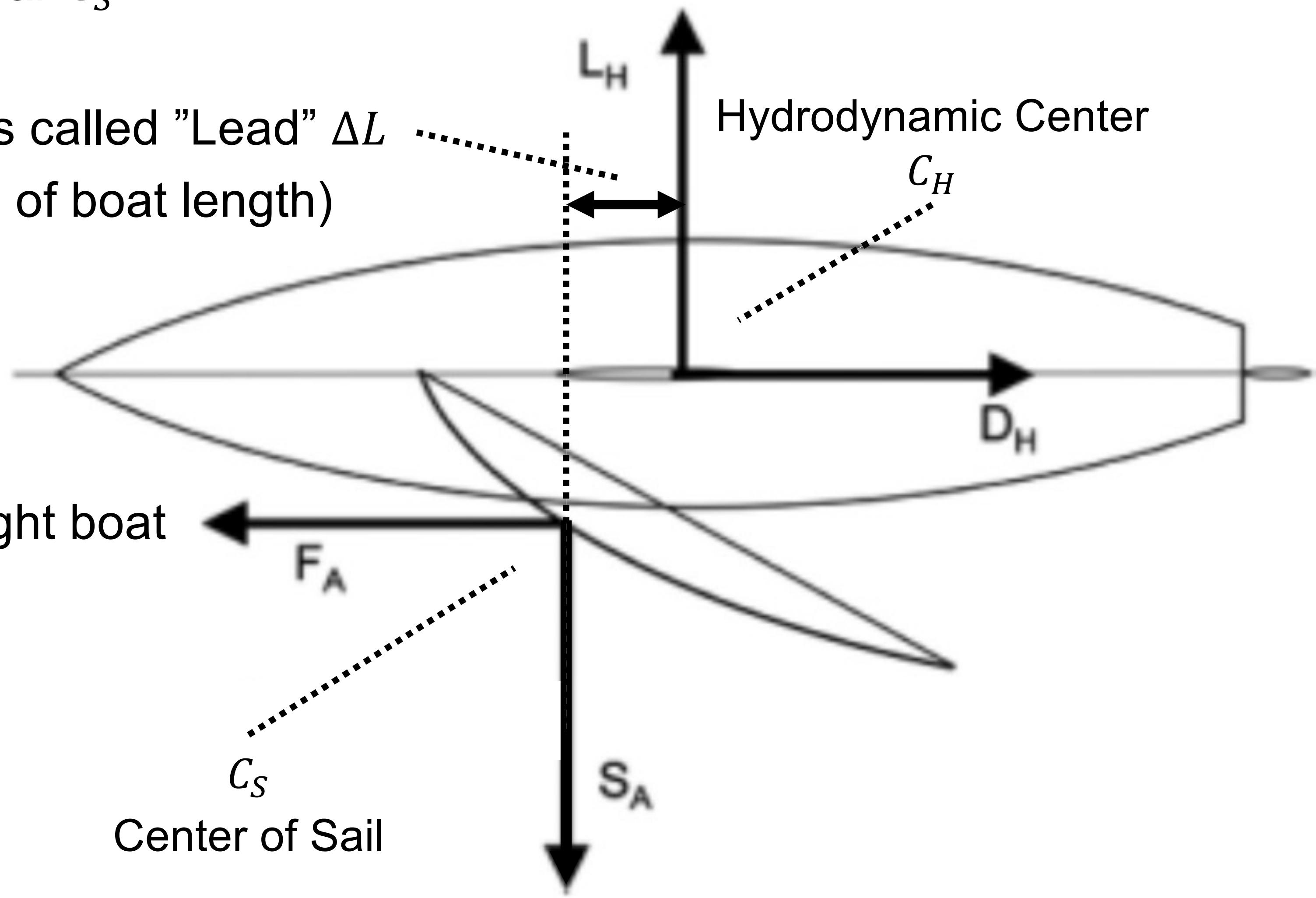


The forces on the sail act further in the lee, the more open the sail is and the more the boat heels.

Boat design considerations

Put the Hydrodynamic Center C_H behind the Center of Sail C_S.

This distance is called ”Lead” ΔL (usually 5-10% of boat length)



One disadvantage of Heeling: Weather Helm

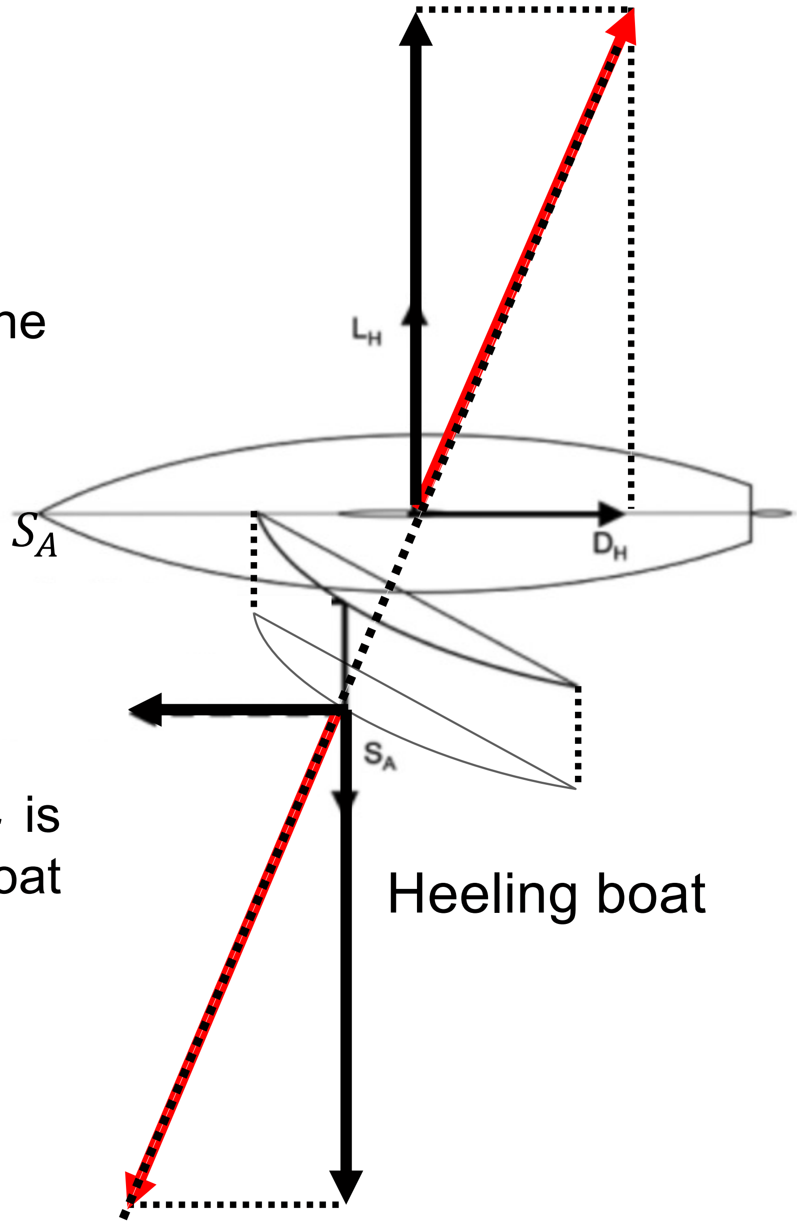
1) Torque around the vertical axis.

The forces on the sail act further in the lee, the more the boat heels.

However, increased lateral forces L_H and S_A can partially balance out the torque.

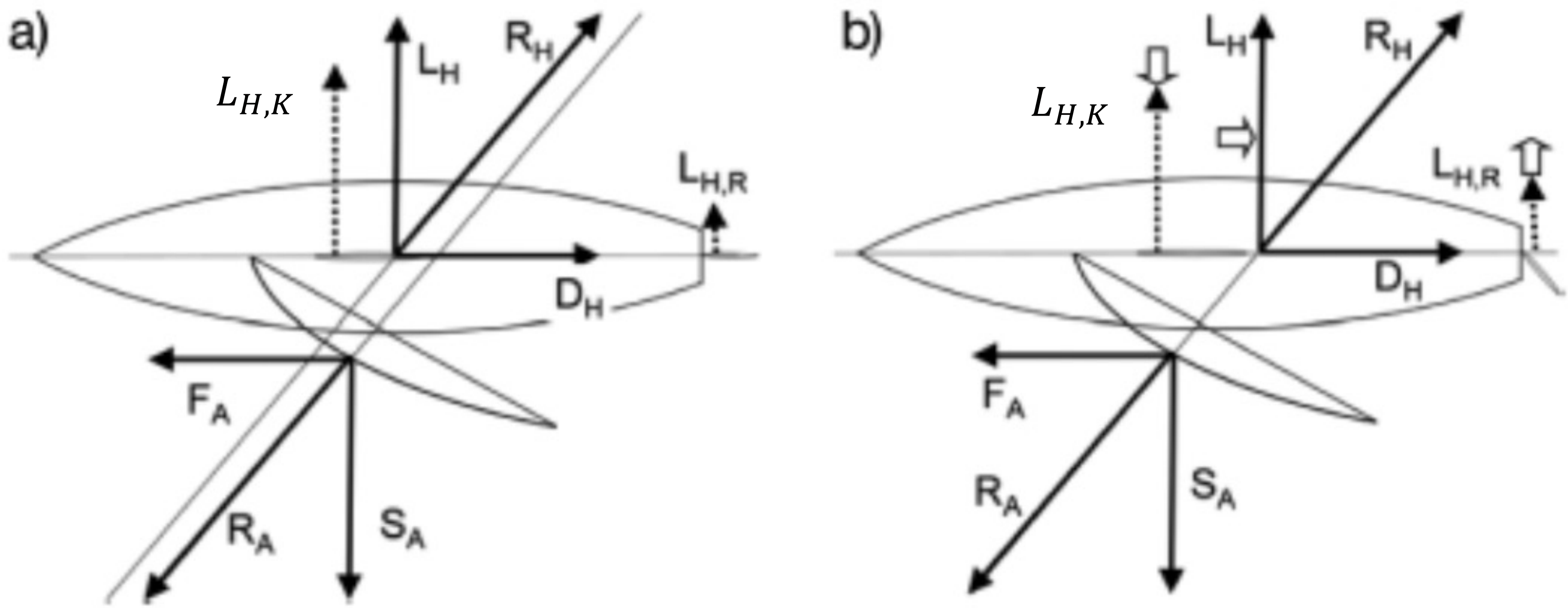
If the outward shift of The Center of Sail C_S is more than required for compensation, the boat pulls windward, i.e., it has Weather Helm.

If it heels less, the boat has Lee Helm.



Torque Correction with the Rudder

1) Torque around the vertical axis.



a) The keel and the rudder produce lift: $L_H = L_{H,K} + L_{H,R}$

b) Weather helm tries to turns the boat windward

c) Increased lift on the rudder, and decreased lift on the keel shifts C_H backward. ➔ torques are balanced again!

The Broach: when steering is not enough

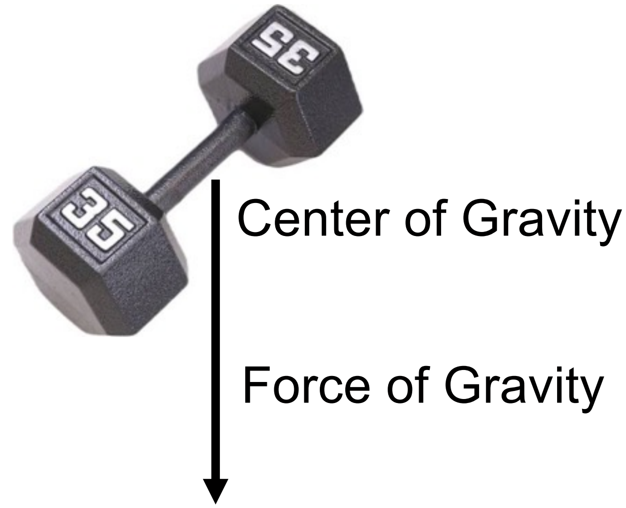
When the torque becomes so large that one cannot correct by rudder:

(a) the torque from the heeling sail is too large,

(b) the heeled rudder produces too little lift.

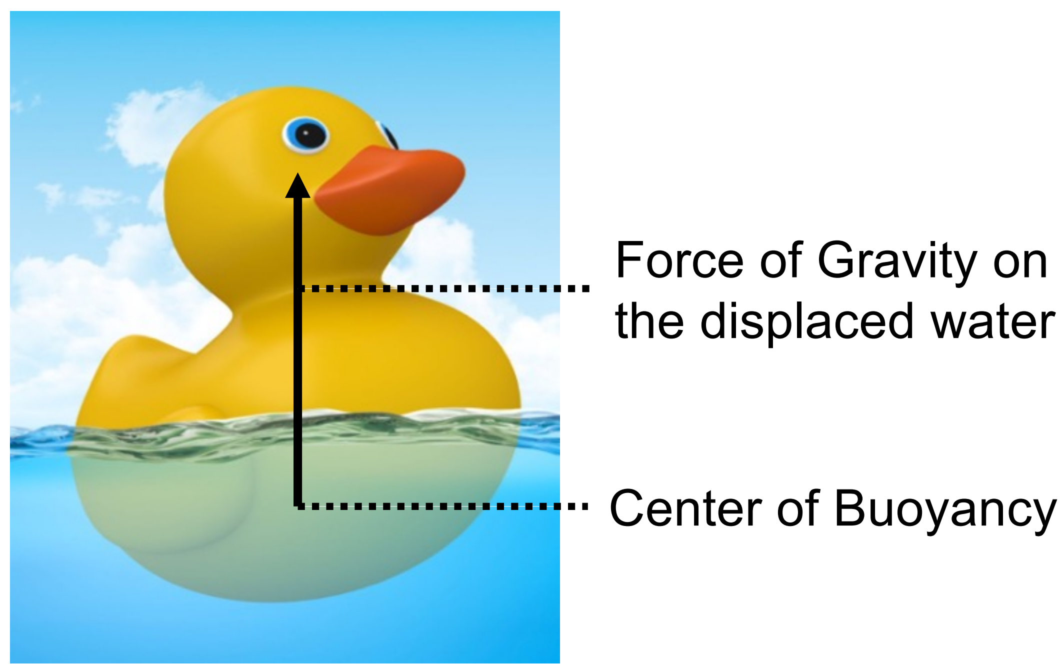
Observation on Flotation

1) The Center of Gravity wants to go down



2) Center of Buoyancy wants to go up.

The Center of Buoyancy is the Center of Gravity of the displaced water.



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Homework 6

1) If an ice sailor's drag on the ice were exactly zero, could it accelerate indefinitely? Explain why not.

2) A boat of 100kg rests in port. How large is its buoyancy force?

3) A boat rests upright port. Explain why the center of gravity and the center of buoyancy lie on a vertical line. Is the center of buoyancy above or below the center of gravity?

4) A boat heels while sailing uniformly under strong winds. Do center of gravity and center of buoyancy still lie on a vertical line?

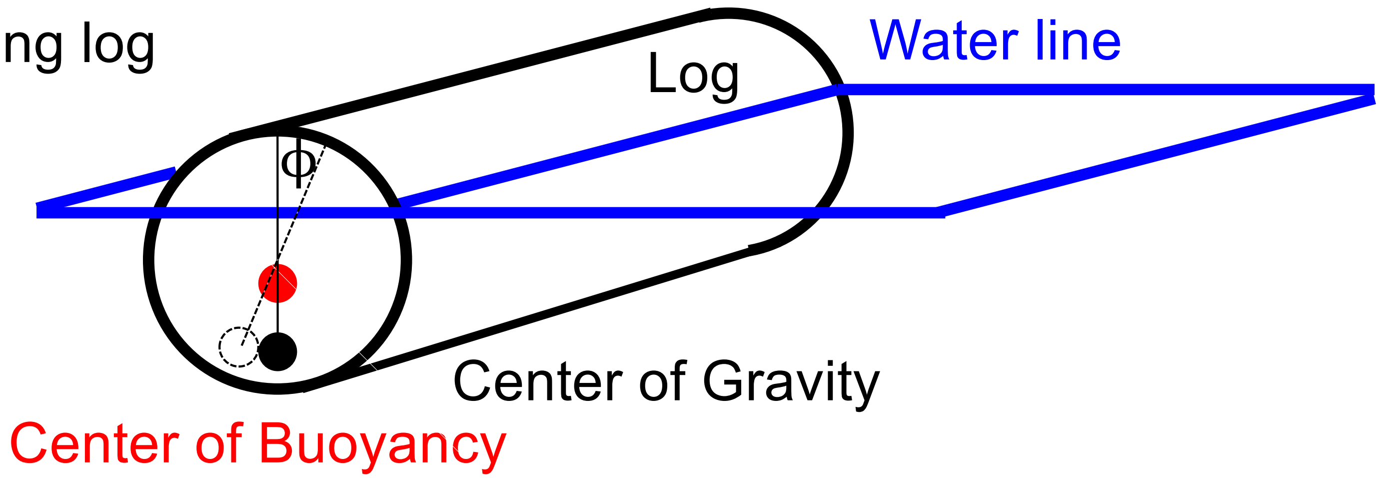
5) A catamaran rests in port, where approximately is its center of buoyancy?

5) You are on starboard tack sailing in a straight line. If you now steer to port, is the hydrodynamic center moving forward or backward? If you steer to starboard, how does the hydrodynamic center then move?

Righting of a Tilted Boat, cylinder model

2) Torque around the longitudinal axis

a) A floating log

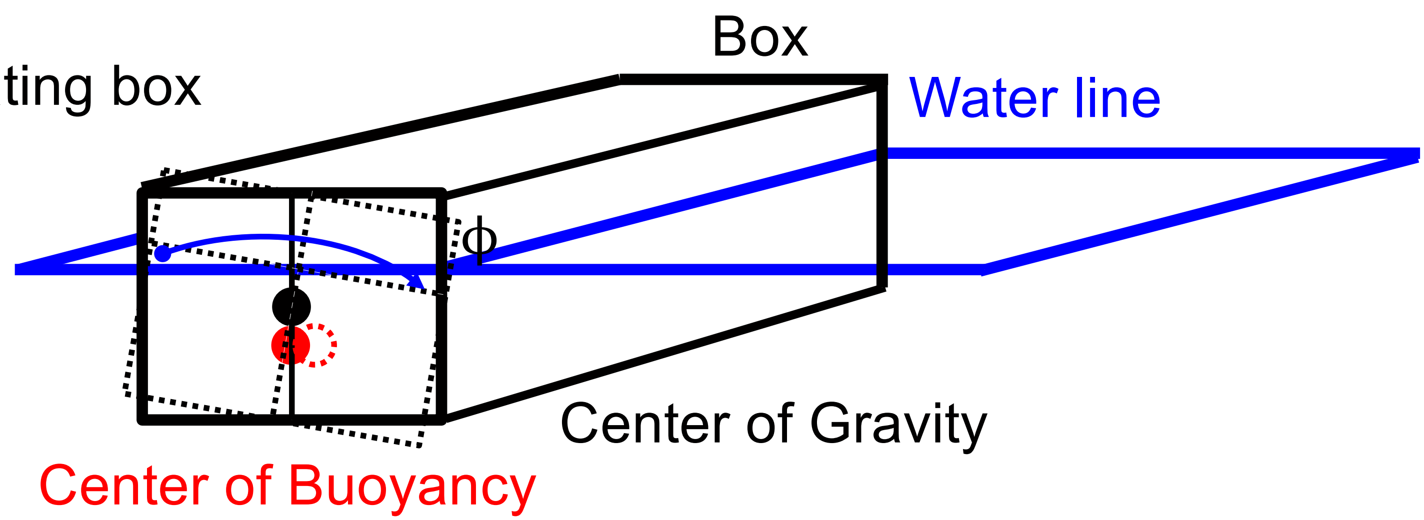


- For a perfectly round cylinder, the Center of Buoyancy does not depend on rotation by ϕ.
- If the Center of Gravity is in the middle, the log can rotate freely.
- If the Center of Gravity is below the middle, it goes up under rotation.
- The Center of Gravity is then pulled down to is lowest point.
- The Center of Gravity will then be exactly above or below the center of buoyancy to counteract the gravitational force.

Righting of a Tilted Boat, box model

2) Torque around the longitudinal axis

b) A floating box



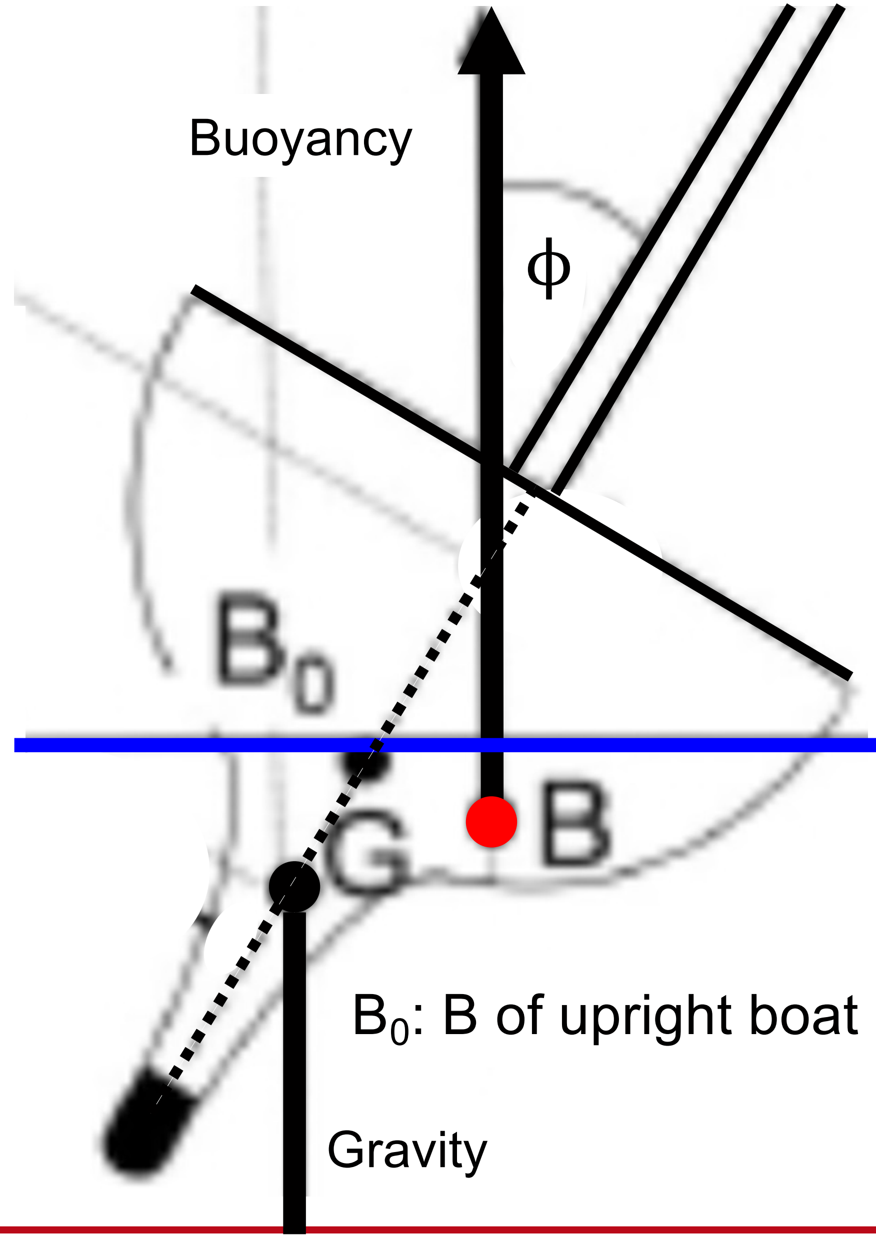
- The height of the Center of Gravity does not depend on rotation ϕ.
- The Center of Buoyancy moves under rotation, mostly sideways.
- The Center of Buoyancy is then pushed up, restoring the boat's angle.
- The Center of Gravity will then be exactly above or below the center of buoyancy to counteract the gravitational force.

Righting of a Tilted Boat

2) Torque around the longitudinal axis

c) Boat

- The height of the Center of Gravity generally changes with angle ϕ.
- The Center of Buoyancy generally changes with angle.
- The boat is twisted in the direction where the height of G-B decreases.
- At equilibrium, these centers are exactly above each other to counteract the gravitational force.

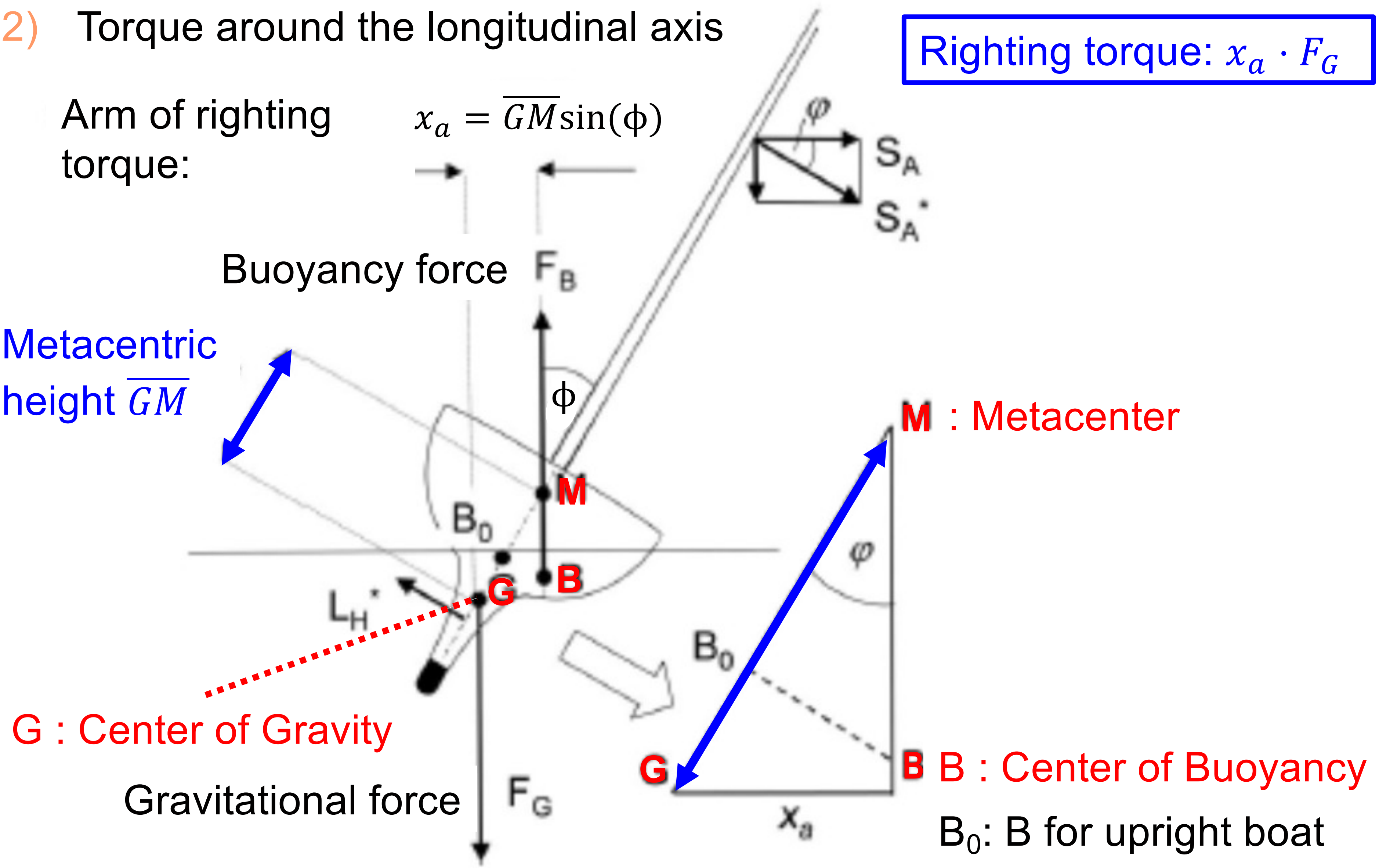


The Metacenter – Center of Oscillation

2) Torque around the longitudinal axis

Righting torque: $x_a \cdot F_G$

Arm of righting torque: $x_a = \overline{GM}\sin(\phi)$

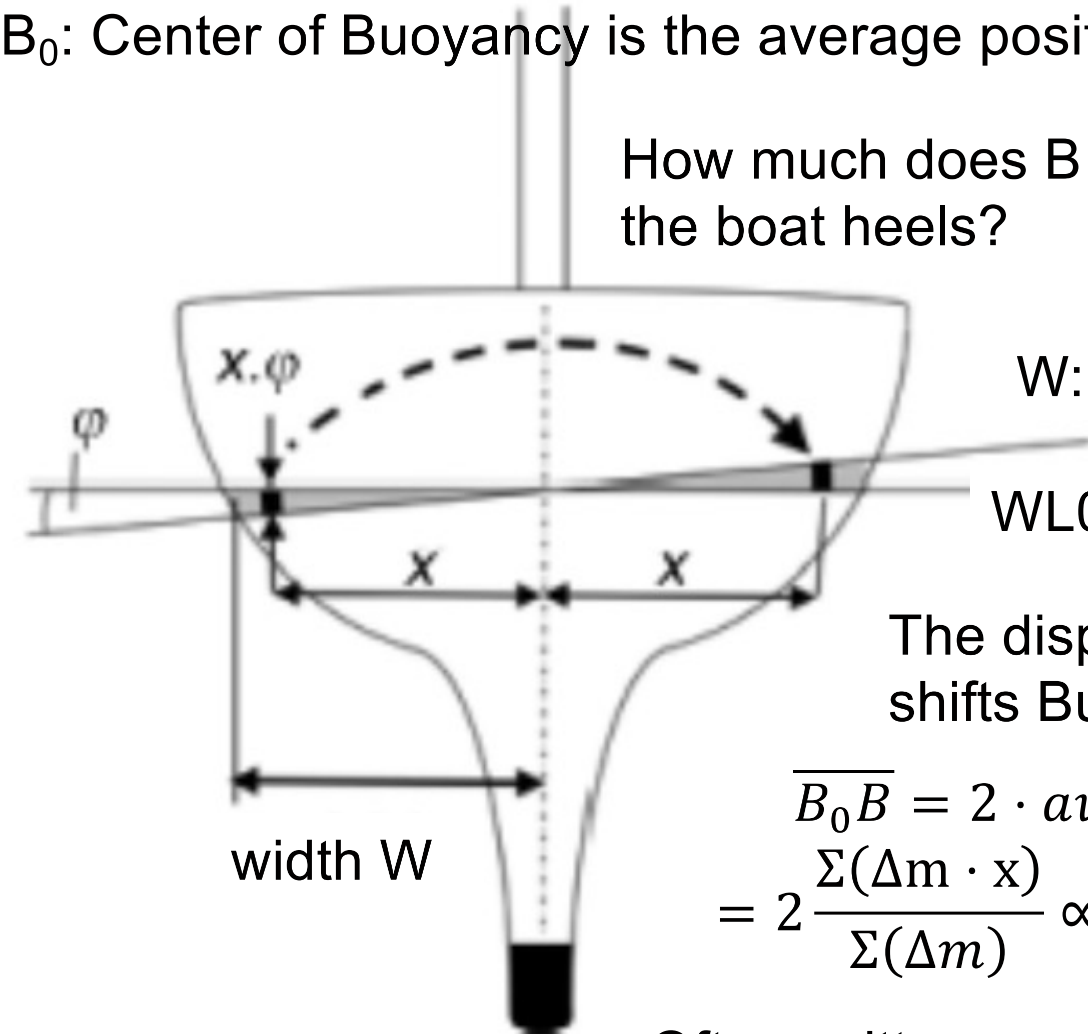


B_0: B for upright boat

Determination of Shift in Buoyency

G: Center of Gravity is the average position of all masses of the boat.

B_0: Center of Buoyancy is the average position of all displaced water.



How much does B change when the boat heels?

W: Water line of the heeling boat

WL0: Water line of the upright boat

The displaced water mass $\Sigma(m)$ shifts Buoyancy by

$$\overline{B_0B} = 2 \cdot average\ x\ of\ the\ triangles$$

$$= 2\frac{\Sigma(\Delta\mathrm{m} \cdot \mathrm{x})}{\Sigma(\Delta m)} \propto Length \cdot W^3 \sin(\phi)/Volume$$

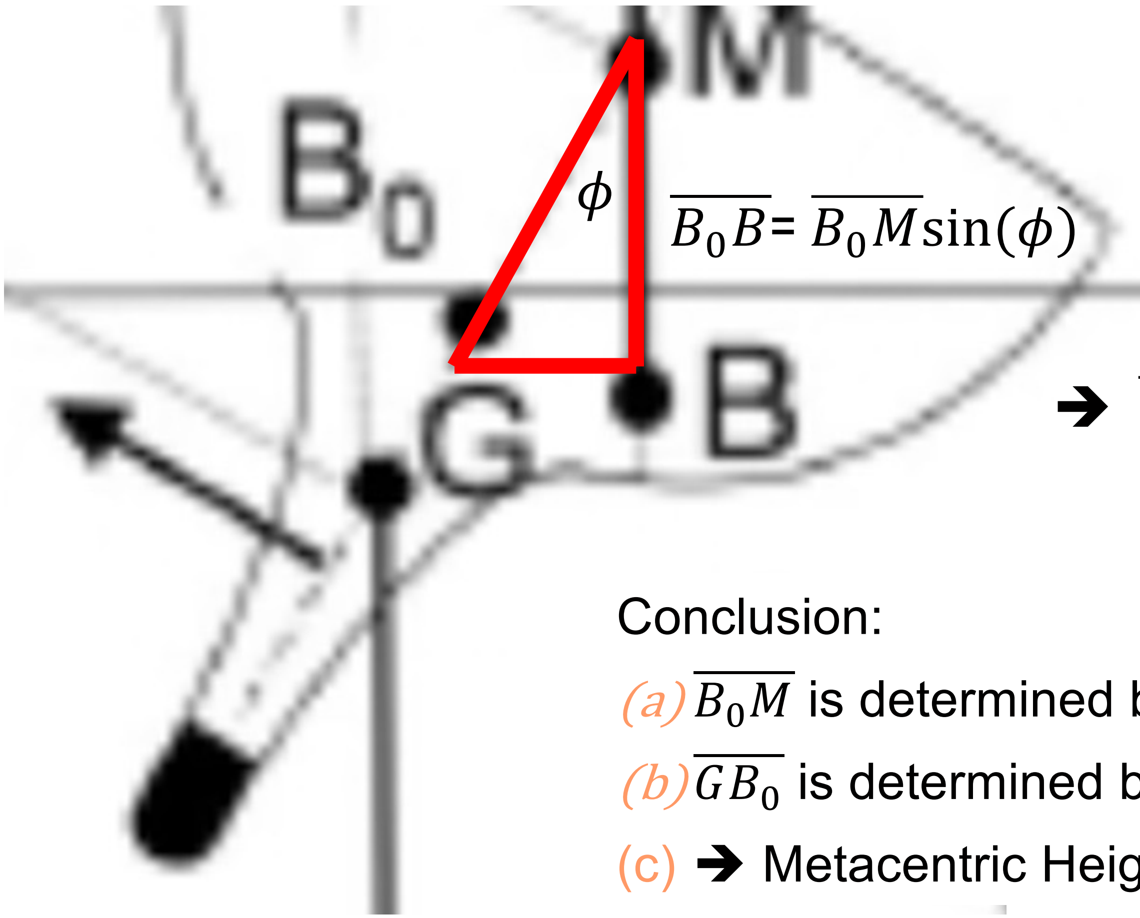
Often written as $\overline{B_0B} = \frac{\mathrm{I_y}}{V}\sin(\phi)$

Determination of the Metacentric Height

Often written as. $\overline{B_0B} = \frac{I_y}{V}\sin(\phi)$

Note: V : the volume of the displaced water

I_y : surface moment of inertia of the water-line surface



➔ $\overline{B_0M} = \frac{I_y}{V}$

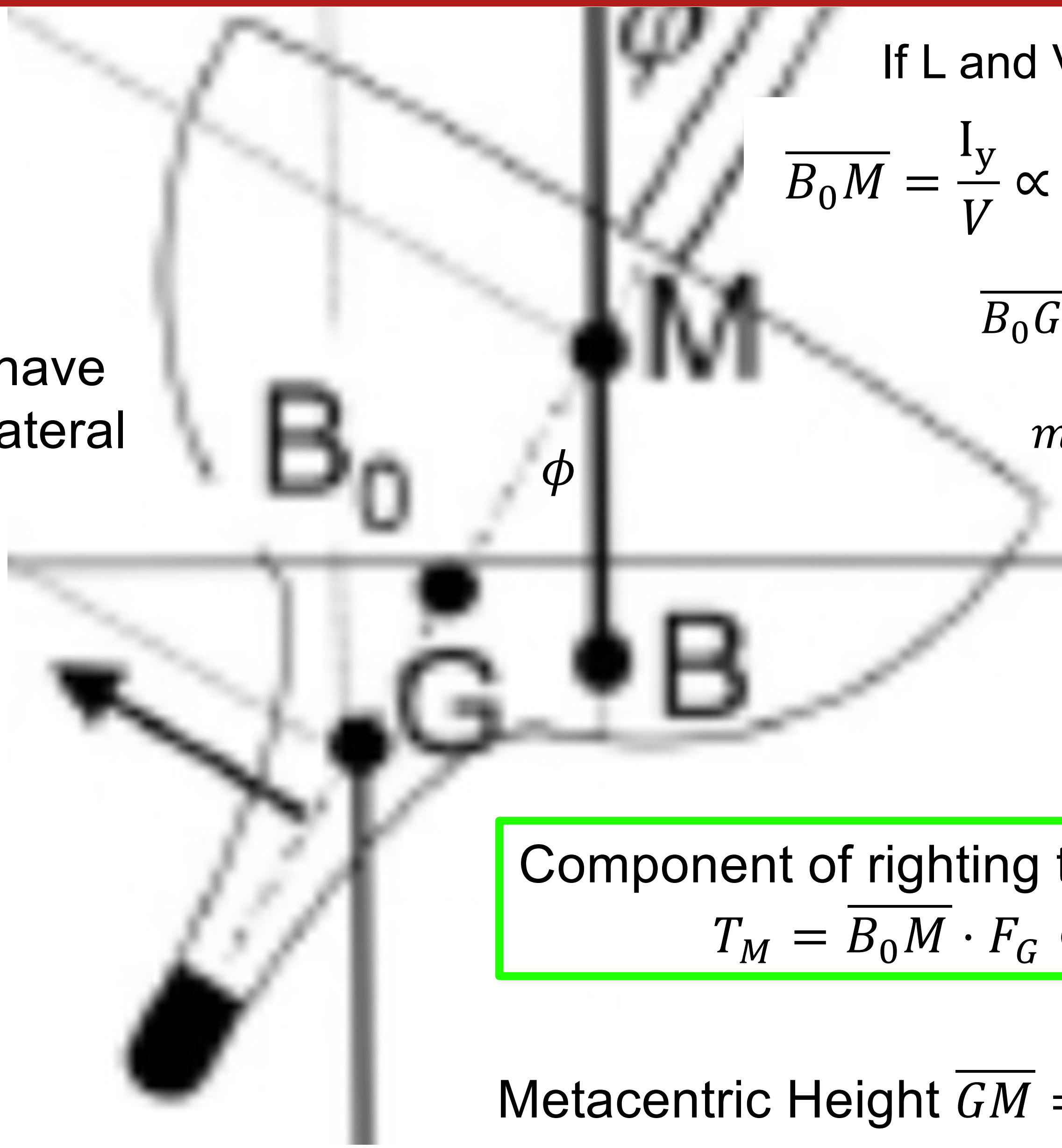
Conclusion:

(a) $\overline{B_0M}$ is determined by analyzing the water line.

(b) $\overline{GB_0}$ is determined by analyzing the upright boat.

(c) ➔ Metacentric Height $\overline{GM} = \overline{B_0M} \pm \overline{B_0G}$

Scaling of Boat Stability with Boat Width



If L and V are constant.

$$\overline{B_0M} = \frac{I_y}{V} \propto \frac{L \cdot W^3}{V} \propto W^2$$

$\overline{B_0G} \approx$ unchanged

$m \propto$ W

Conclusion:

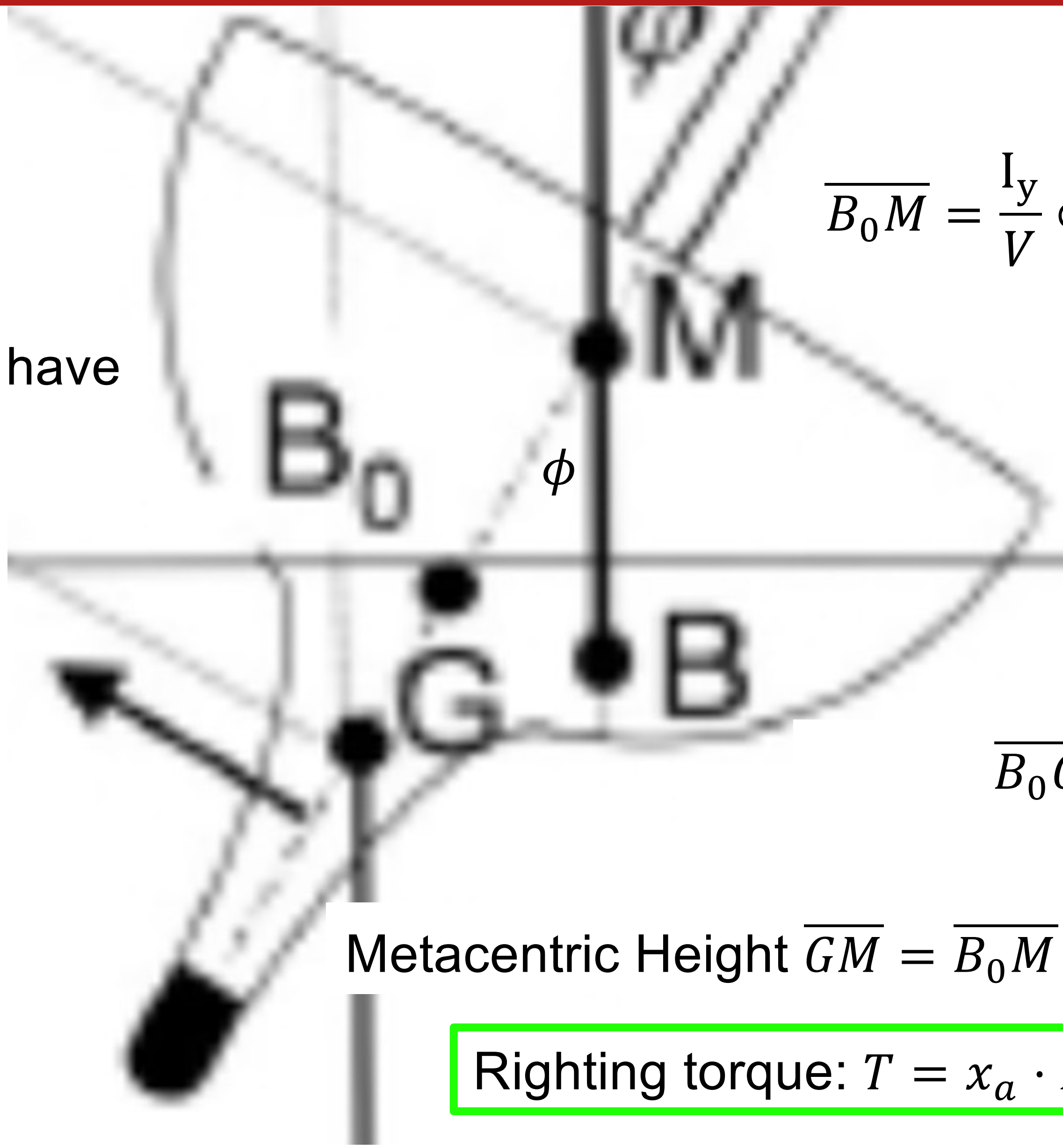
Wider Boats have much better lateral stability.

Component of righting torque:

$$T_M = \overline{B_0M} \cdot F_G \propto W^3$$

Metacentric Height $\overline{GM} = \overline{B_0M} \pm \overline{B_0G}$

Scaling of Boat Stability with Boat Size



$$\overline{B_0M} = \frac{I_y}{V} \propto \frac{L \cdot W^3}{V} \propto size$$

Conclusion:

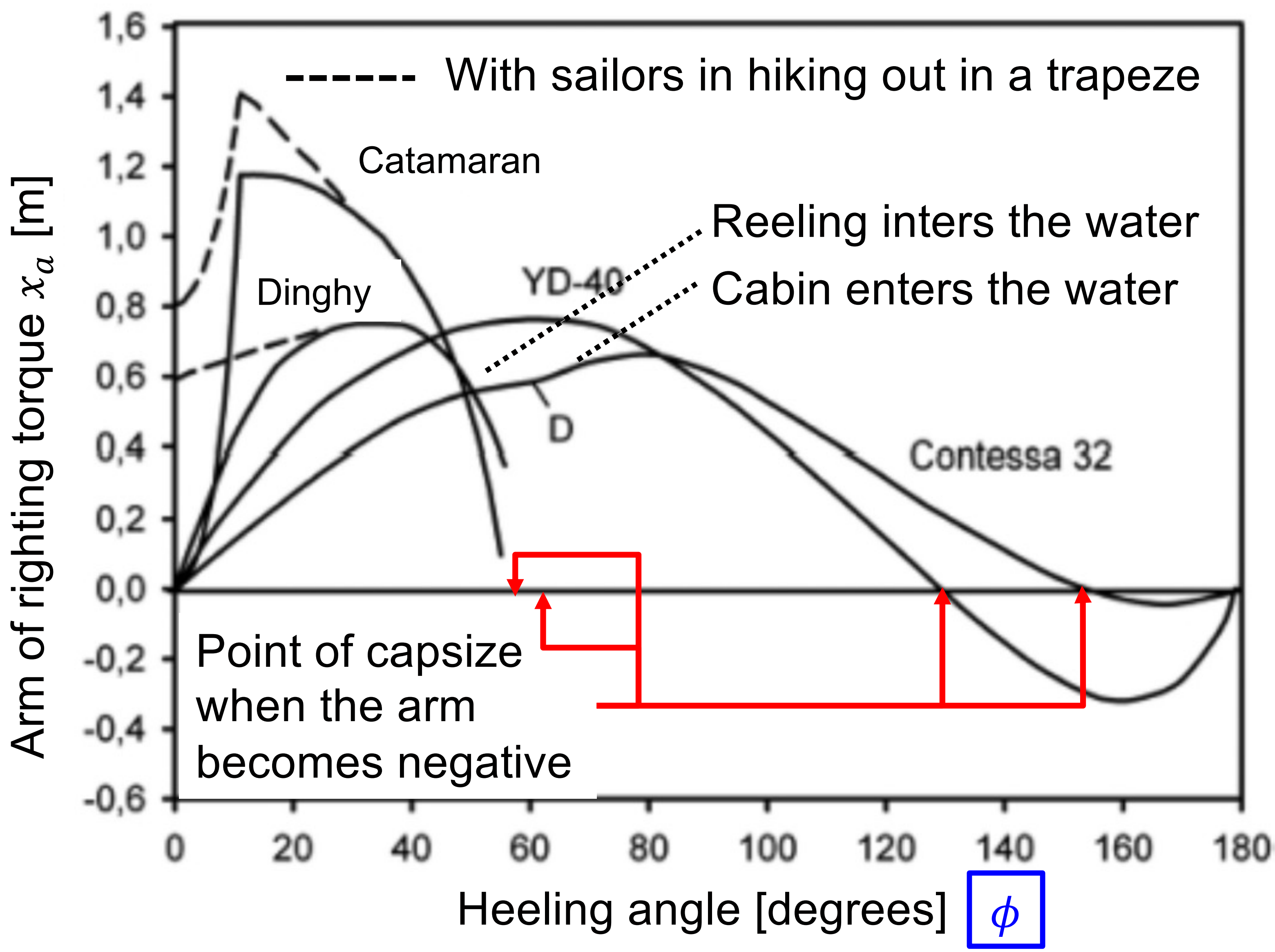
Larger Boats have better lateral stability.

$$\overline{B_0G} \propto size$$

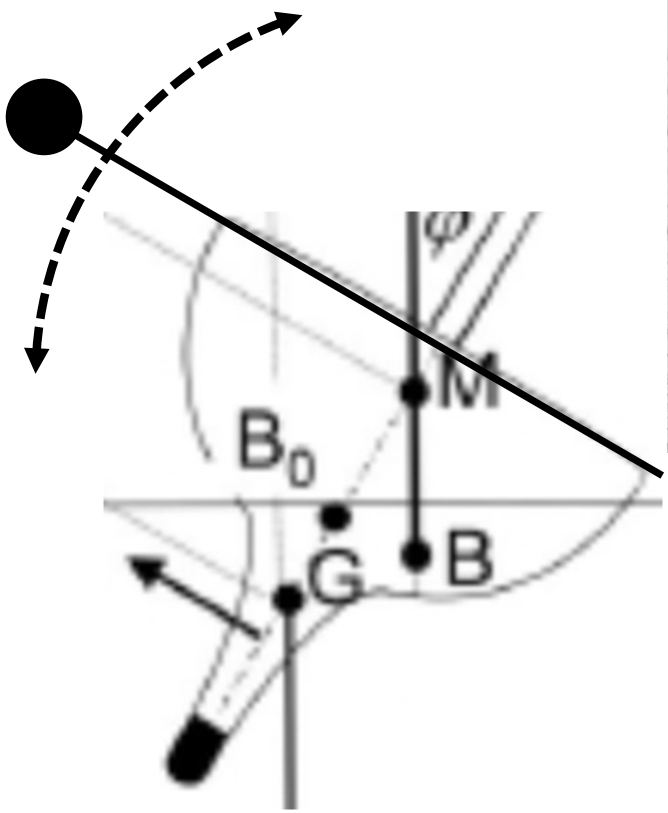
Metacentric Height $\overline{GM} = \overline{B_0M} \pm \overline{B_0G} \propto size$

Righting torque: $T = x_a \cdot F_G \propto size^4$

Stability Plot for a Boat's Lateral Rotations

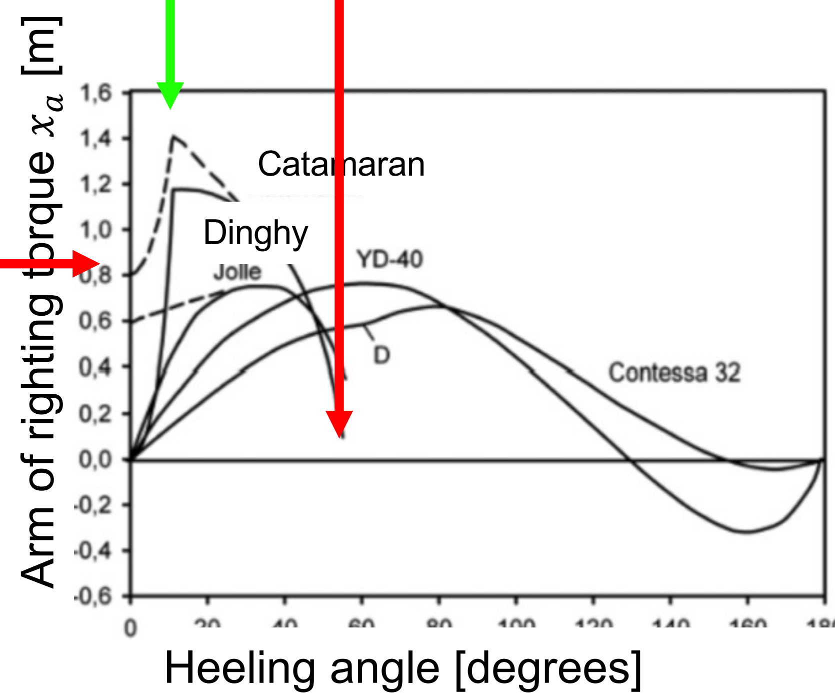


Hiking out in a Trapeze



- There is already a righting arm, even for the not heeling boat.
- At small angles, the righting arm changes little.

Catamaran stability



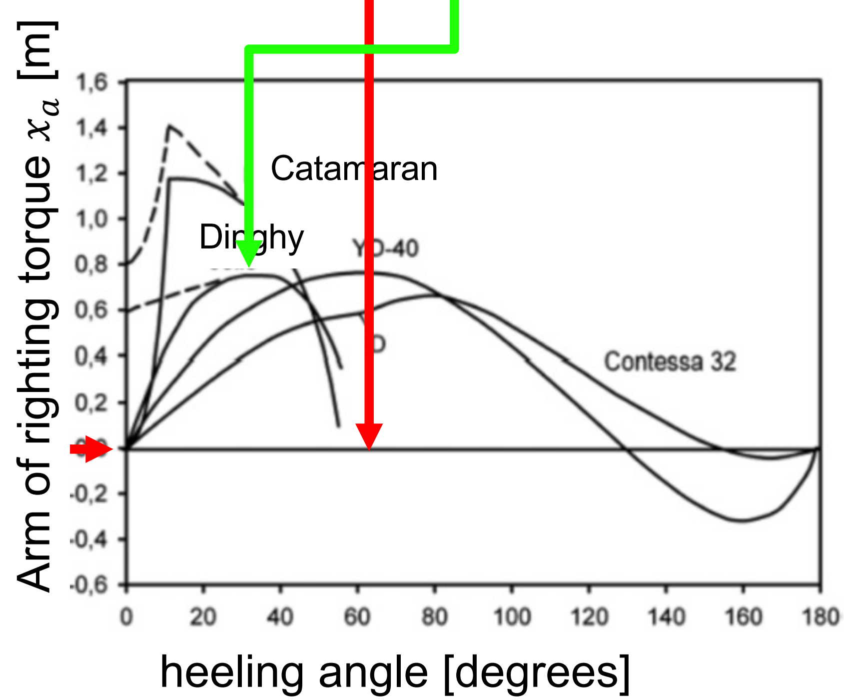
With small angles the righting arm hardly changes letting catamarans fell less smooth.

Dinghy stability

Form Stability: Center of Buoyancy moves down because the flat boat shape (not because the keel moves by an angle ϕ).

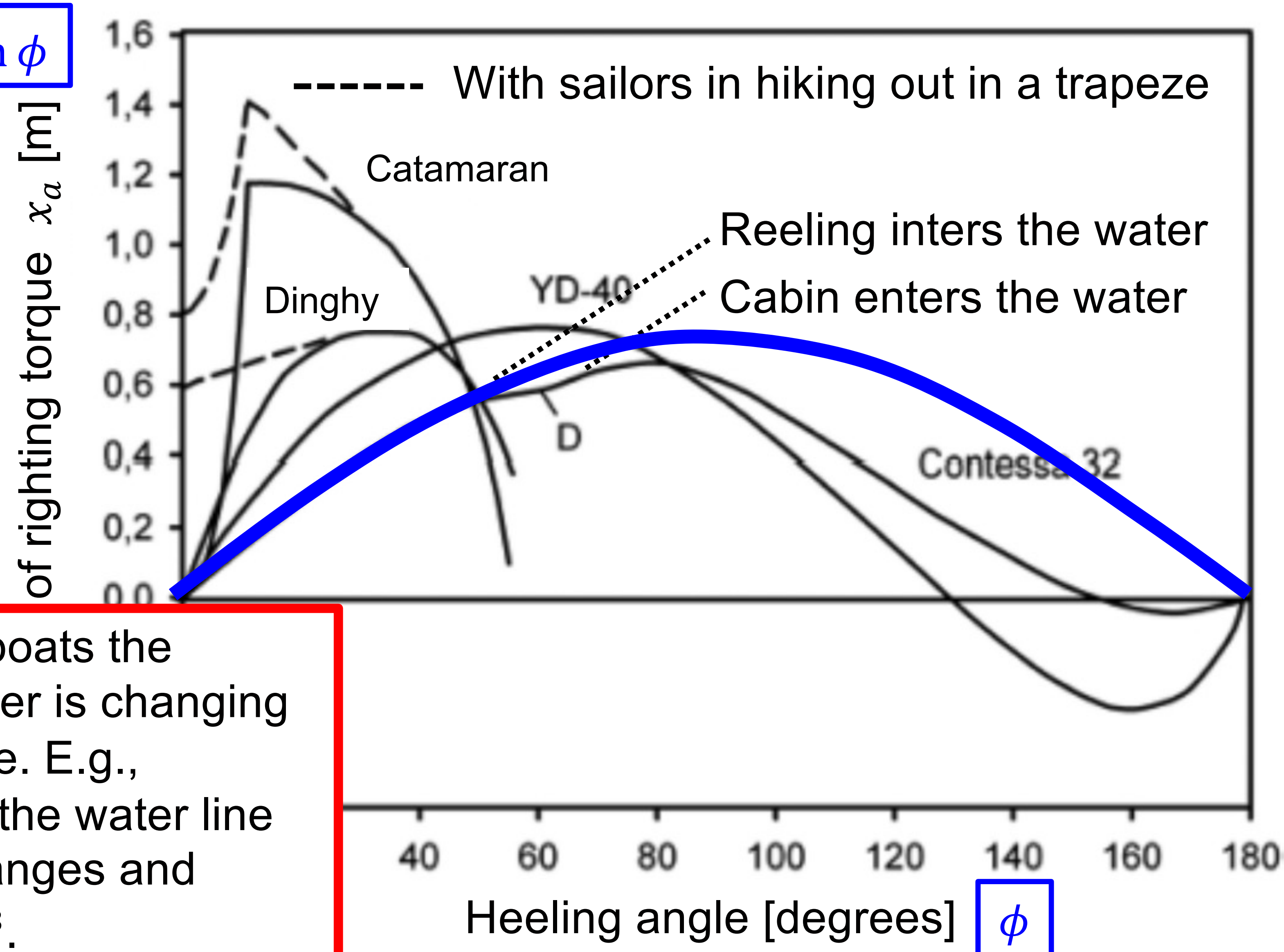
Center of Gravity is usually above Center of Buoyancy.

Catamarans and Dinghy's are form stable.



Stability Plot for a Boat's Lateral Rotations

$x_a = \overline{GM} \sin \phi$

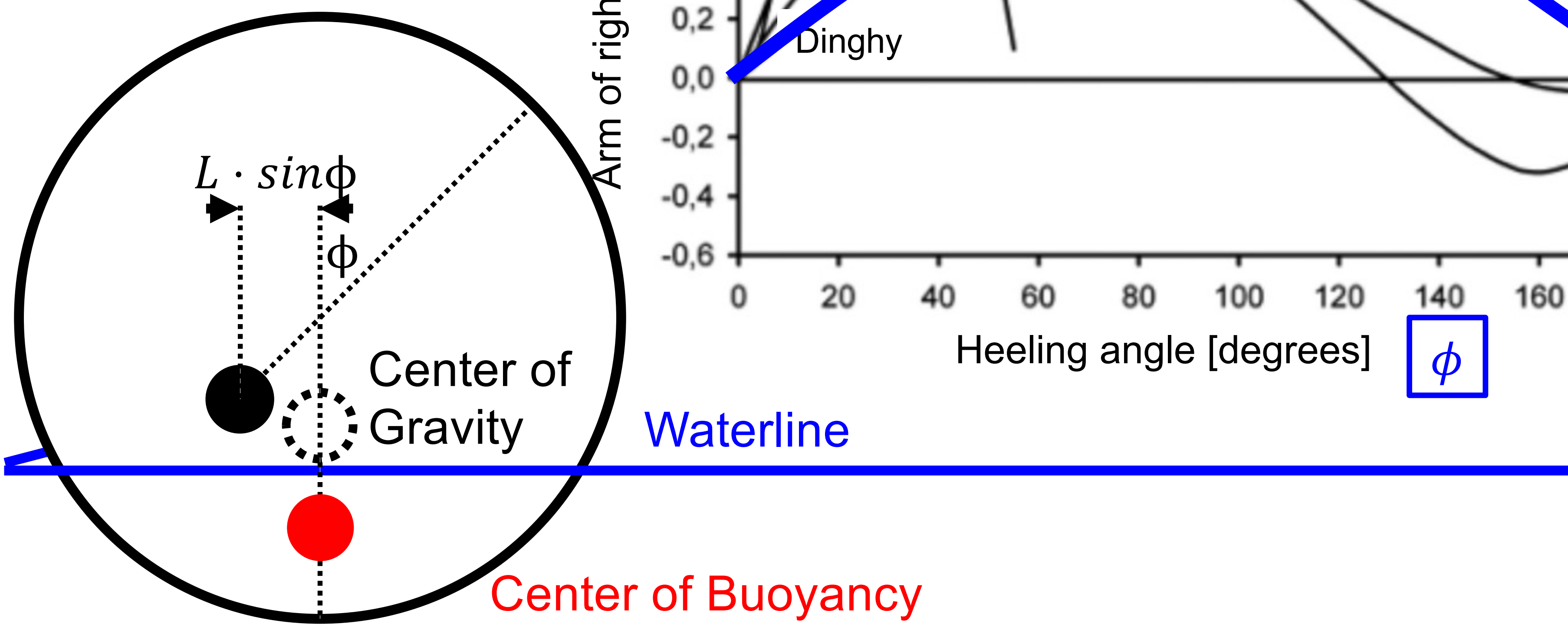


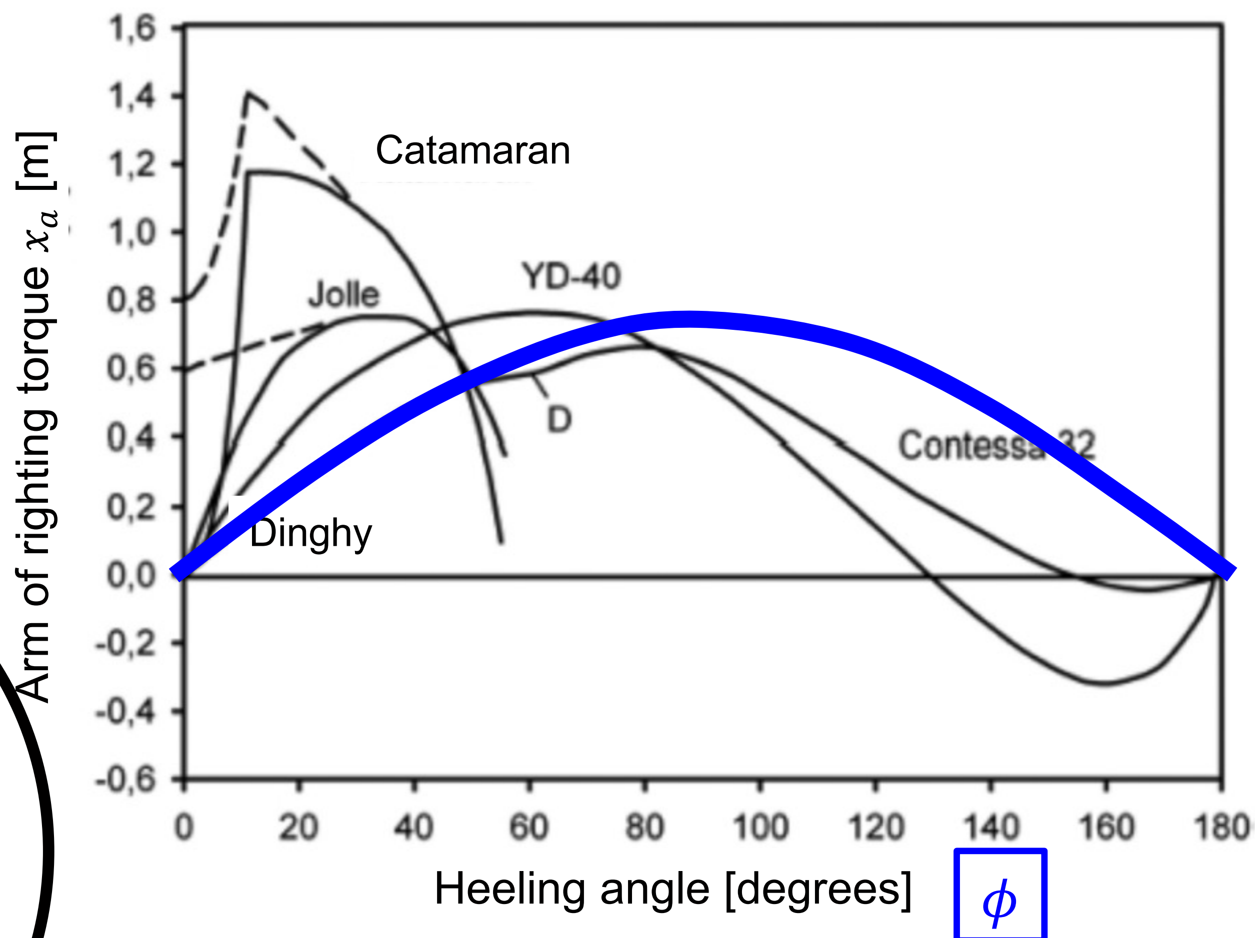
- For real boats the metacenter is changing with angle. E.g., because the water line width changes and $\overline{BM} \propto W^3$.

Stability Plot for a Boat's Lateral Rotations

$$x_a = \overline{GM}\,\sin\phi$$

Stability for Center of Gravity above or below Center of Buoyancy.

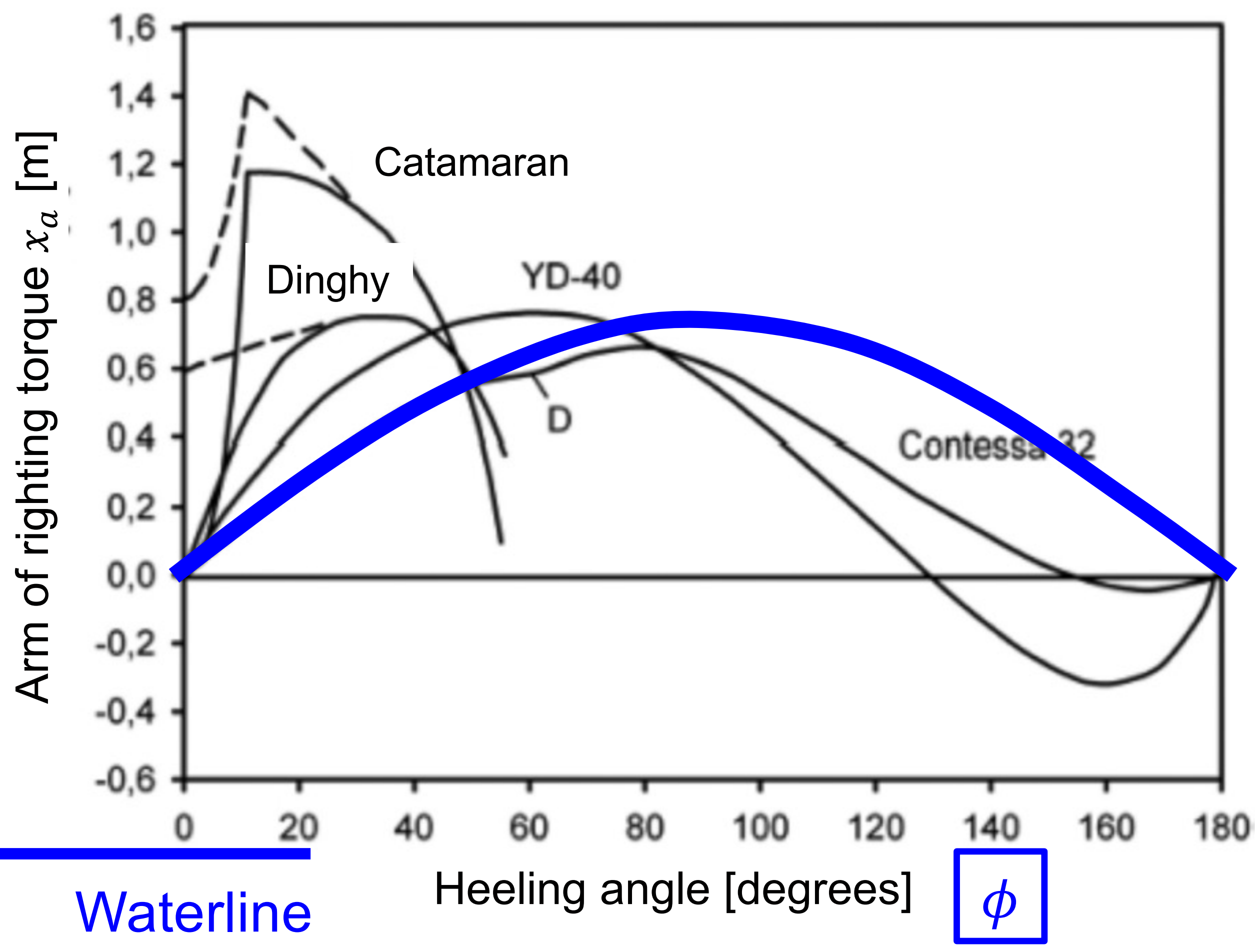


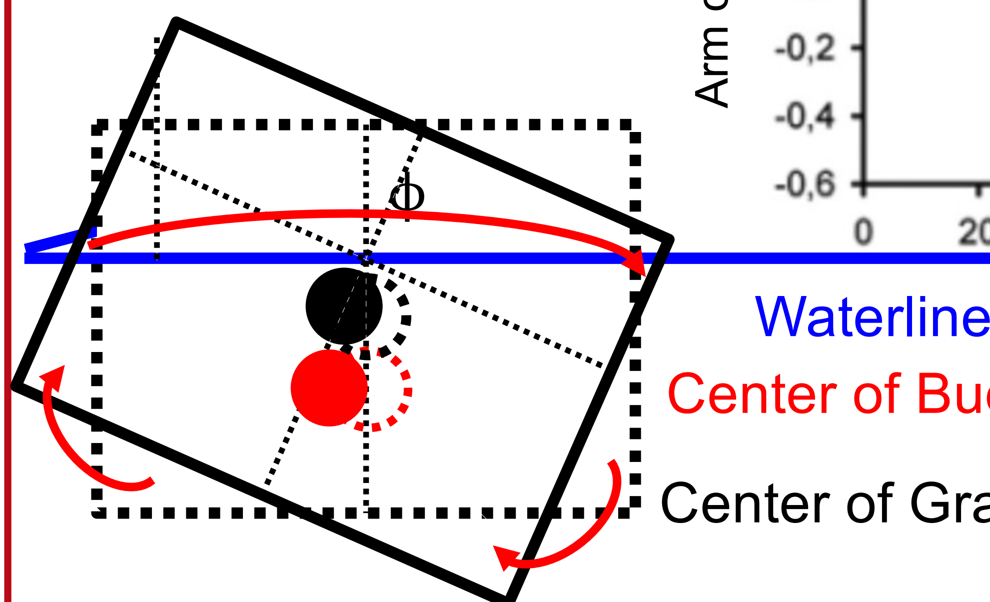


Stability Plot for a Boat's Lateral Rotations

$$x_a = \overline{GM}\,\sin\phi$$

- $\overline{GM}$ decreases a little with ϕ.
- For stability, G cannot be much above B.
- This example would be unstable!





Weight Stability of Keelboats, with cabin

Weight Stability: Center of Buoyancy moves because the weight is lifted by the angle ϕ.

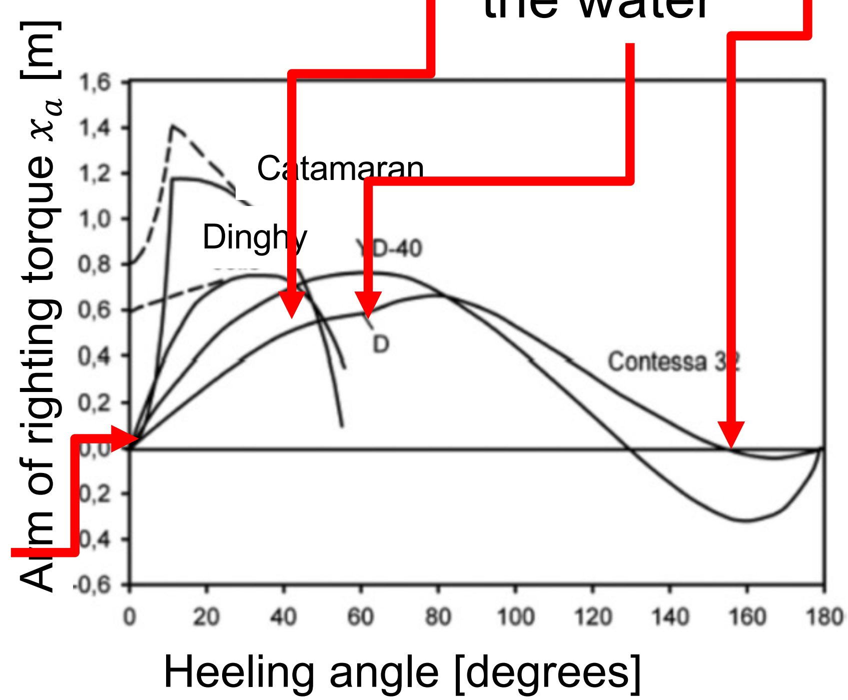
Center of Gravity is usually below Center of Buoyancy.

Keel boats are weight stable.

Can hardly capsize

Reeling inters the water

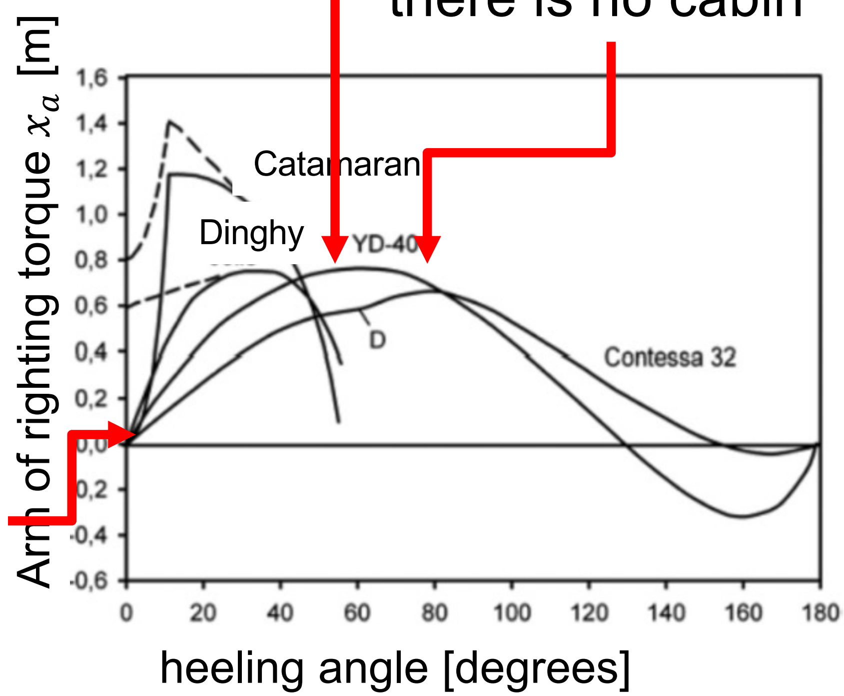
Cabin enter the water



Weight Stability of Keelboats, without cabin

Reeling inters the water

No kink because there is no cabin



Weight Stability: Center of Buoyancy moves because the weight is lifted by the angle ϕ.

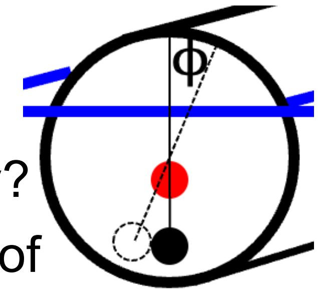
Center of Gravity is usually below Center of Buoyancy.

Keel boats are weight stable.

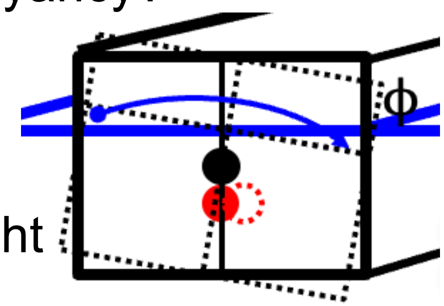
Cornell University

Homework 7

1) A perfectly symmetric, uniformly filled cylinder swims in water.



   (a) Where is its center of gravity G?
   (b) Is its center of buoyancy B_0 above or below its center of gravity?
   (c) When the cylinder rotates, how do center of gravity and center of buoyancy B change?
   (d) A metal wire is glued along one side of the cylinder, how does this change the center of gravity? How will it change the center of buoyancy?
   (e) Where is the metacenter M? What is the metacentric height $\overline{GM}$?

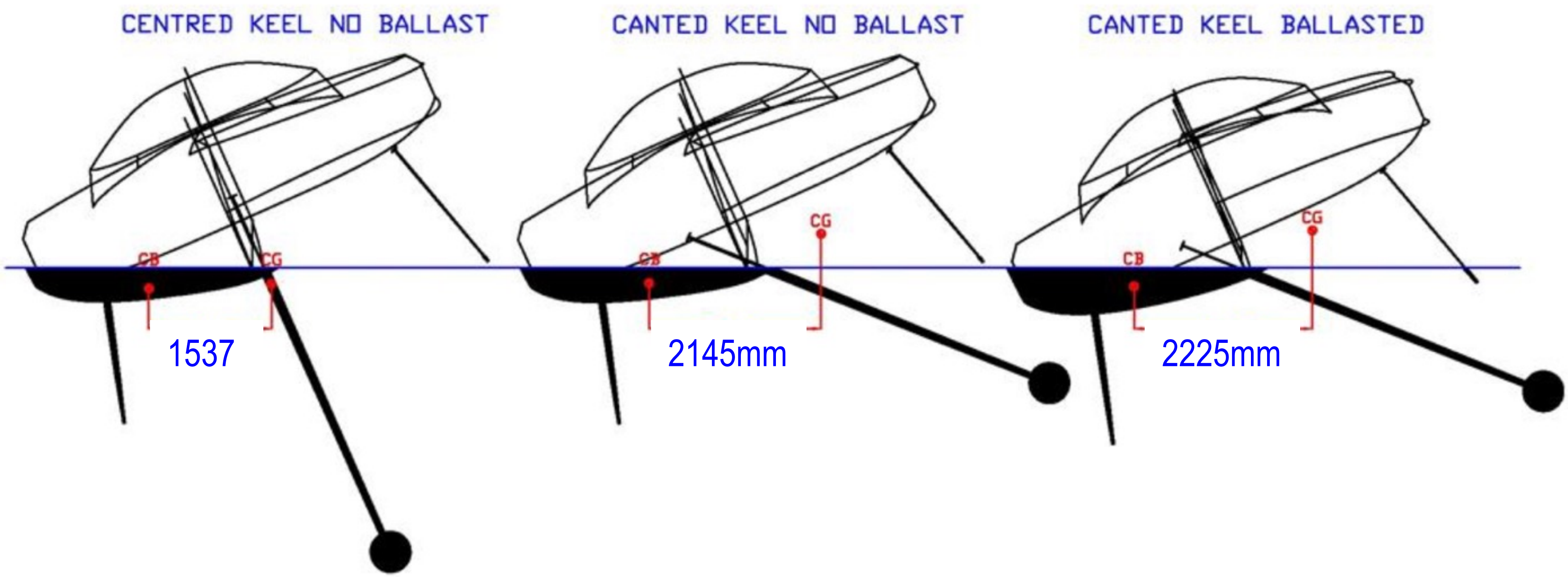


2) A perfectly symmetric uniformly filled rectangular box of height b swims to 2/3 submerged in water.
   (a) Where is its center of gravity G and the center of buoyancy B_0?
   (b) When the box rotates along its longest axis c, which is horizontal, how do center of gravity and center of buoyancy change?
   (c) If the center of buoyancy moves horizontally by $\overline{B_0B} = \Delta B$ when the box rotates by an angle ϕ, where is the metacenter, and what is the metacentric height?

Swing Keels to increase the Metacentric Height

Advanced racing boat, e.g., of the Open 60 class have keels that can be canted laterally to move the center of gravity further away from the center of buoyancy.

Additionally, they can add water ballast the flows to the deep part of the boat to further move the center of buoyancy.



Derivation of Formulas

1) Measure the torque as a function of angle

$$T = H \cdot F \cdot \sin(\theta) = m \cdot g \cdot \overline{GM} \cdot \sin(\phi)$$

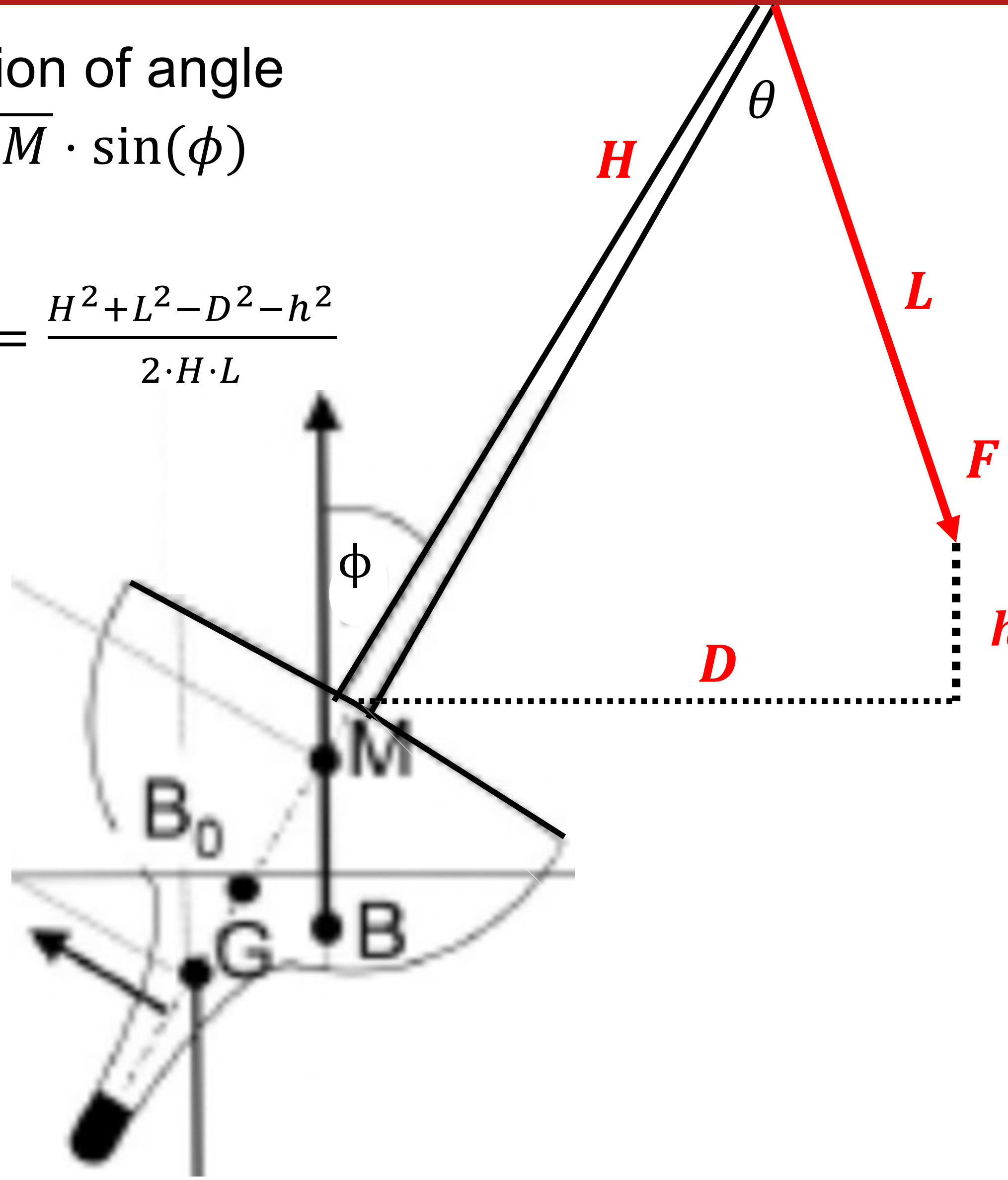
2) Determine the angle θ ➔ T

$$\vec{L} - \vec{H} = D\vec{e}_x + h\vec{e}_y \text{ ➔ } \cos(\theta) = \frac{H^2+L^2-D^2-h^2}{2 \cdot H \cdot L}$$

3) Determine the angle ϕ.

➔ $\sin(\phi) = \frac{D\,H - D\,L\,\cos(\theta) - h\,L\,\sin(\theta)}{D^2+h^2}$

➔ $\overline{GM} = \frac{T}{g\,m\,\sin(\phi)}$



Measure the force for many different L and compute $m \cdot \overline{GM}$ and ϕ each time. Plot $m \cdot \overline{GM}$ against ϕ.

Measuring the Metacentric Height times Mass

1) Measure the torque as a function of angle

$$T = H \cdot F \cdot \sin(\theta) = m \cdot g \cdot \overline{GM} \cdot \sin(\phi)$$

2) Determine the angle θ.

$$\vec{L} - \vec{H} = \vec{D} + \vec{h} \rightarrow (\vec{L} - \vec{H})^2 = (\vec{D} + \vec{h})^2$$

$$\rightarrow H^2 + L^2 - 2 \cdot L \cdot H \cdot cos\theta = D^2 + h^2$$

$$\rightarrow \cos(\theta) = \frac{H^2+L^2-D^2-h^2}{2 \cdot H \cdot L}$$

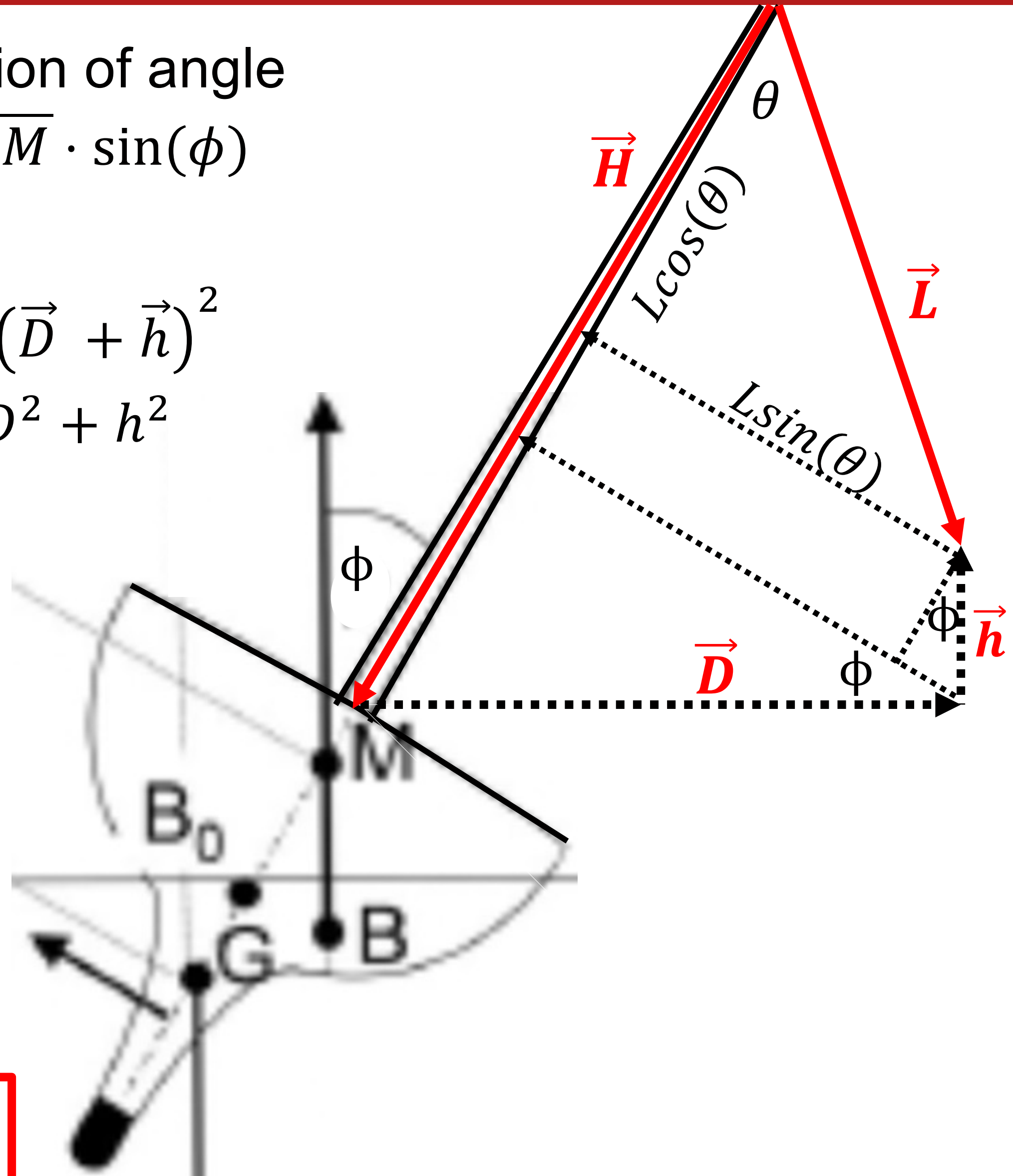
3) Determine the angle ϕ.

a) $Lsin(\theta) = Dcos\phi - hsin\phi$

b) $Lcos(\theta) = H - Dsin\phi - hcos\phi$

$(a) \cdot h + (b) \cdot D \rightarrow$

$$\rightarrow sin(\phi) = \frac{D\ H - D\ L\ \cos(\theta) - h\ L\ \sin(\theta)}{D^2+h^2}$$



Measuring the GM with different mass distributions

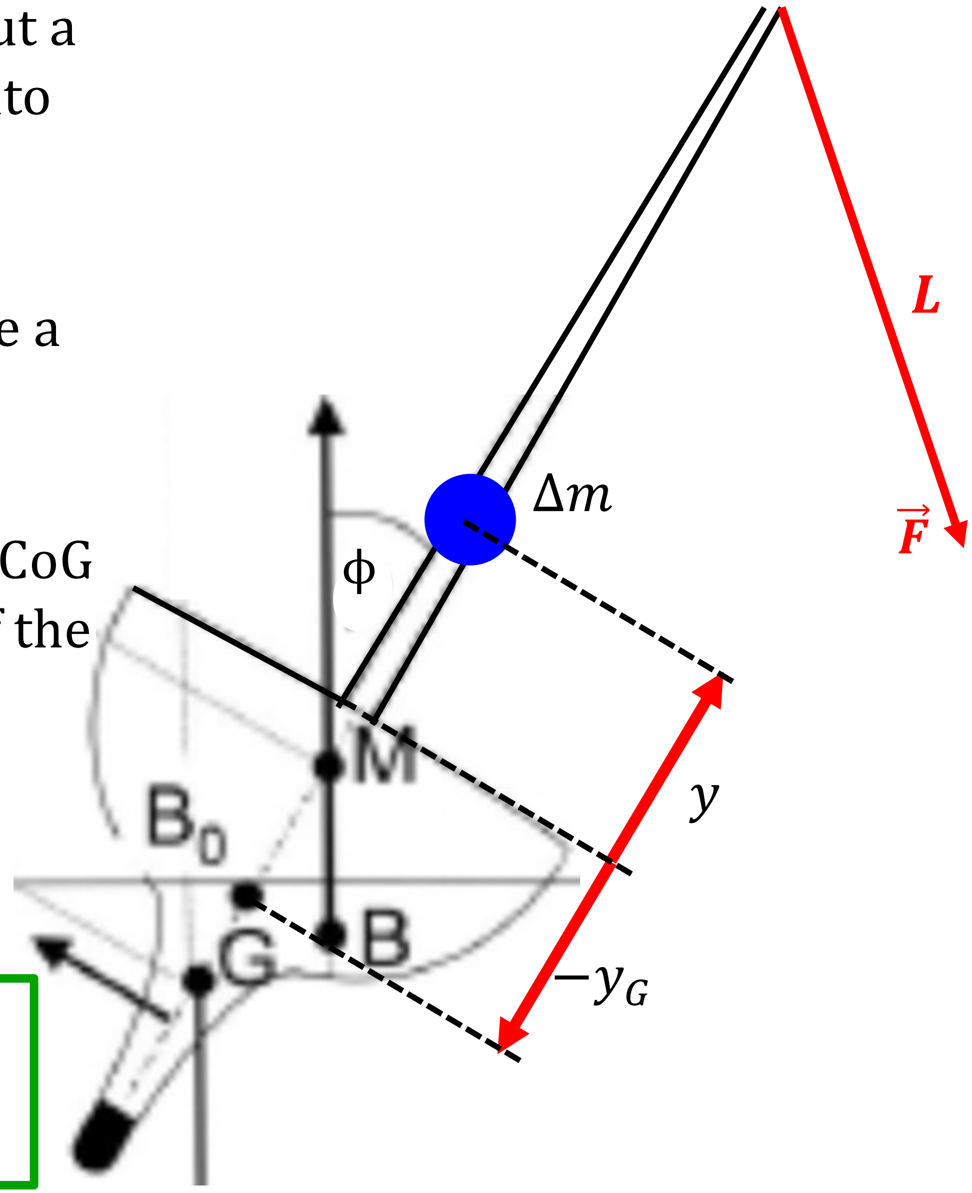
Measure $\overline{GM}$ and $\overline{GM}_0$ with and without a person standing next to and holding onto the mast.

Let the boat's center of gravity (CoG) be a distance y_G above the deck.

A person of mass Δm standing with its CoG distance y above deck raises the CoG of the boat by $\frac{\Delta m}{m+\Delta m}(y - y_G) = \overline{GM}_0 - \overline{GM}$.

$$y_G = y - \frac{m + \Delta m}{\Delta m}(\overline{GM}_0 - \overline{GM})$$

Having (even many) sailors congregate a distance y_G above deck will therefore not diminish stability of the boat.



Elliptical approximation of $\overline{B_0M}$

1) Measure the width W and length L of the waterline.
2) An Ellipse has the surface moment of inertia of $I_V = \frac{\pi}{64} L \cdot W^3$

Note on the side: This comes from $\mathrm{L} = 2a$, $\mathrm{W} = 2b$,

$$I_V = \int_L \int_W x^2 dxdy = \frac{2}{3} b^3 \int_L \sqrt{1 - \left(\frac{y}{a}\right)^2}^3 dy = \frac{2}{3} a\, b^3 \int_{-\pi/2}^{\pi/2} \cos^4 \phi \, d\phi = \frac{2}{3} a\, b^3 \frac{3}{8} \pi$$

3) Use $\overline{B_0M} = \frac{I_V}{V} \approx \frac{\pi}{64} L \cdot W^3 \frac{\rho_{water}}{m}$
4) Measure $\mathrm{m}\,\overline{GM}$ and look up the weight m of the boat.
5) Use $\overline{GM}$= $\overline{B_0M} \pm \overline{GB_0}$ to check whether the Center of Gravity is above or below the Center of Buoyancy, and check validity of the elliptical model.

Longitudinal equilibrium of forces and torques

3) Torque around the transverse axis

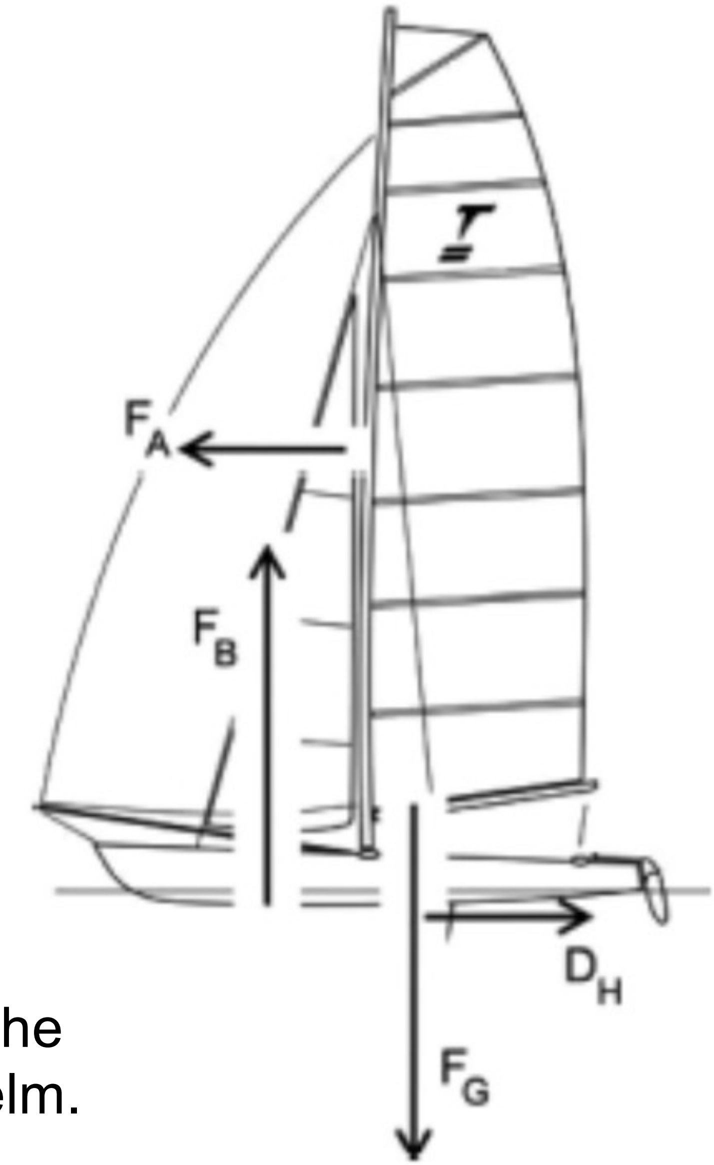
The Center of Buoyancy must be in front of the Center of Gravity.

The longitudinal Metacentric Hight is computed as $\overline{GM}$. It is much larger than the transverse one, because the surface moment of inertia is much larger around the transverse axis.

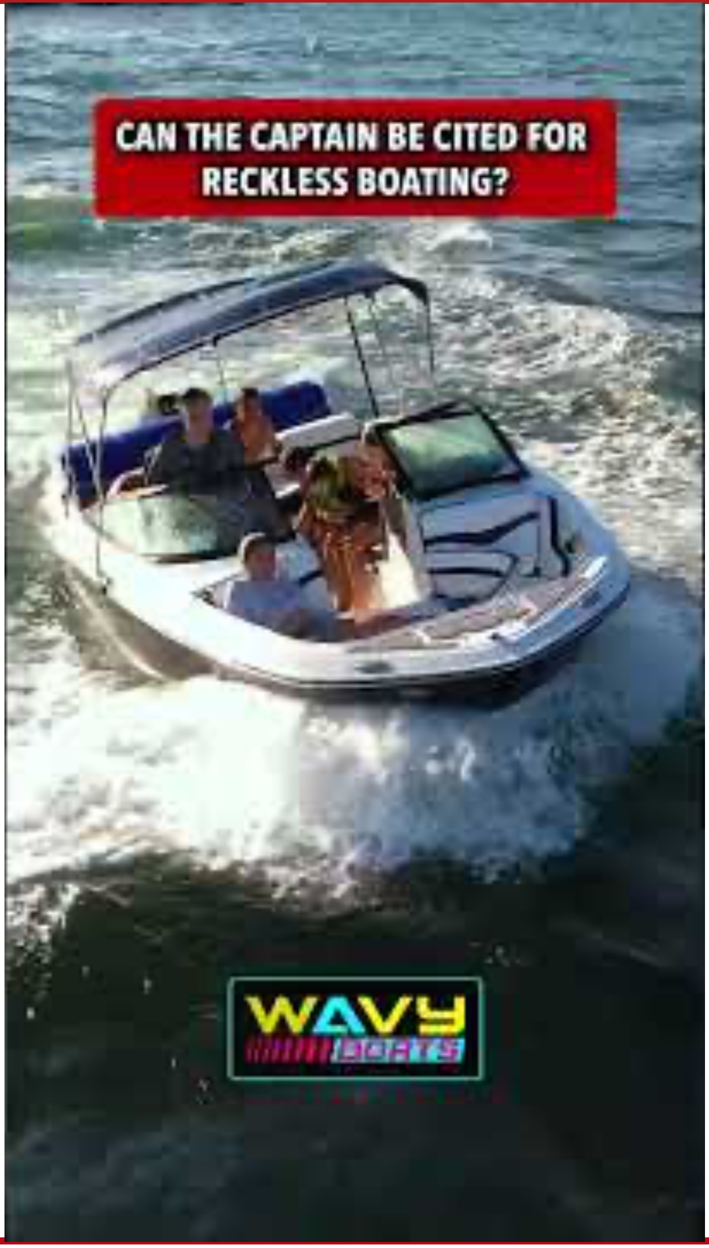
Typical longitudinal $\overline{GM}$: several meter

Typical transverse $\overline{GM}$: less than one meter

More wind pushes the bow deeper, moving the hydrodynamic center forward ➔ Weather helm.



Resonant Longitudinal Oscillations (Pitching)



Resonant Longitudinal Oscillations (Pitching)

Natural (or free) Oscillations

A swing has a natural oscillation period. It is given by $T_{swing} = 2\pi\sqrt{\frac{L}{g}}$. Where g is the gravitational acceleration of earth of $g = 9.8\,\frac{m}{s^2}$.

Note 1: If a grand father's clock has a pendulum of 1m length, it ticks back and forth every second, because $\sqrt{9.8} \approx \pi$ to 0.4%.

Note 2: One meter (3.3 feet) was initially defined by a Frenchmen as $10{,}000^{th}$ of the distance from north to south pole, on the longitude going through France.

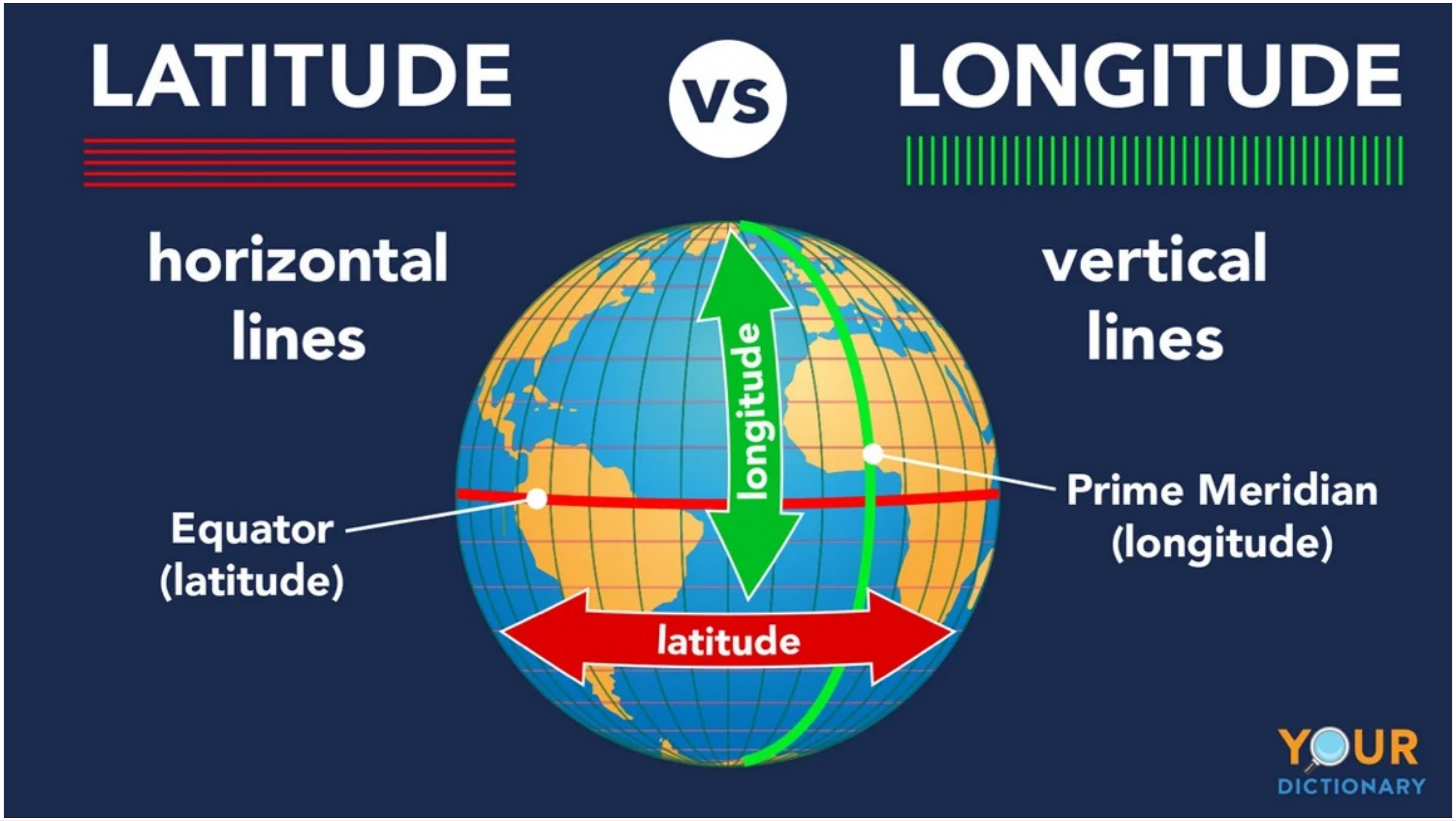
Note 3: A longitude is a circle on earth that includes both poles.

Latitudes and Longitudes

Latitudes (or Parallels) and Longitudes are used to define locations on earth.

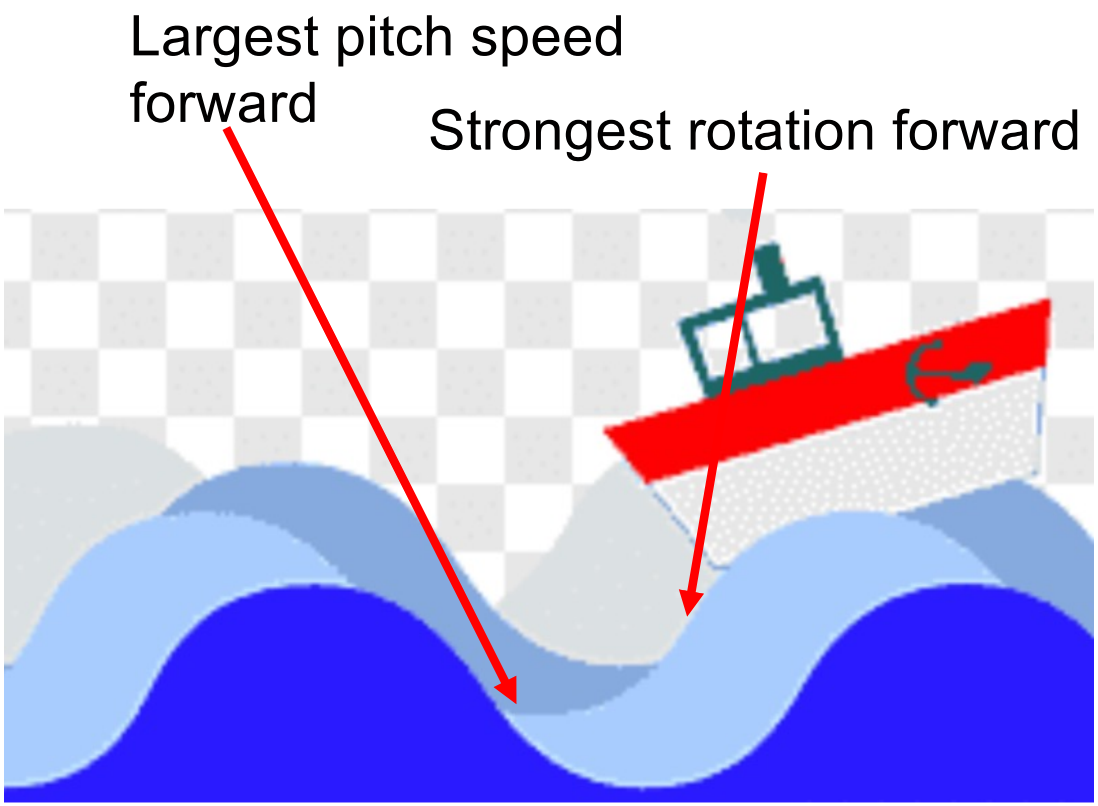
Latitudes go from 0^0 through Greenwich/UK to 180^0 East and 180^0 West.

Longitudes go from 0^0 on the equator to 180^0 South and 180^0 North.



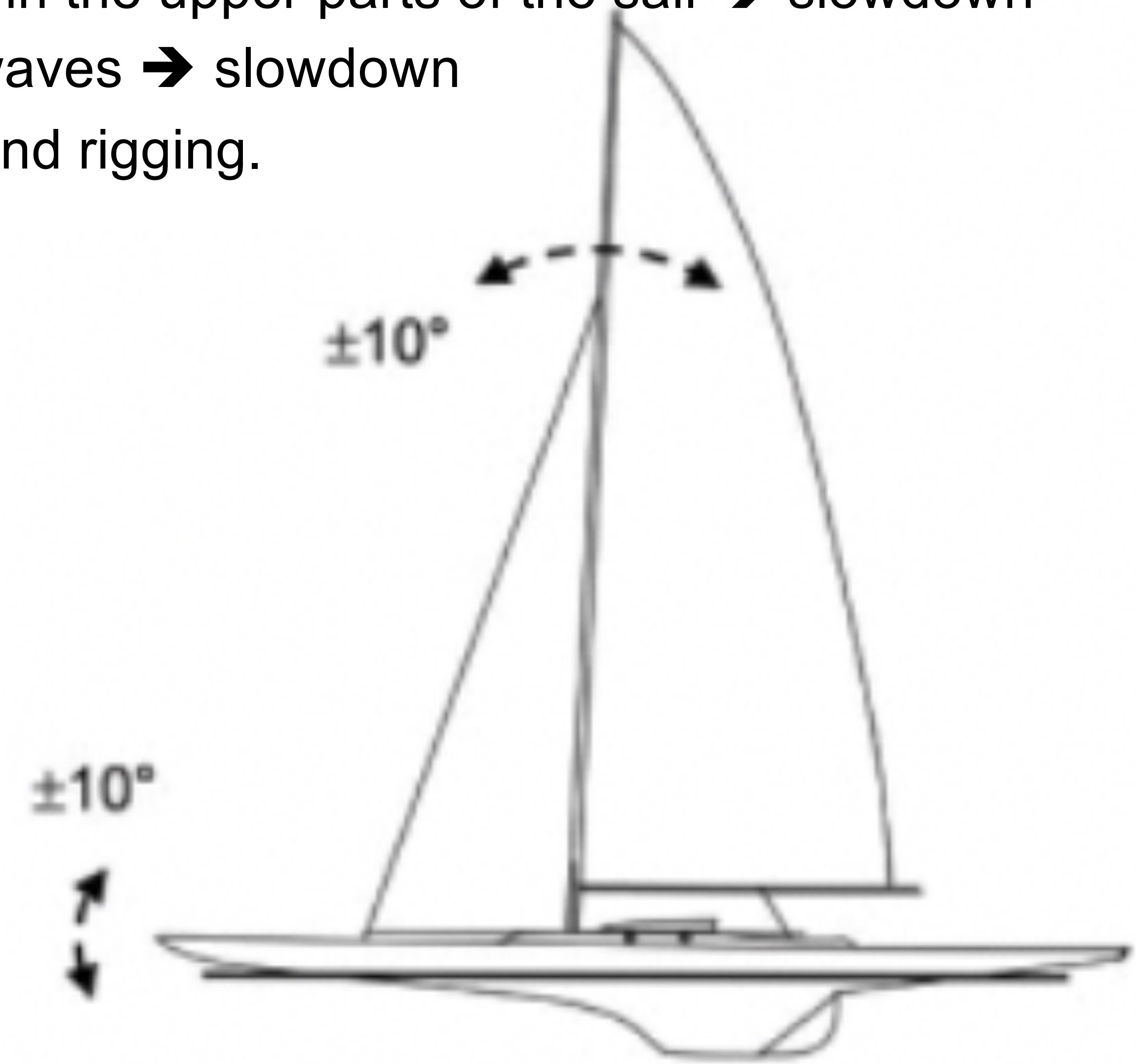
Driven oscillations, e.g., of Pitch Rotations

1) When pushing a swing at its natural oscillation period, it reaches the highest amplitude.
2) The pushing is then maximally out of phase with the velocity, i.e., the strongest push happens when the velocity is 0, at the peak of the swing. The largest velocity is ¼ oscillation after the push.



Problems with Pitch Oscillations

1) Unpleasant, albeit possibly exciting.
2) Periodic change of wind in the upper parts of the sail ➔ slowdown
3) Breaking action by the waves ➔ slowdown
4) Wear and tear on sails and rigging.



Resonant Transverse Oscillations (Rolling)

When the time period of the pitching or rolling is close to the waver period, the oscillations are resonantly reinforced to large amplitudes.

Cornell University

Homework 8

Start writing your experimental report for measuring the Metacentric Height by pulling over the mast and measuring $F(\phi)$ by including the following sections:

- **Abstract:** A short text that gives enough information to let the reader decide whether they should be interested in the report. Include why the topic is important, what you measure, and your result including its precision.
- **Introduction:** A section that gives background material, e.g., what is the topic important, where does it find application, how can relevant quantities be measured and why do you use the presented method of measurement.
- **Measurement:** Describe your method of measurement, derive relevant formulas, and present your data. Evaluate your data and present your result, including its precision.
- **Conclusion:** This summarizes your result and analyzes how the measurement could be improved or what it could be used for in the future.
- **Acknowledgement:** Thank people who have helped you, including funding.
- **References:** A list of literature that you have used. Each item of the list has a number, often in square brackets, e.g., [1]. And this number is placed in the text where the literature has been used.

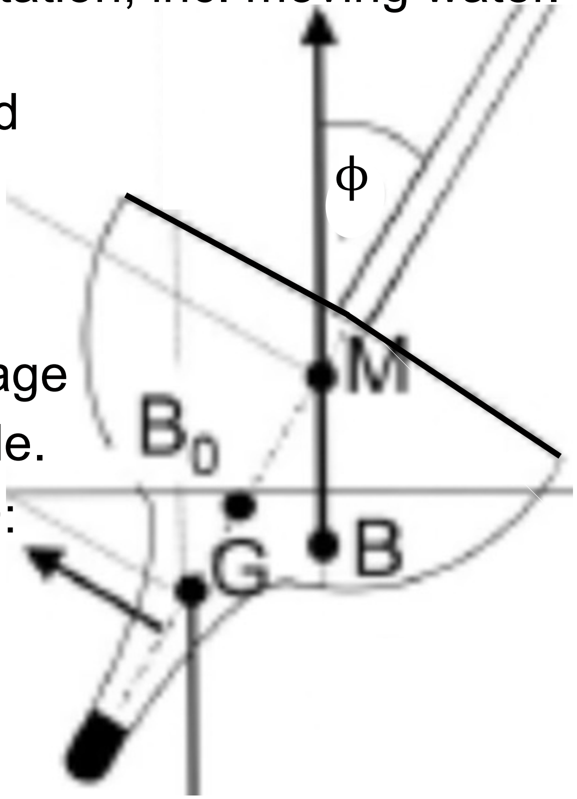
As **Homework**, hand in a draft report for which you will receive helpful suggestions.

Frequency and stability

1) The righting torque T accelerators the angle ϕ:

$$T = m \cdot g \cdot \overline{GM} \cdot \sin(\phi) = \Theta_{\text{eff}} \frac{d^2\phi}{dt^2}$$

Θ_{eff}: effective moment of inertia around the center of rotation, inc. moving water.



2) This leads to a swinging motion with the time period

$$\mathrm{T}_{swing} = 2\pi \sqrt{\frac{\Theta_{\text{eff}}}{m \cdot g \cdot \overline{GM}}}$$

3) Long periods provide comfort and cause less damage on mast and material. But too small $\overline{GM}$ is not stable.

4) For a ½ filled ellipsoid with W, length L, and draft D:

$m = \frac{1}{8}\frac{4\pi}{3}\rho\, WLD$ and $\Theta = \frac{\pi}{15 \cdot 8}\rho\, LDW^3$

$$\mathrm{T}_{swing} \approx \frac{2\pi}{\sqrt{20g}} f_{eff} \frac{W}{\sqrt{\overline{GM}}} = 0.4 f_{eff} \frac{W}{\sqrt{\overline{GM}}} \frac{\text{seconds}}{\sqrt{meters}}$$

5) The fudge factor is often chosen to be $f_{eff} = 2$. $\overline{GM}$ is required to be sufficiently large. Commonly used is 0.9m, if not more precisely specified.

Fishermen's Stability Check

1) Measure transverse periods to determine transverse $\overline{GM}$ (usually around 1m).

2) Similarly measure longitudinal $\overline{GM}$ (usually several m).

3) Pulling the mast to an angle measures $m\,\overline{GM}$, which can be compared to (1) and (2) if the mass of the boat is known.

Safety at Sea, Stability Check

HOW TO CHECK THE STABILITY

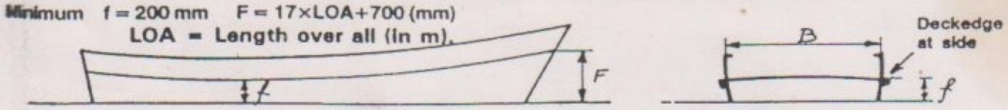
4

1. MINIMUM FREEBOARD

Measured with maximum load on board

Minimum f = 200 mm F = 17×LOA+700 (mm)

LOA = Length over all (in m).



B is measured inside fenders

2. MAXIMUM ROLLING PERIOD

The rolling period, measured in seconds, is an indication of the boats' stability. Comparing two boats with the same beam, the one with the lower rolling period is more stable. The maximum rolling period acceptable for good stability is dependent on the beam and the freeboard of the boat. It is given in the table below.

It is based on the formula: $Tr = \frac{K \times B}{\sqrt{GM}}$

(where K = 0.8, Minimum GM = 0.60 + 0.05B − 0.25f)

How to measure the rolling period

① The fish-hold must be empty but the boat must carry a normal amount of fuel and freshwater. The fishing gear and crew must be on deck. The boat must be away from the quay with the mooring lines slack.

② Start the boat rolling by making the crew run from side to side.

③ When the boat is rolling freely, stop the crew amidships quickly and start the stopwatch when the mast is furthest to one side. Count five rolls and take the time. Divide this time by 5 to get the time for one roll. Repeat the same procedure three times and calculate the average time for one roll. Measure freeboard = f (mm) and beam = B (m). If the *measured rolling time is less than what is shown in the table below*, the stability is acceptable.

NOTE: This is only a check. If possible, a complete stability investigation should be done by a naval architect.

		BEAM "B" (in metres)										
		2	2.2	2.4	2.6	2.8	3	3.2	3.4	3.6	3.8	4
		ROLLING PERIOD (in seconds)										
FREEBOARD "f" in mm	200	2	2.2	2.4	2.5	2.7	2.9	3.1	3.2	3.4	3.5	3.7
	400	2.1	2.3	2.5	2.6	2.8	3	3.2	3.3	3.5	3.7	3.8
	600	2.2	2.4	2.6	2.7	2.9	3.1	3.3	3.4	3.6	3.8	4
	800	2.3	2.5	2.7	2.9	3.1	3.3	3.4	3.6	3.8	4	4.2

Heave Oscillations (bouncing up and down)

Waves can resonantly excite a boats bouncing up and down in the water.

Problems:

Wear and tear, e.g. in drill rigs.

Slowing down of the boat.

$$T_{heave} = 2\pi\sqrt{\frac{V_H}{A_H\, g}}$$

$$\approx 0.7s\sqrt{D_H[ft]}$$

A_H = waterline area

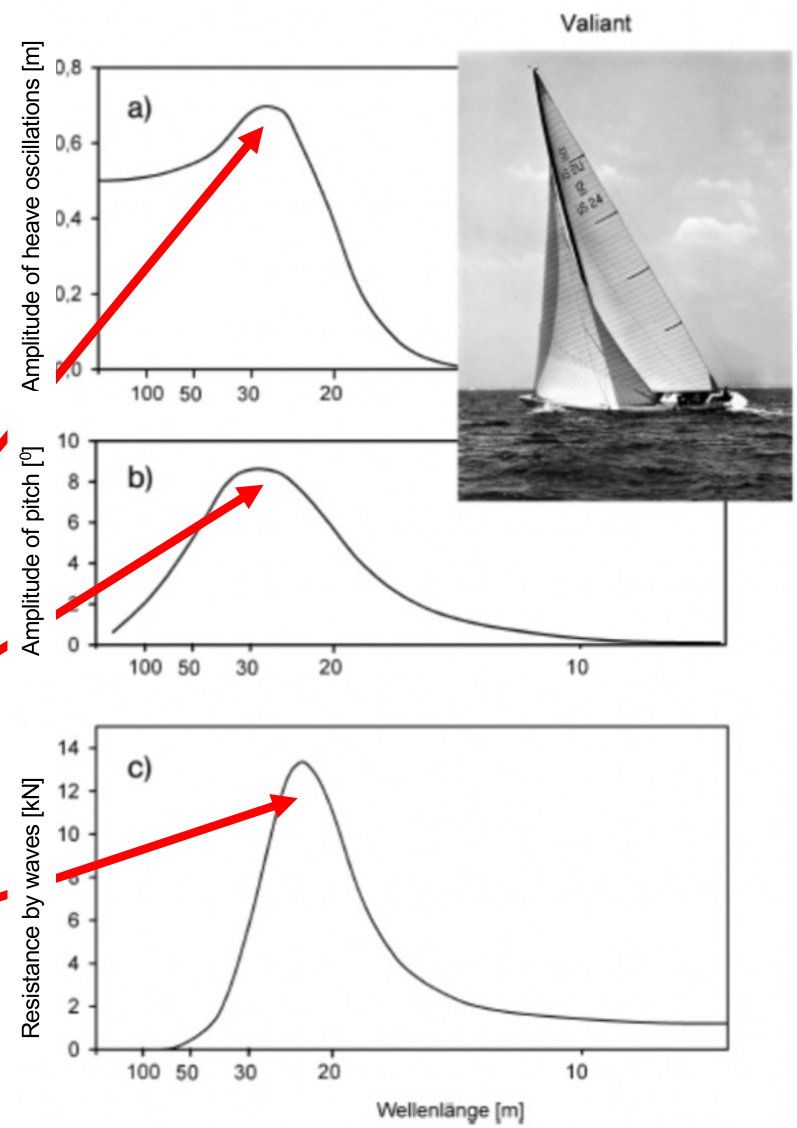
V_H = water volume

D_H = water depth

Resonant waves for heave oscillations.

Resonant waves for pitch oscillations.

➔ Combined resonant resistance and slowdown.



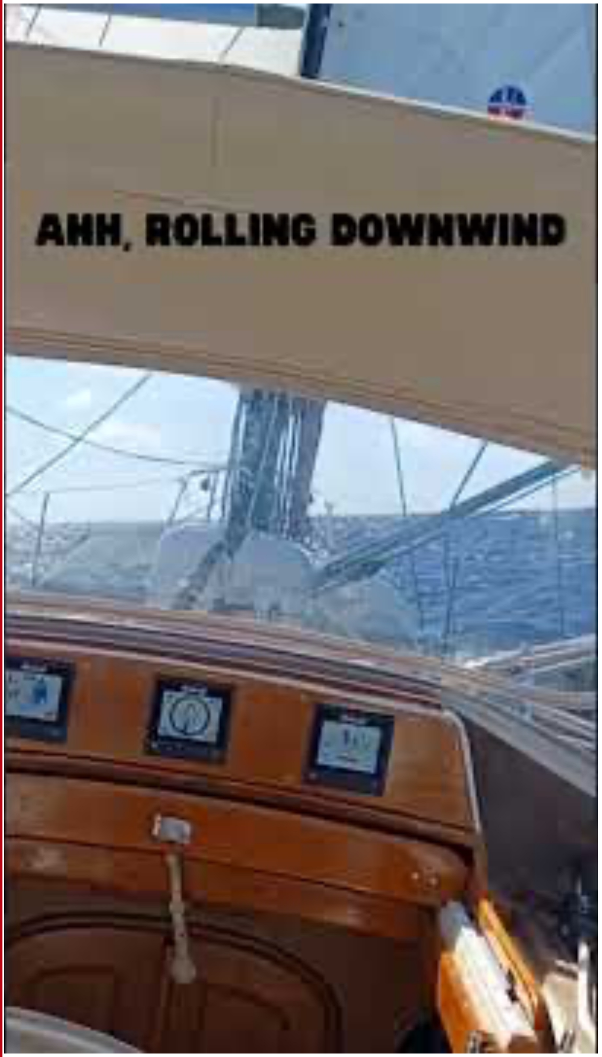
Resonant Oscillations (Rolling and Pitching)

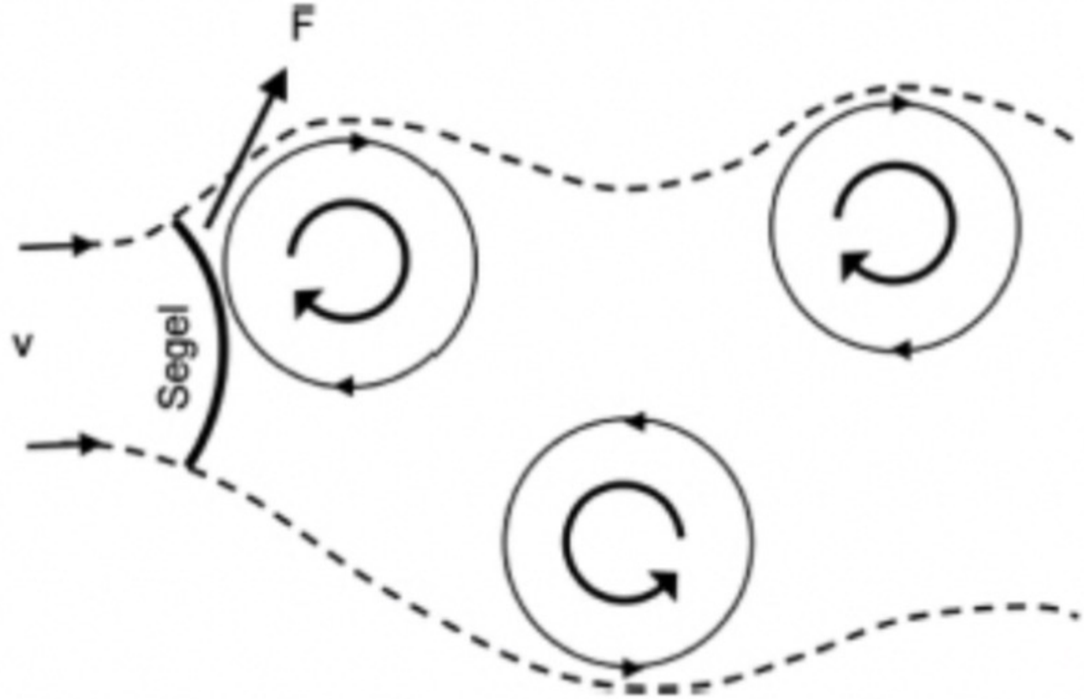
When the time period of the pitching, rolling, or heaving is close to the wave period, the oscillations are resonantly reinforced to large amplitudes.

If this happens to you:

a) Change the time period of the waves by changing your direction or your speed.
b) Change the time period of your boat by changing $\overline{GM}$ by moving masses, typically to lower in the boat.
c) Change the time period of your boat by changing the moment of inertia, typically moving masses more to the center of the boat.
d) Put up sail to **damp** oscillations, e.g., in a motor sailboat.

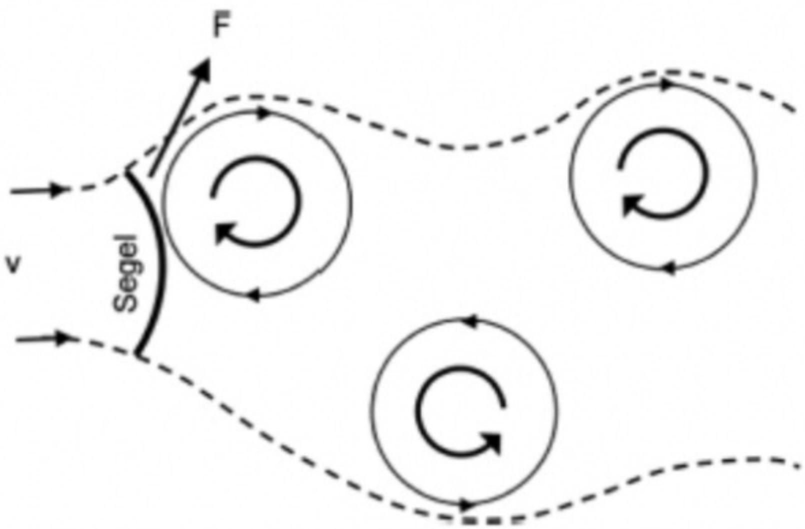
Downwind Rolling – self excited without waves





1) The edges of the sail create turbulences. The one that is a little strong creates a force in its direction and the sail starts moving there.
2) This motion creates an apparent wind that increases this force but reduces the turbulence.
3) When the turbulence on the other side becomes stronger the motion reverses.

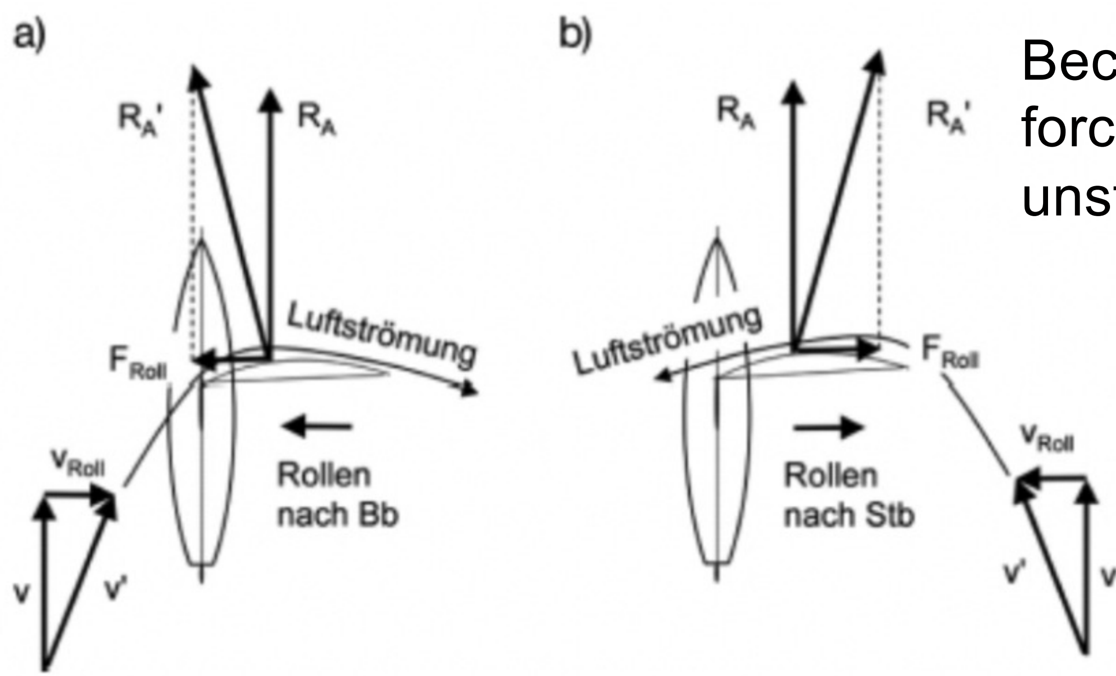
Rolling into Broach



- In a broach one roles so much that the ruder becomes too ineffective to control the boat angle.
- The increased heel increases weather helm that rotates into the wind.
- The wind from the beam and the the added lift from the rotation increases the heel further.

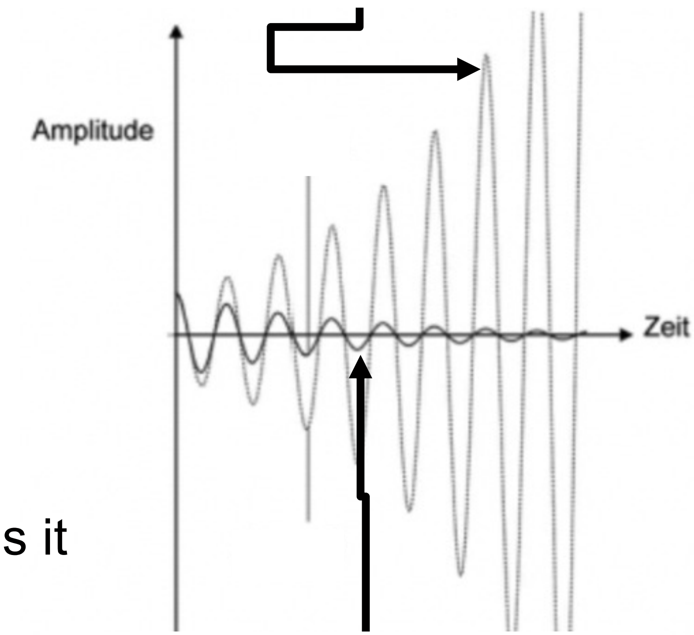
Downwind Rolling

Self excited downwind rolling at $\gamma_w = 90^{\circ}$, with sail angle $\alpha = 90^{\circ}$.



Because the motion increases the force that created it, it builds an unstable feedback system.

Unstable oscillation



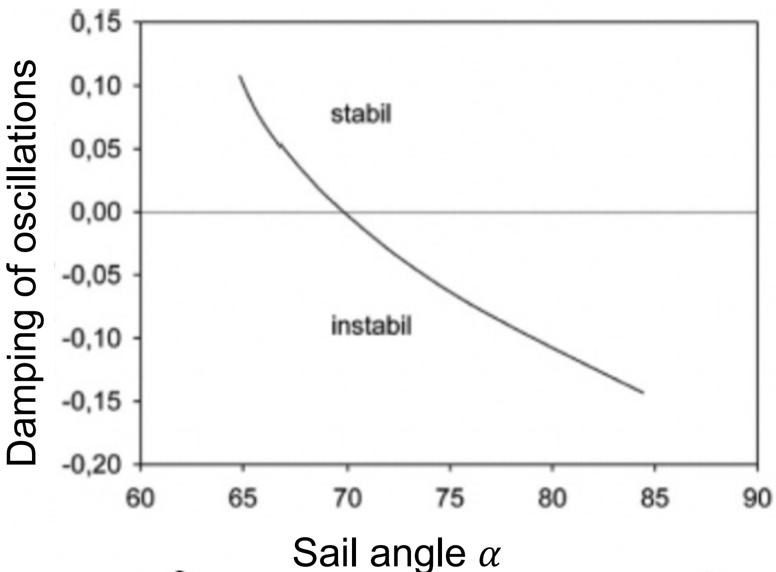
Damped oscillation

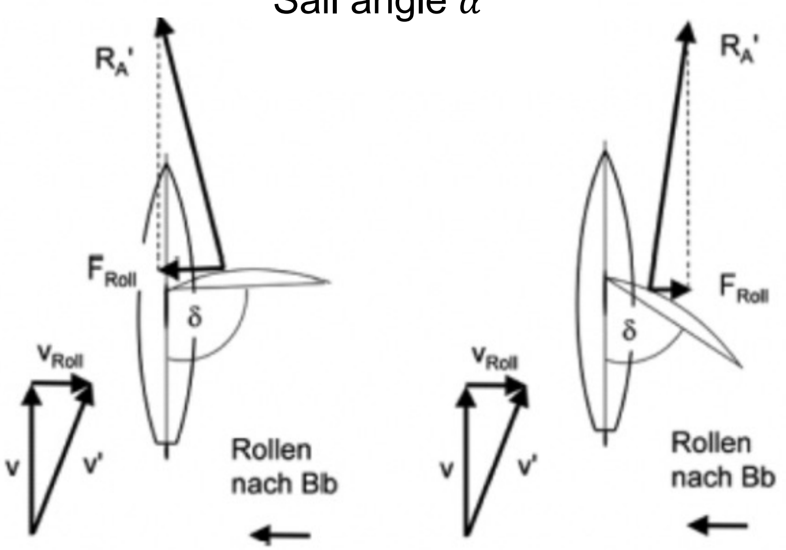
1) To avoid instability, one can change either course (γ_w) or sail angle (α).

2) Sailing with these oscillations can be faster, as it uses self created apparent wind.

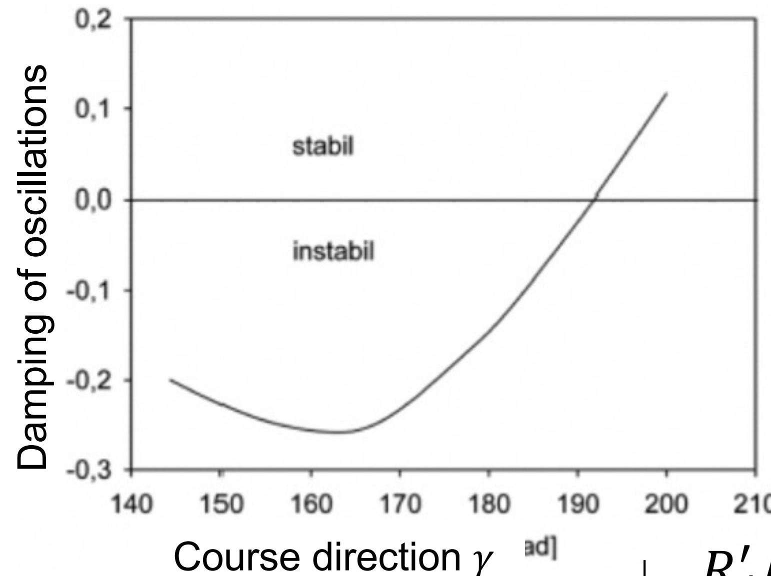
Avoiding Unstable Downwind Rolling

Instability control by sail angle:

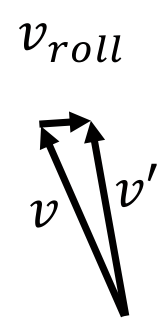


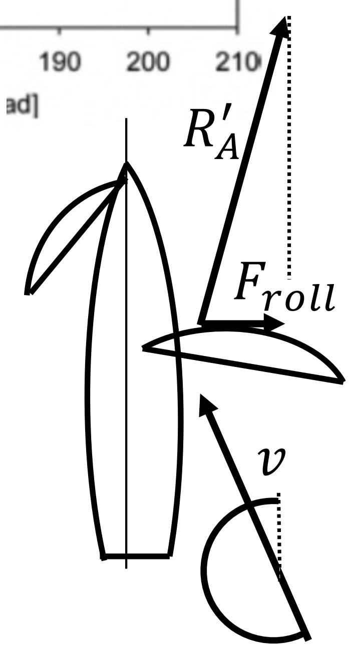


Instability control by wind angle:



This can be used for butterfly sailing.

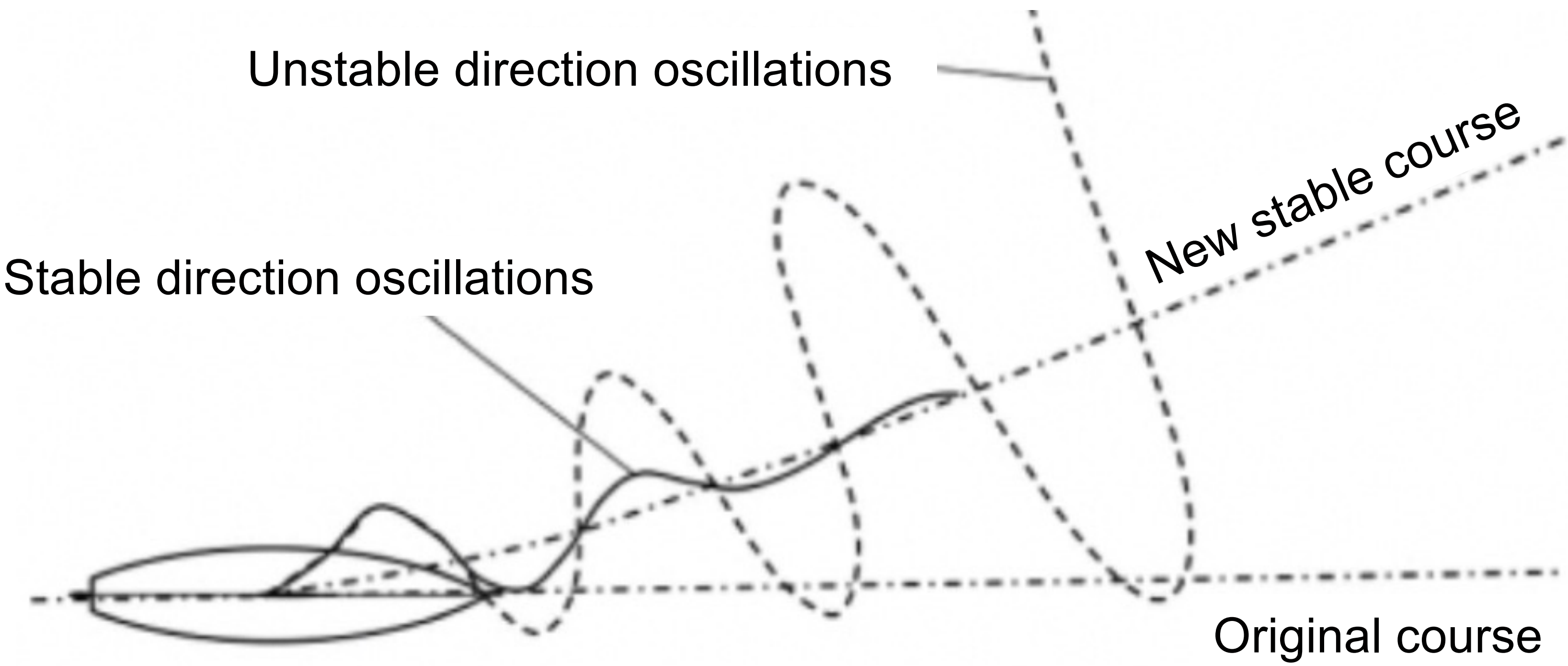




Direction instabilities – around the vertical axis

Joshua Slocum, the first to sail alone around the world, had no auto steering and simply fixed his rudder for a desired course. This only works for long-keeled sailboats.

Narrow deep keels produce faster boats but produce less directional stability. Continuous human steering or autosteering are then needed to avoid directional instabilities after a sudden change of wind.



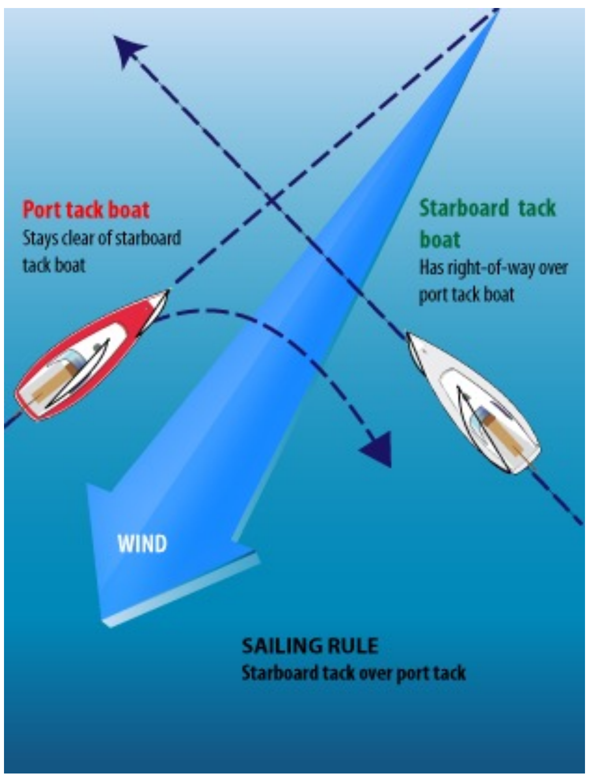
Right of Way on Water

The boat with right of way is called the **stand-on vessel**.

The opposing boat is called the **give-way vessel**.

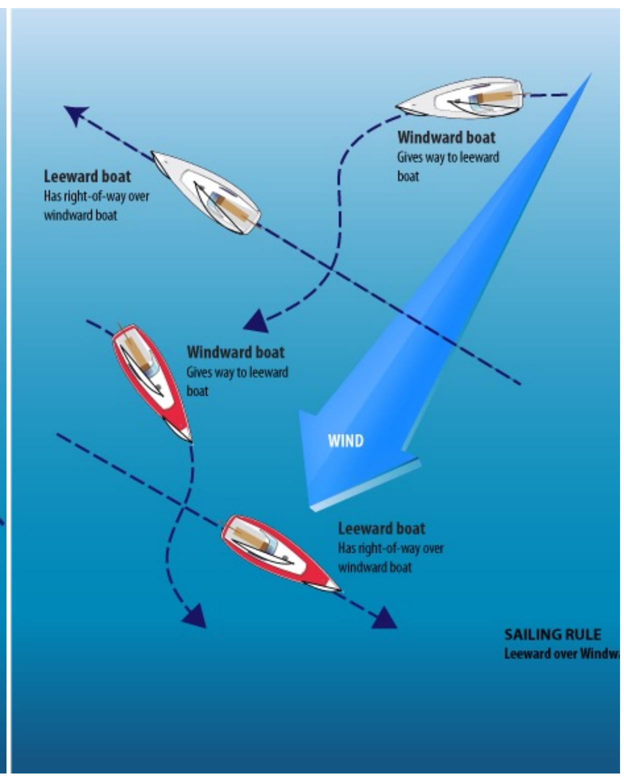
Opposite tack rule:

Starboard tack has right of way.



Same tack rule:

Leeward boat has right of way.



- **Overtaking rule**: The boat ahead has right of way, even if it is a power vessel.
- Vessels when under sail have right of way over power boats.
- Ships, tugboats with tows, commercial ferries and fishing vessels have right of way over sailing vessels.
- Sailing vessel using its motor is considered a power vessel even when sails are hoisted.

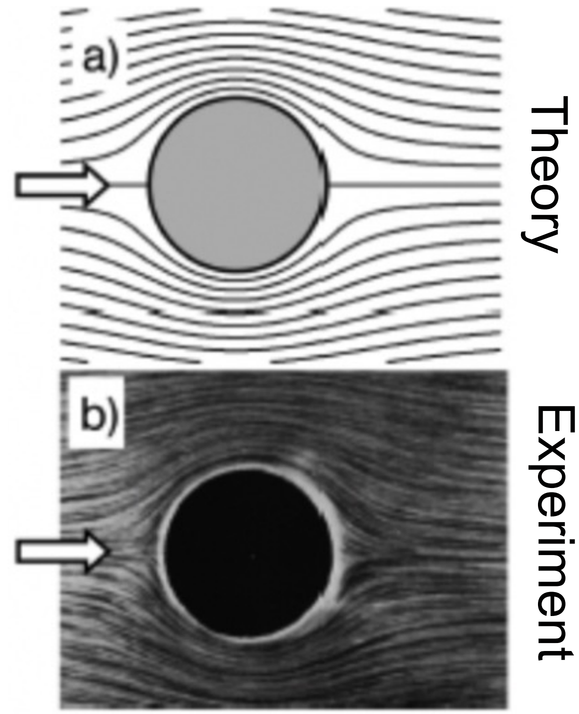
Aerodynamics and Hydrodynamics

The water we sail on can be treated as an uncompressible fluid. Its volume does not change significantly during the pressure variations of weather and environment.

While air is compressible in pumps, and air pressure waves transport sound, if we average over times longer than sound periods, also air can be treated as uncompressible in the natural environment.

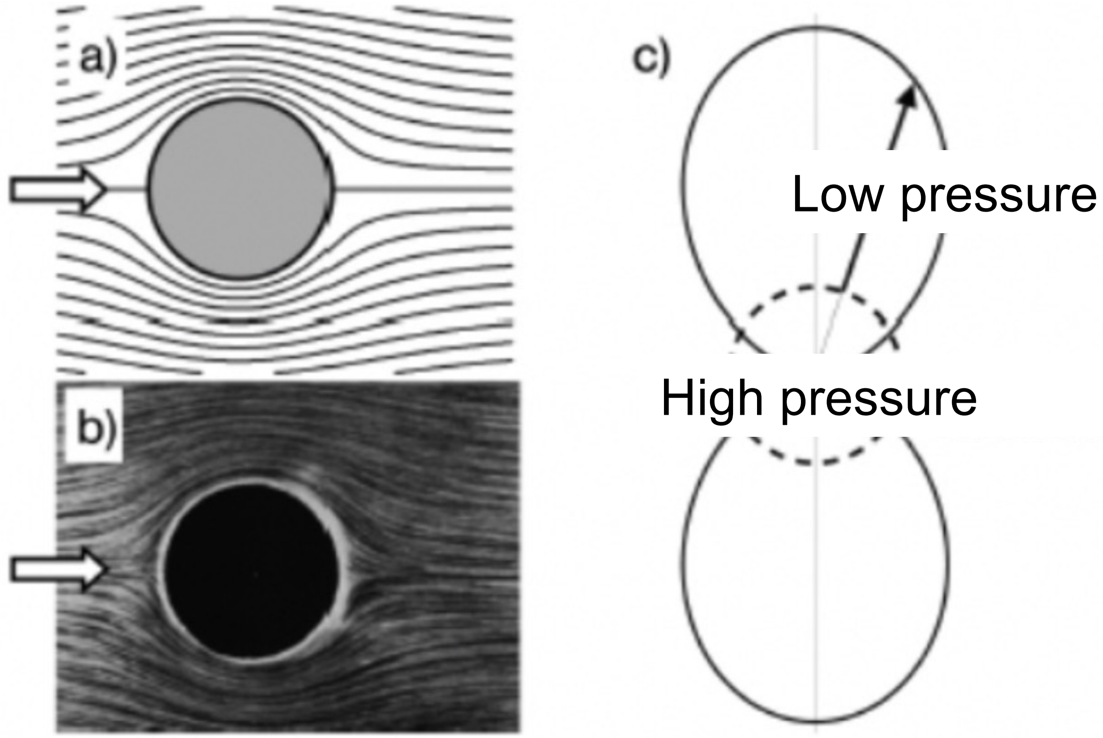
The phenomenon of flowing air around the sail and of flowing water around the keel become identical.

This flow has no sources or drains
where flow lines would end or start.
And flowlines cannot cross.



Aerodynamic paradox (a)

The fluid is pressed from the high-pressure to the low-pressure region.



If pressure from velocity variations were the sole reason for forces on sail and hull, there would be no force on the cylinder, because velocities are the same in front and behind of it.

➔ A rotating spoon in a coffee would not twist and mix the coffee.

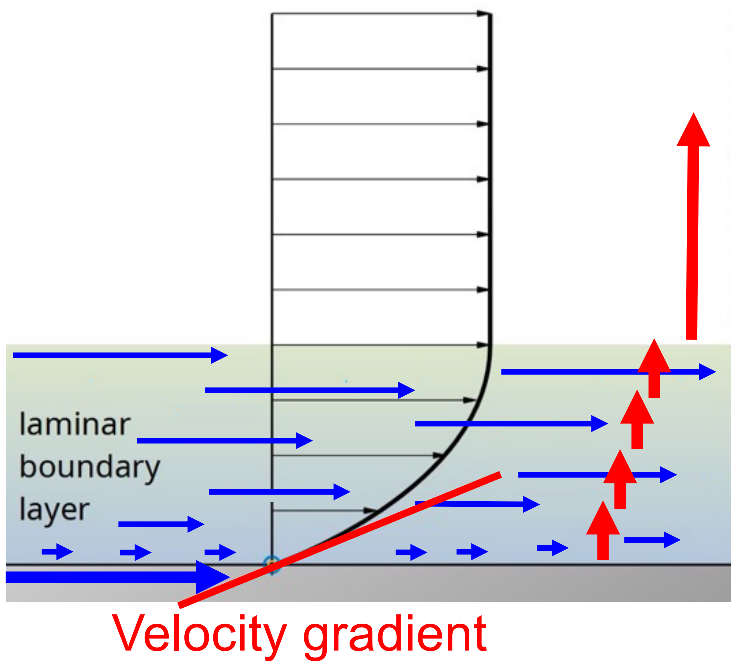
Internal Friction

For forces from the incompressible fluid (air or water) to be transferred to the sail or keel, two things need to happen:

(a) The fluid particles attach themselves to the boat parts.

(b) There is friction between the fluid particles

- ➔ The fluid right next to the boat parts gets dragged along with the boat.
- ➔ The fluid some distance from the boat has the velocity of wind or water.
- ➔ Displacing or rubbing one layer of fluid against the other costs energy and therefore puts forces on the boat.



- ➔ Friction forces & velocity associated pressure forces

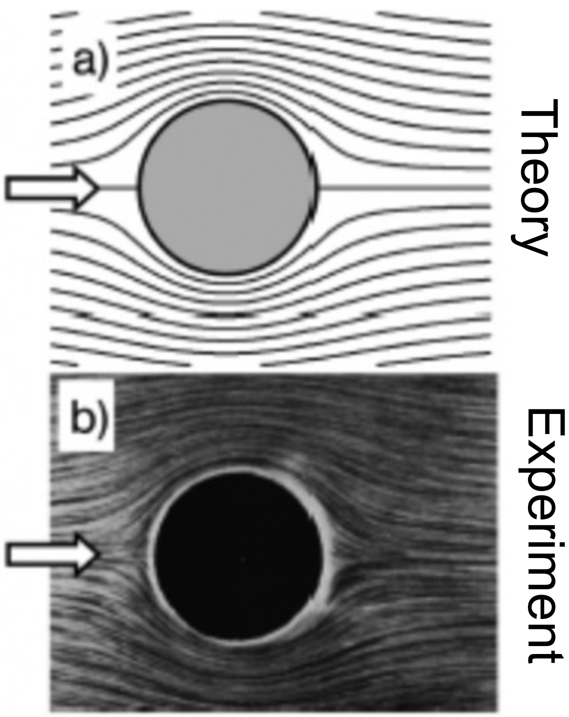
Aerodynamic paradox (b)

1) There are velocity-associated pressure forces on the boat, even thought there is no liquid velocity directly at the boat.

   Explanation: Small pressure forces between liquid layers add up.

2) There are friction related drag forces even though each molecule only has a tiny attachment force to the boat.

   Explanation: The small forces from many molecules add up.



➔ Friction and pressure forces in laminar flow.

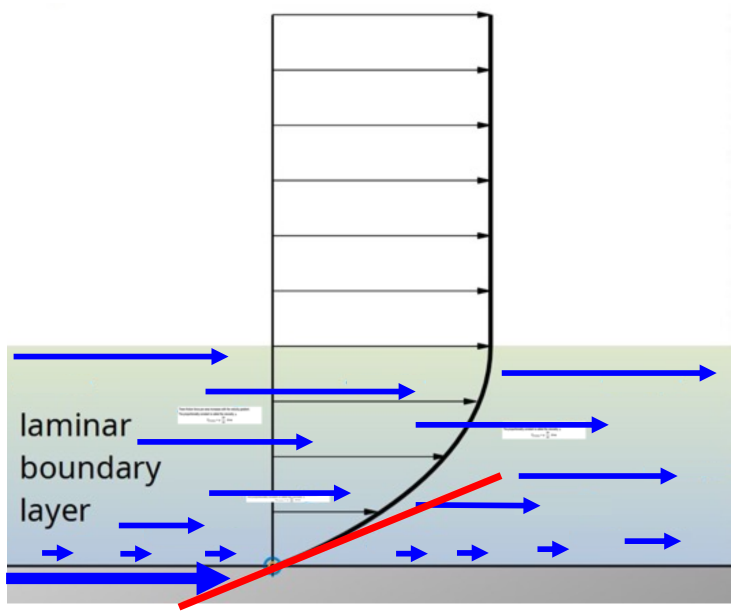
➔ In laminar flow, friction forces tend to contribute more to the drag than pressure forces

Internal Forces and Viscosity

There friction force per area increases with the velocity gradient.

The proportionality constant is called the viscosity: μ.

$$F_{friction} = \mu \cdot \frac{\Delta v}{\Delta x} \cdot Area$$



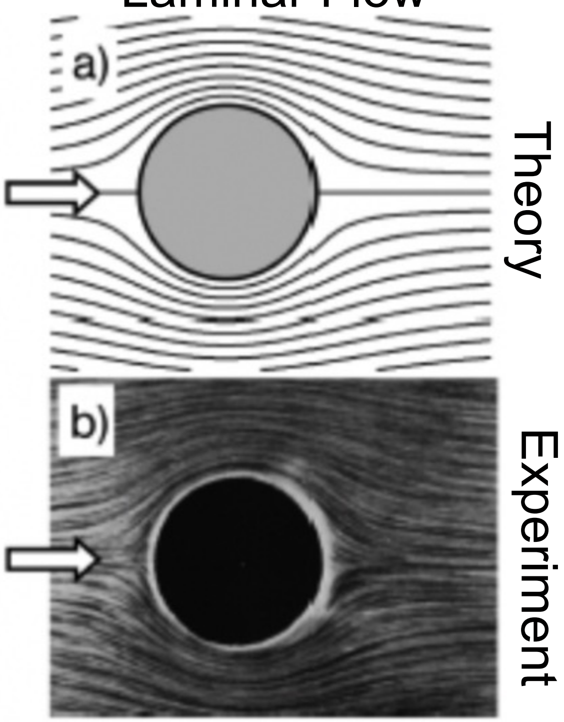
Faster flow ➔ More friction force

Note: Often one plots $F_{friction}/v^2$ which reduces approximately with $1/v$.

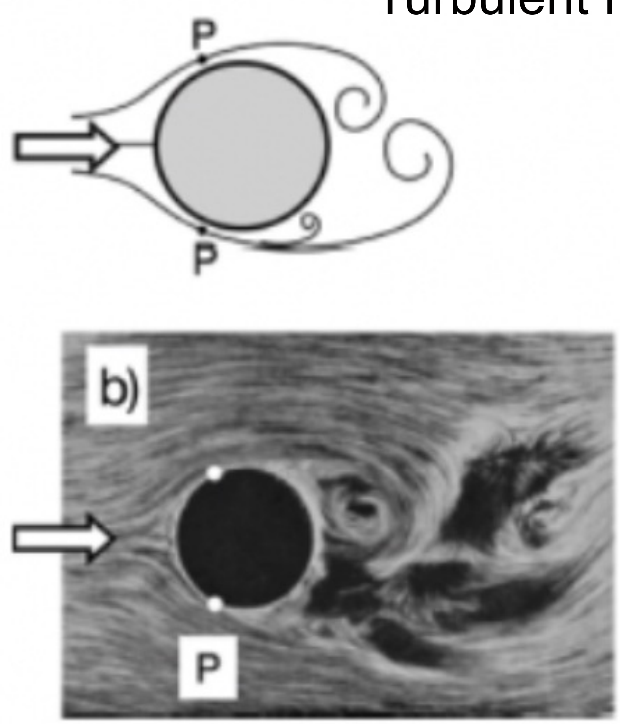
Turbulence and its Forces

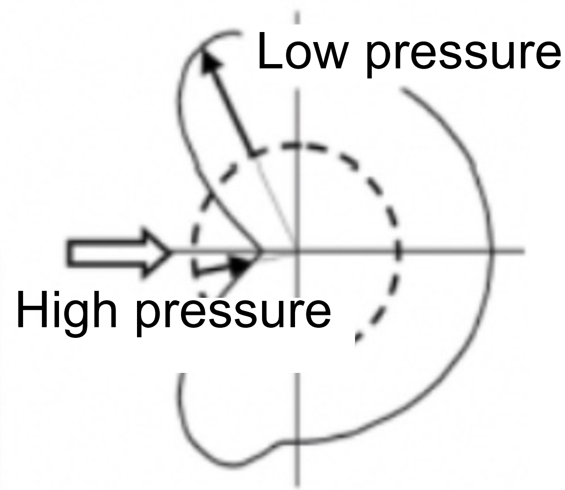
1) Internal forces in the fluid and attachment to the surface give rise to turbulences.
2) The velocities in turbulences produce additional pressures.
3) The associated energy goes into rotation of the vortexes.
4) Pressure forces tends to contribute more to the drag than friction.

Laminar Flow



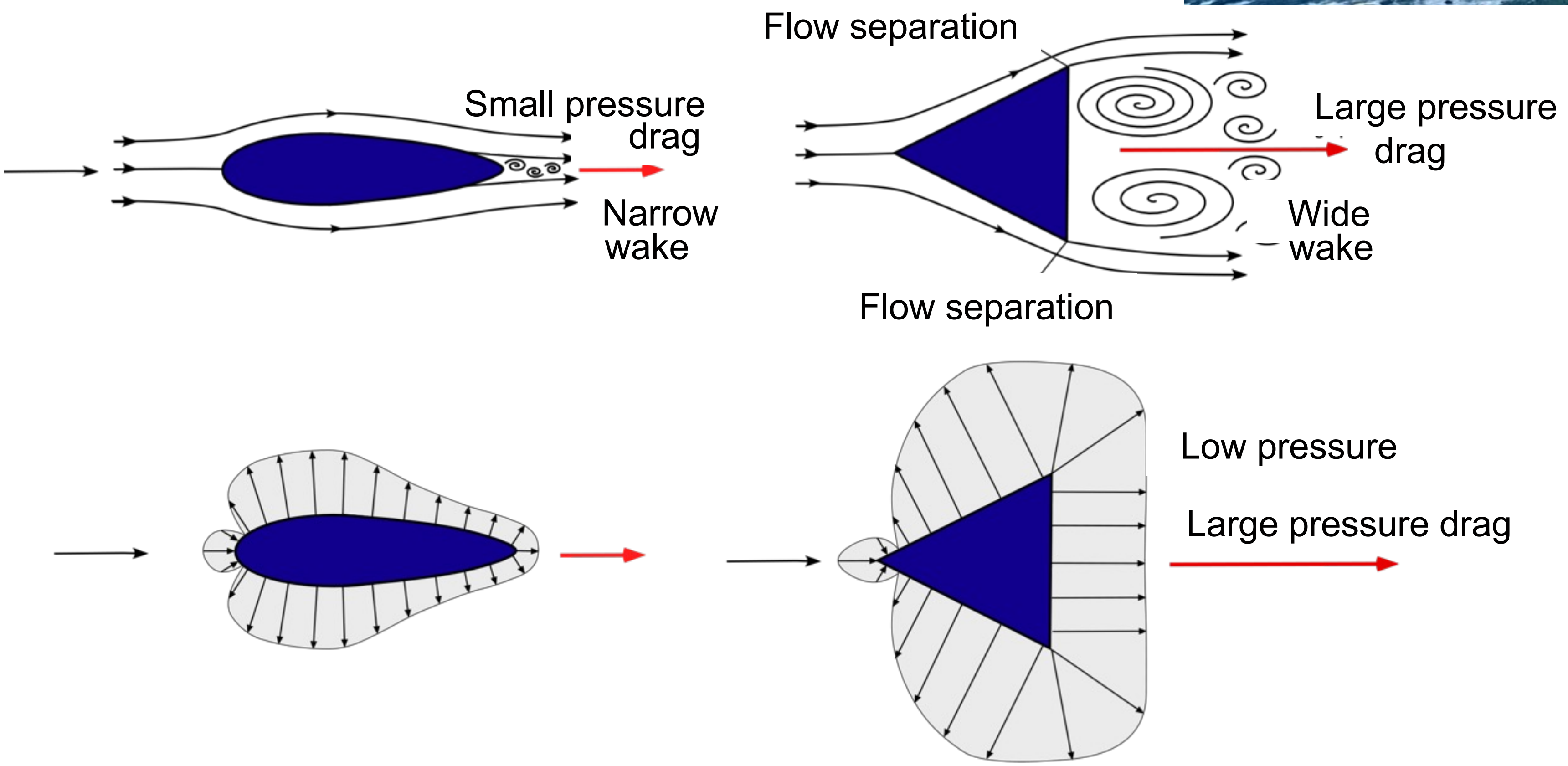
Turbulent Flow





Boat designs against turbulences

1) Long, narrow, streamlined boats have a narrow region of turbulences and thus loose less energy into vortexes.
2) Compromise: A longer hull has more wet area and more drag from laminar flow.

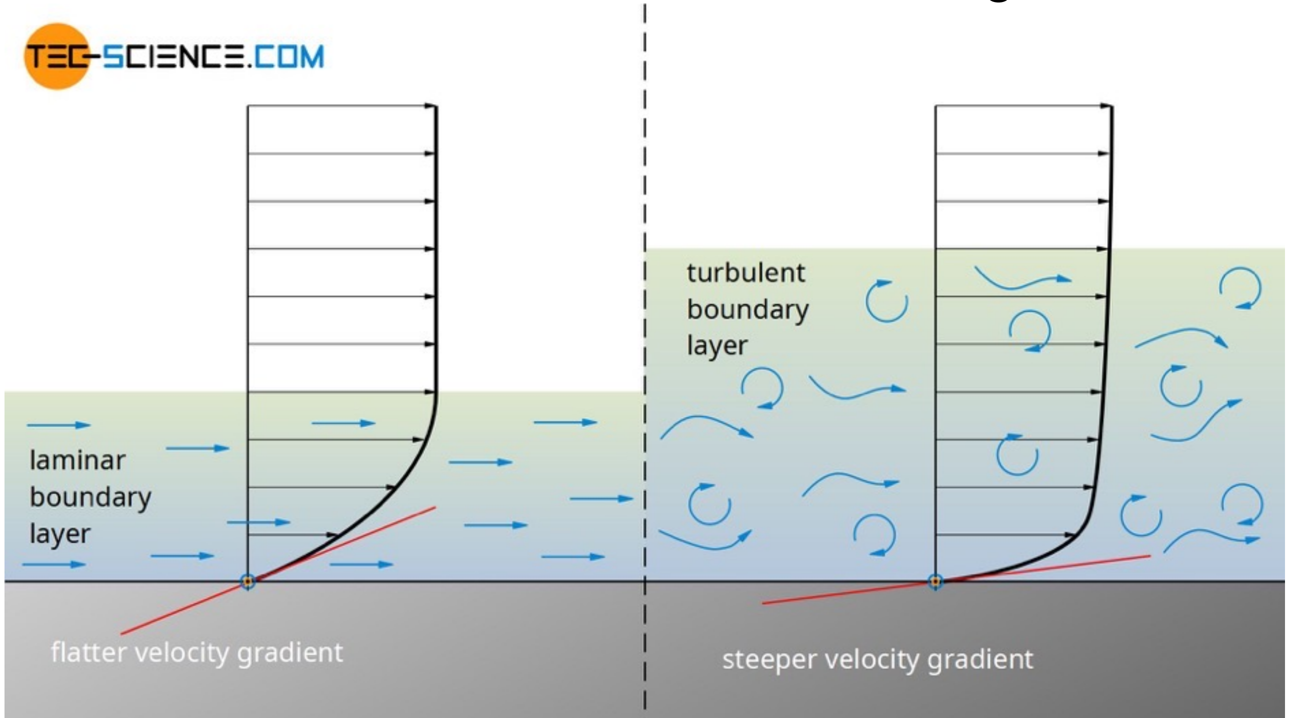


Thickness of Boundary Layers

Turbulences communicate velocities between regions of the fluid.

→ The distance between fluid regions that go with the boat and those that have the wind or water velocity is thinner with turbulences.

→ The region with turbulences is thicker than the region with laminar flow.

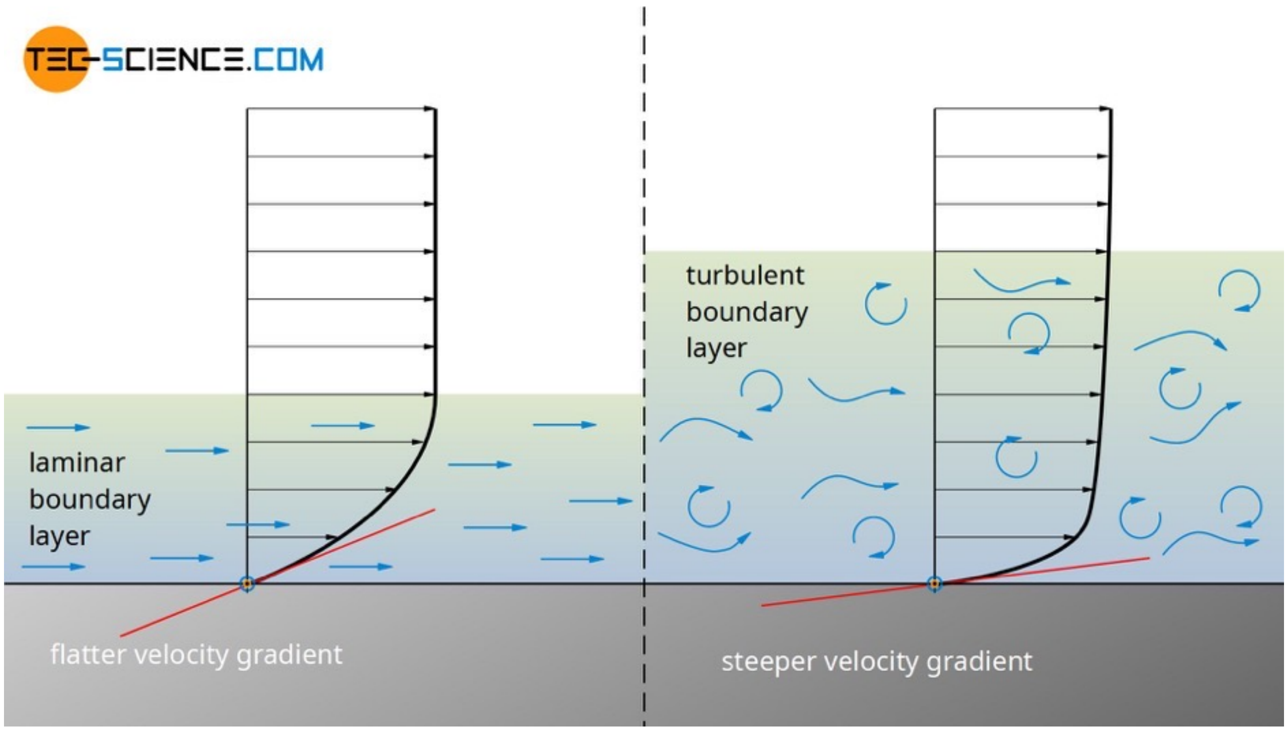


Internal Forces and Viscosity

There friction force per area increases with the velocity gradient.

The proportionality constant is called the viscosity.

$$F_{friction} = \mu \cdot \frac{\Delta v}{\Delta x} \cdot Area$$



Faster ➔ More friction force

Turbulence ➔ More $\frac{\Delta v}{\Delta x}$ ➔ More friction force

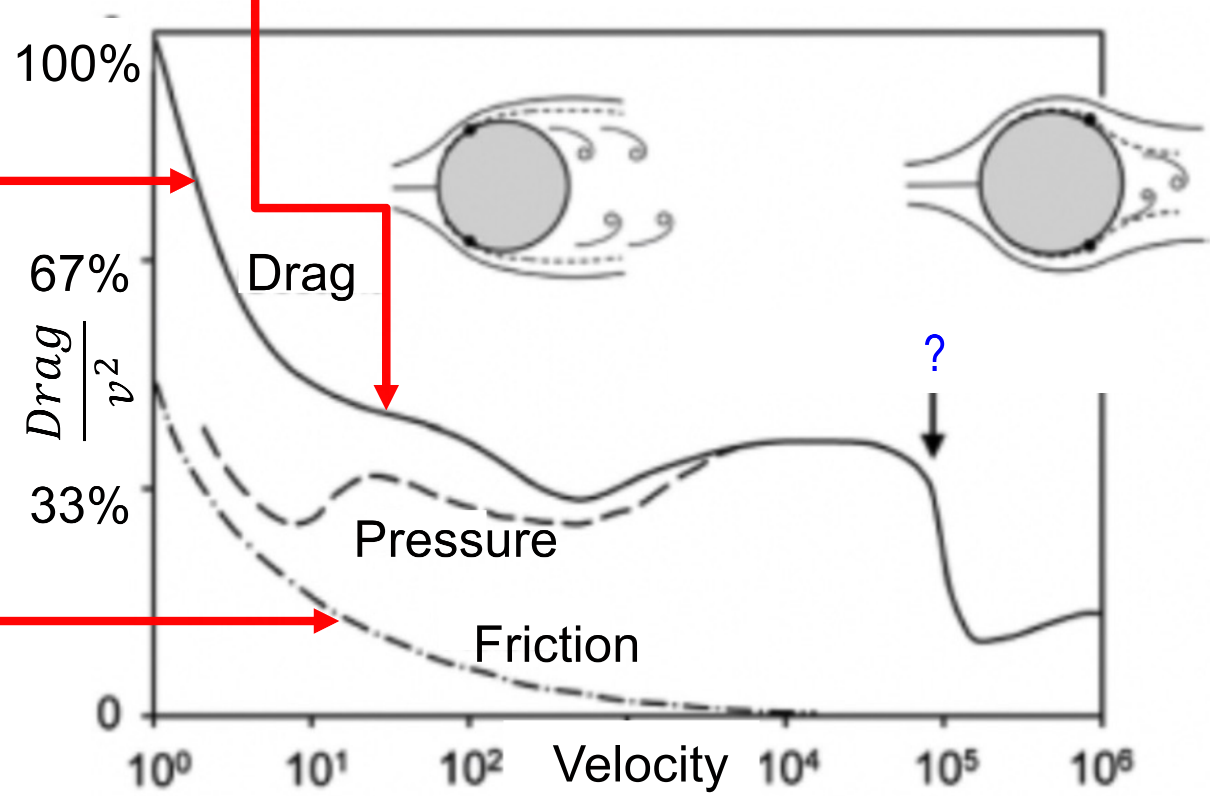
Friction when flow separates from the surface

Laminar flow: $F_{friction}/v^2$ goes down with $1/v$.

Turbulent flow along the full surface:

$F_{friction}/v^2$ goes down with $1/v$.

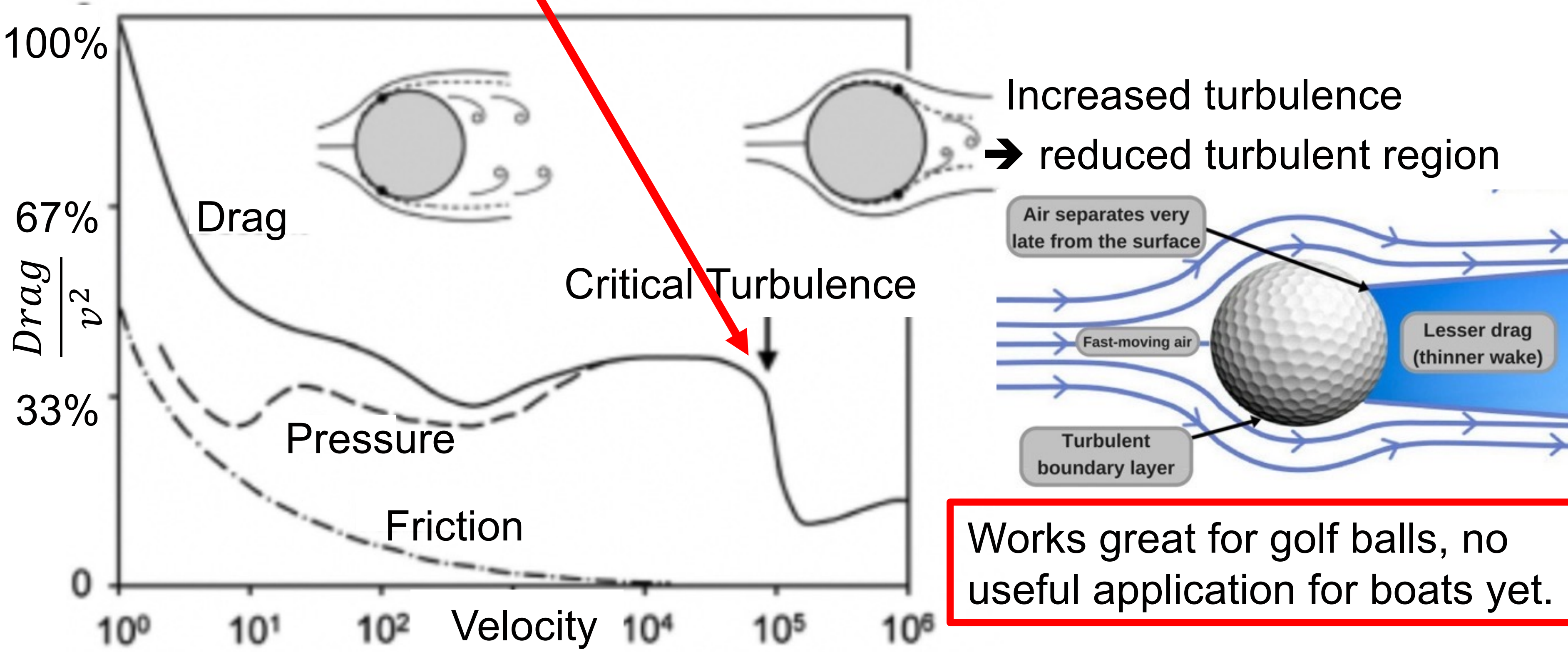
Turbulence separates from surface increasing lost energy



Rare advantages of Turbulences

Because the outside fluid velocity of air or water is achieved closer to the surface, the overall flow pattern can look more like laminar flow with increased turbulence.

➔ The reduced turbulent region can reduce the overall drag.



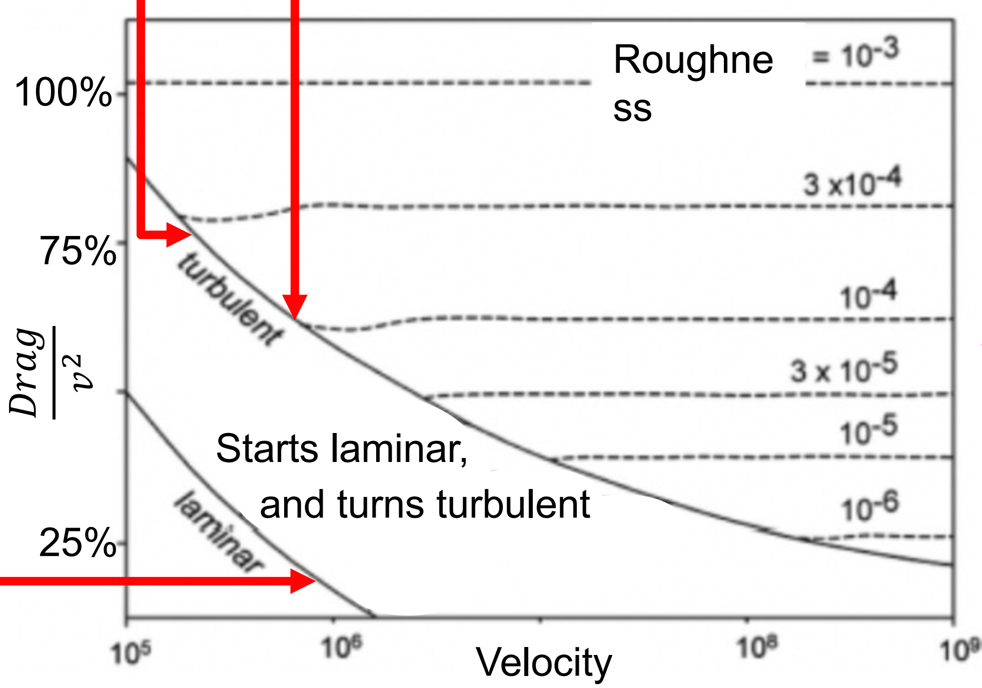
Works great for golf balls, no useful application for boats yet.

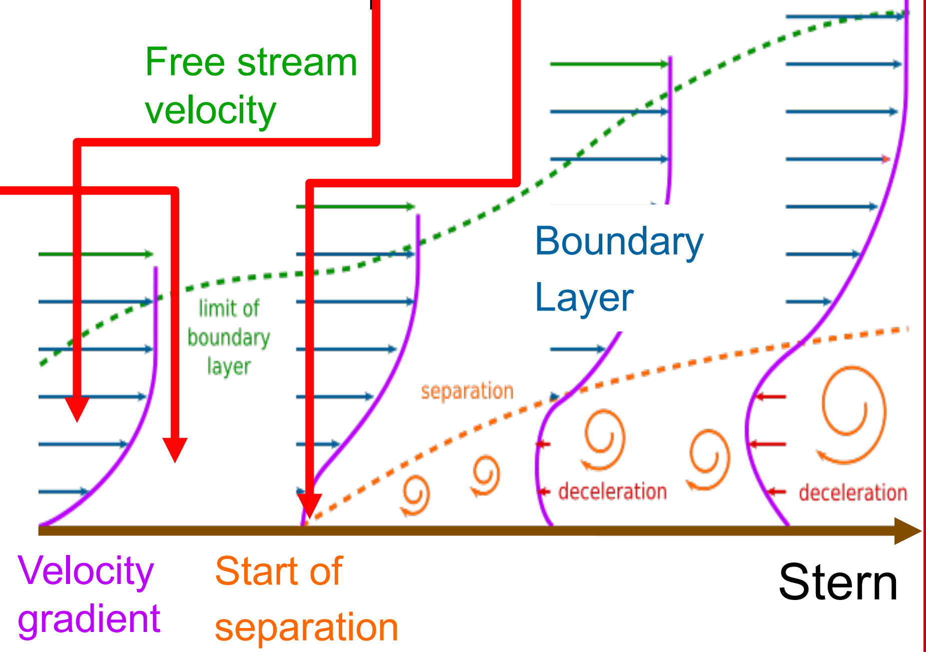
Separation of turbulent flow along the boat length

Laminar flow: $F_{friction}/v^2$ goes down with $1/v$.

Boundary layer along the full surface

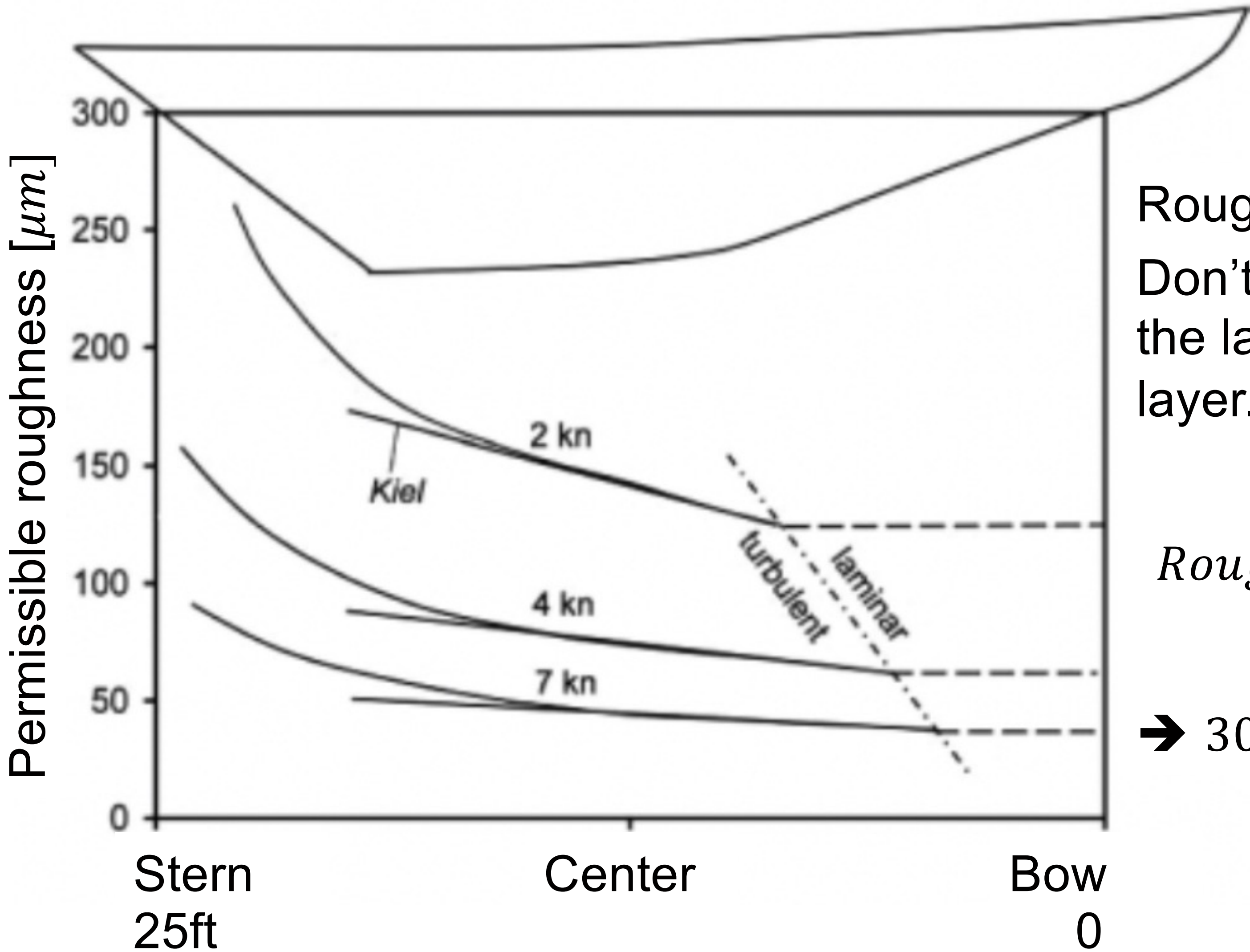
Turbulence separates from surface





With increasing v, the separation moves to the bow and the turbulent layer gets wider ➔ $\text{drag} \propto v^2$

Advisable Roughness along the Hull



Rough rule:

Don't penetrate out of the laminar boundary layer.

$$Roughness < \frac{0.2}{v}\frac{mm}{knot}$$

➔ 30μ for 7kn.

Note: the diameter of one hair is about 100 μm.

Turbulences Transport Energy

Vincent van Gogh's masterpiece "The Starry Night" 1889

“(A study) examined the scale (the) 14 main whirling shapes to understand whether they aligned with physical theories that describe the transfer of energy from large- to small”

“the sizes of the 14 whirls or eddies in “The Starry Night,” and their relative distance and intensity, follow a physical law (of) fluid dynamics, Kolmogorov’s theory of turbulence.”

https://www.cnn.com/2024/09/19/science/starry-night-van-gogh-hidden-math/index.html

Scaling sails in air

The forces that produce the flow pattern are due to the moving mass of the particles and due to internal friction.

Question: Can one produce a small model of a sail or a hull but change the velocity of the air or the water, so that the ratio between these forces stays the same? The flow pattern would then stay the same.

Result:

1) When the velocity is doubled
   a) the force from moving masses becomes 4 times larger: Particles have twice the momentum, and twice as many particles move per second. ➔ $F_{mass} \propto v^2$.
   b) Friction forces between two plates double.
2) If the size of the setup is doubles, friction forces increase by 2, because the area doubles – increasing friction by 4, and the distance between air layers doubles, reducing friction by 2. ➔ $F_{friction} \propto v \cdot size$.

➔ Increasing v by the same as decreasing the size makes the same flow.

Sail models in water, or hull models in air ?

Air and Water are both incompressible fluids and should therefore behave the same. But they have different densities and viscosities.

Question: Under what conditions are is the same flow pattern produced by air and water?

How can one change the size of the wing and the speed of the water to get the same flow even though density and viscosity are different?

Or can I test a hull model in an air channel with an air velocity that produces the same flow pattern?

Flow model of an airplane wing in water

The Reynolds Number

Result: If the ratio of viscosity forces (from forces between molecules) and kinetic force (from moving masses) are the same, then the flow pattern is the same.

This ratio is the Reynold's number and scales like: $R = \frac{\rho}{\mu} \cdot v \cdot size$

$size$: a characteristic length, e.g., length of the sail.

v: speed of the fluid

ρ: density

μ: viscosity

Because $F_{mass} \propto \rho \cdot v^2$, $F_{friction} \propto \mu \cdot v/size$

$\frac{\mu}{\rho}$ is 15 times larger in air than in water.

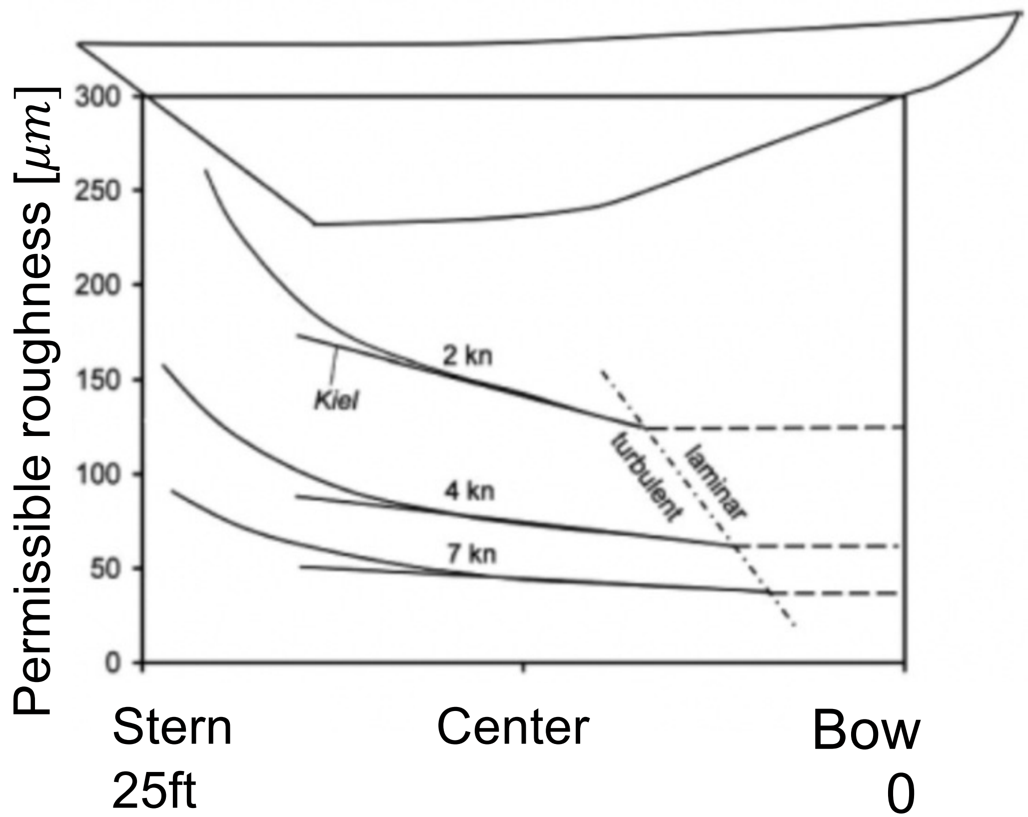
➔ A hull model in air produces the same flow pattern as in water, if the air velocity is 15 times larger.

Flow model of an airplane wing

Advisable Roughness along the Sail

$R = \frac{\rho}{\mu} \cdot v \cdot size$ and $\frac{\rho}{\mu}$ is 15 times larger in water

➔ roughness on the sail can be 15 times larger than on the keel.



Rough rule:

Don't penetrate out of the laminar boundary layer.

$$Roughness < \frac{3}{v}\frac{mm}{knot}$$

➔ $0.5mm$ for 6kn.

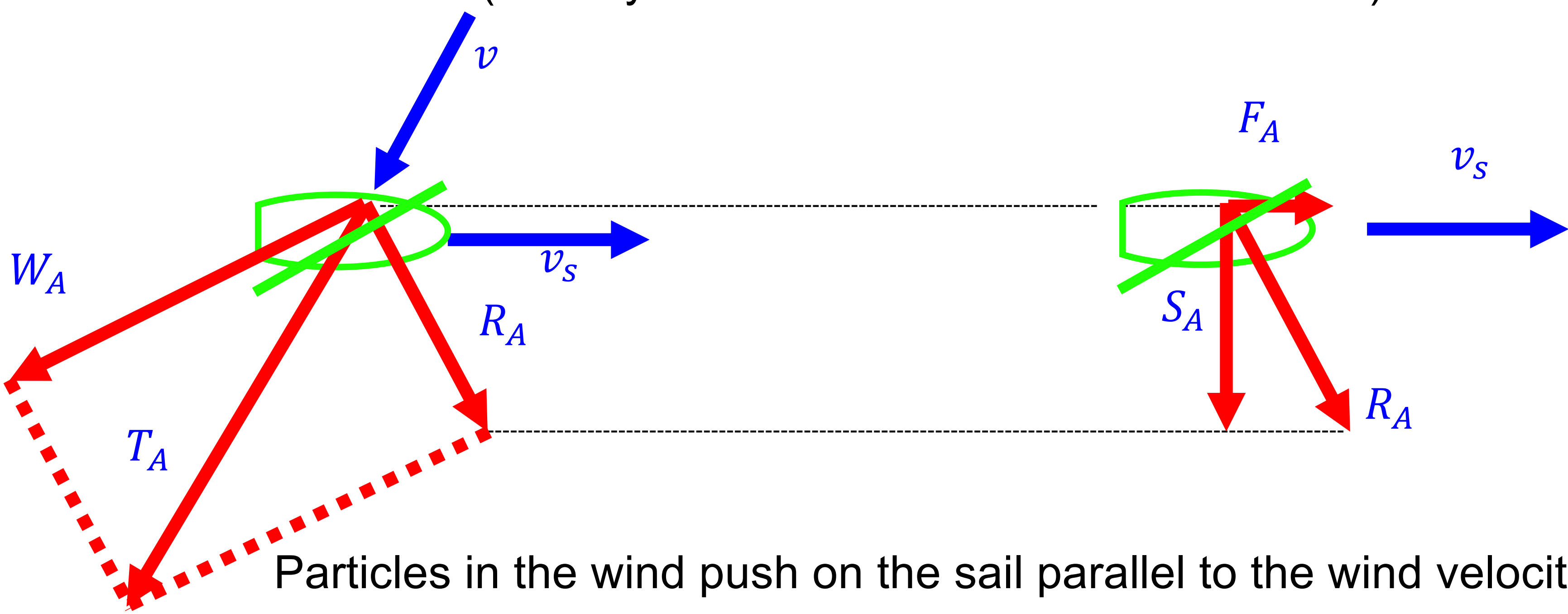
➔ Thread seams on a sail are usually permissible.

Note: the diameter of one hair is about 100 μm.

The wrong Lift and Drag on Wing, Sail, or Keel

Common but **wrong** concept

1) The momentum model (already mentioned in 1673 and often since)



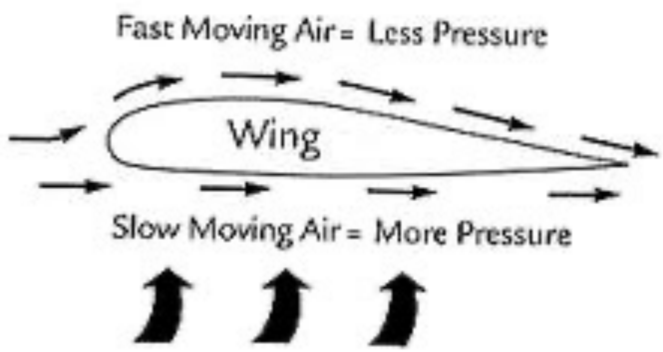
Particles in the wind push on the sail parallel to the wind velocity v, but the wind glides along the sail and only a force R_A perpendicular to the sail is created. This force can be decomposed into two forces, F_{A} and S_{A}.

This model might work in space where occasionally none-interacting particles strike a sail. It is **wrong** in air, where particles interact with each other and establish pressures. An **individual particle treatment is therefore not appropriate**.

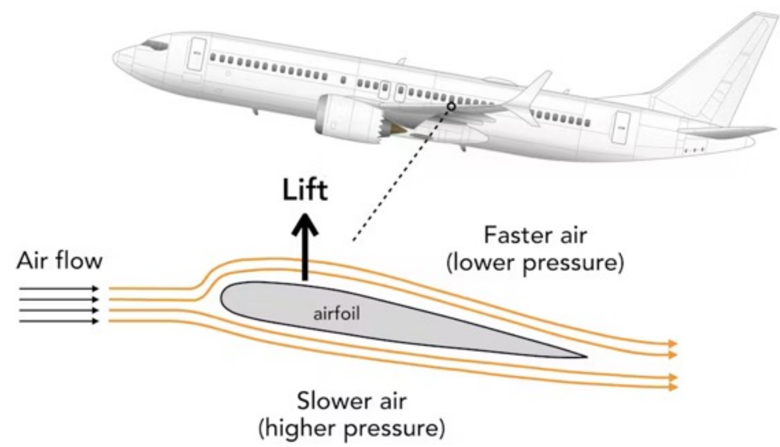
The wrong Lift and Drag on Wing, Sail, or Keel

Common but **wrong** concept

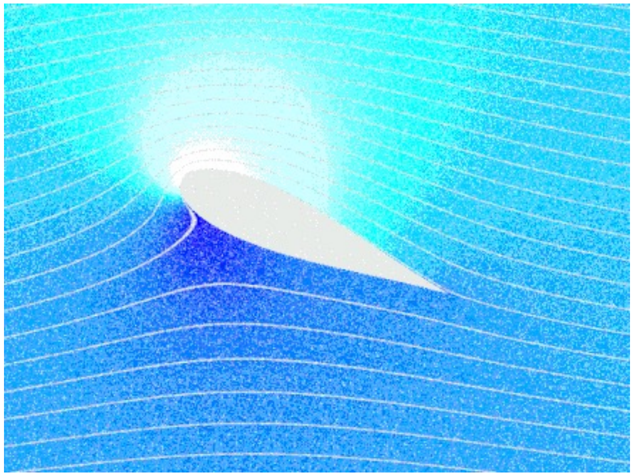
2) The path around the curved part of a wing is longer, air on this path must be faster to arrive at the tail of the wing together with the air on the shorter path. The higher velocity is associated with less pressure.



It is **wrong** because there is no reason why th path along the top and the bottom of the sail needs **to take the same time**. Usually, the top path is much faster. Also, this would not explai how a tilted symmetric wing can producer lift.



The Lift and Drag on Wing, Sail, or Keel – really !

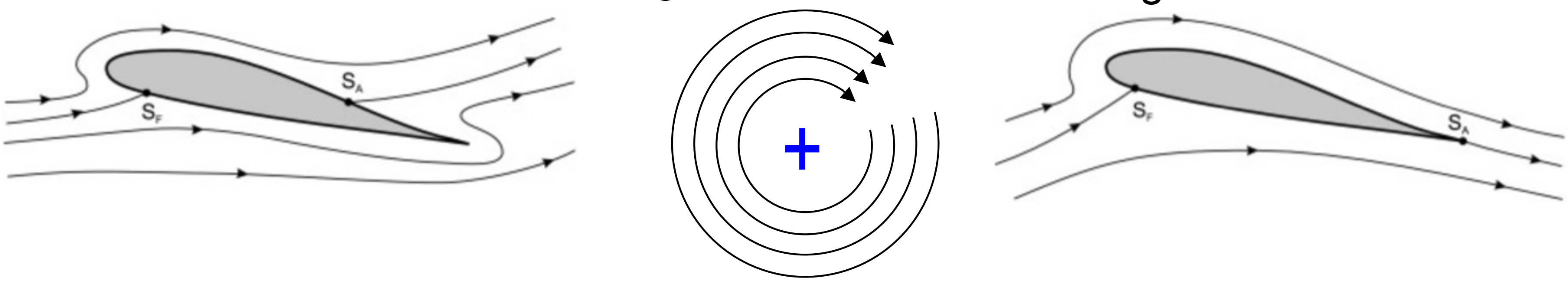


How does a wing really work?

Kutta condition: in steady state, flow leaves a wing's sharp trailing edge smoothly.

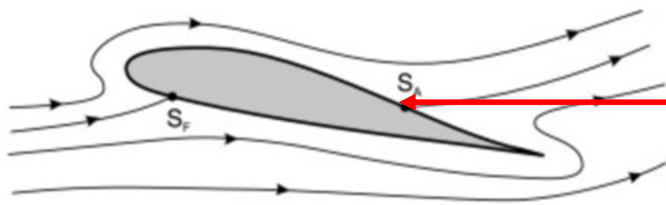
Solution for incompressible fluids without circulation around the wing:

To satisfy Kutta condition, a circulation around the wing needs to be added:

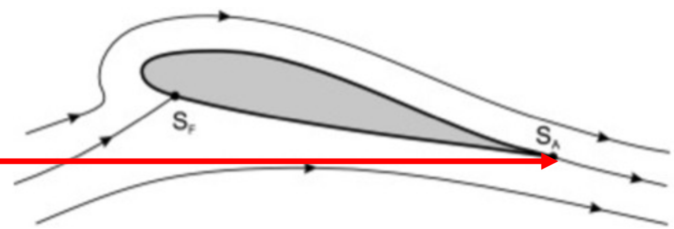


At steady state, there is an air rotation around a wing, which makes particles reach the back much faster over the top than over the bottom. The larger velocity creates less pressure, resulting in lift.

Establishing rotation around Wing, Sail, or Keel

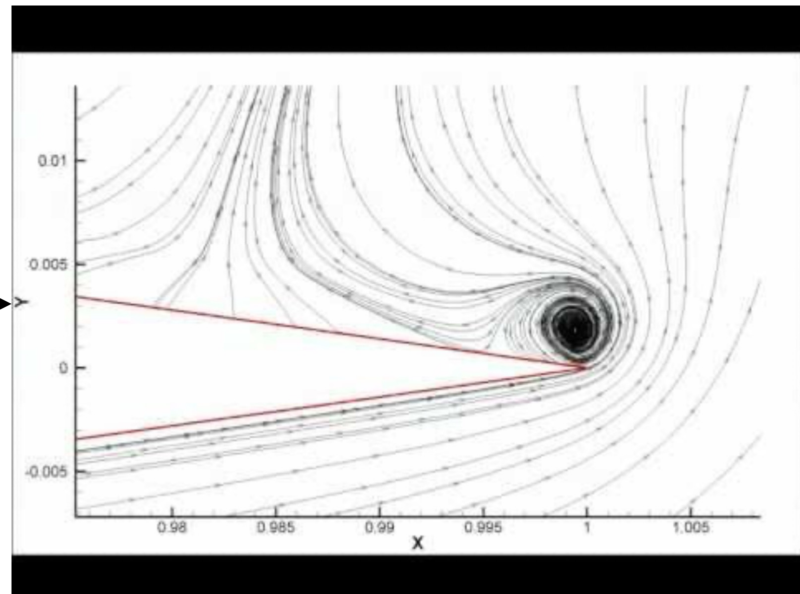


Stagnation point moves back

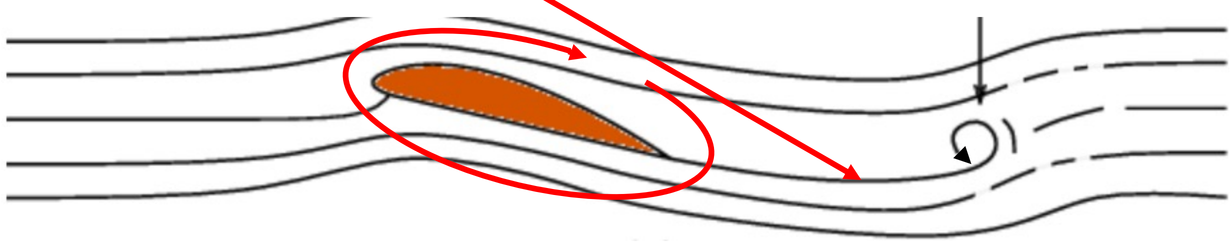


Start of air flow past the wing

Steady state



Turbulent boundary

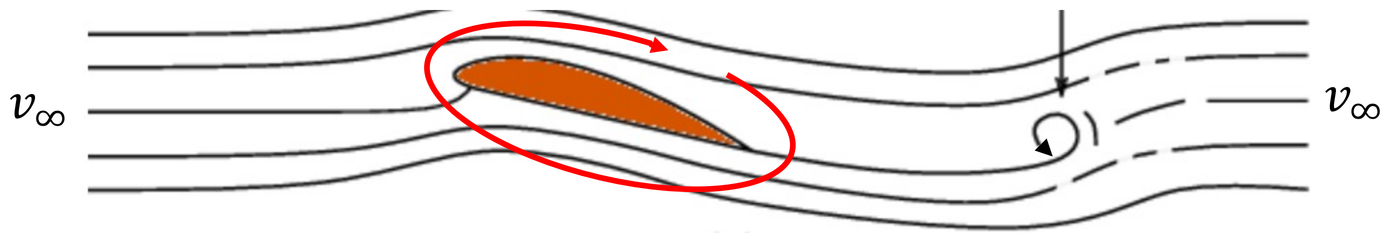


With increasing velocity, the vertex separates to a distance behind the wing.

The Kutta-Joukowski Formula for Lift

Note that it takes a while for the circulation to be established. Every change of the sail changes the stagnation point, which then must move back to the trailing edge again. It takes about 6 sail or rudder length to reach steady state to 90%.

➔ **Sail and rudder changes changes should generally be very slow should generally be very slow** to not disturb the circulation too much.



While lift per length L/b is created, a circulation pattern is established that is independent of the wing shape (density ρ, velocity ρ, circulation $\Gamma = \oint v\, ds$):

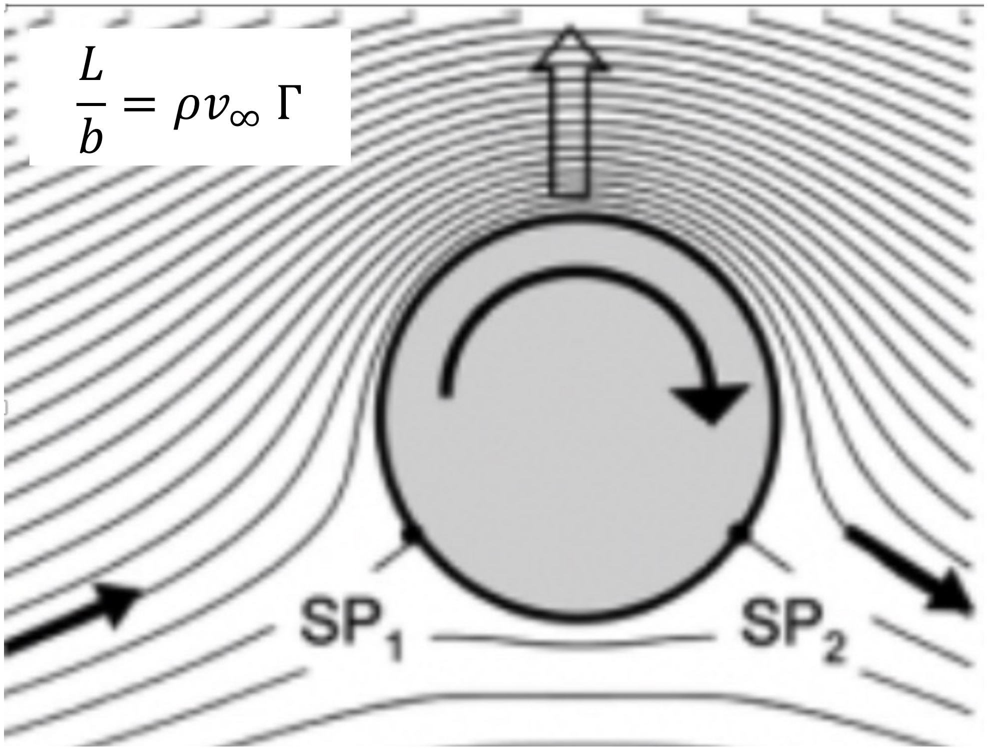
$$\frac{L}{b} = \rho v_\infty \, \Gamma$$

Note: While the circulation pattern and the lift are created by internal and surface forces, the footprint of the lift is a circulation that can be evaluated away from the wing, where these forces do not influence the flow pattern.

The Magnus Effect

A simple way of creating circulation, is rotating a cylinder or ball.

$$\frac{L}{b} = \rho v_\infty \Gamma$$



A Spinning Basketball is Thrown Off a Dam!

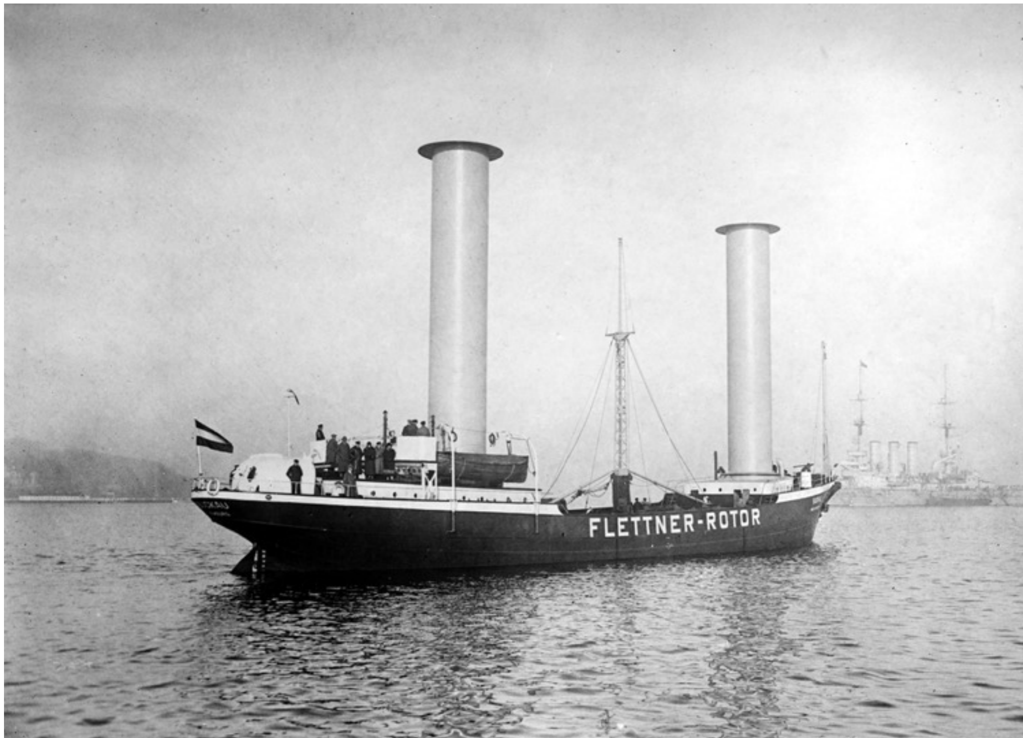
Notes:

(a) The lift is where the total air velocity is increased, not where the velocity against the surface is larger.

(b) It takes some time for the steady state pressure pattern to be established.

(c) The common explanation of acceleration by pushing air away is **wrong !**

Sailing with the Magnus Effect: Flettner Rotors



Bruckau makes an Atlantic crossing 1925

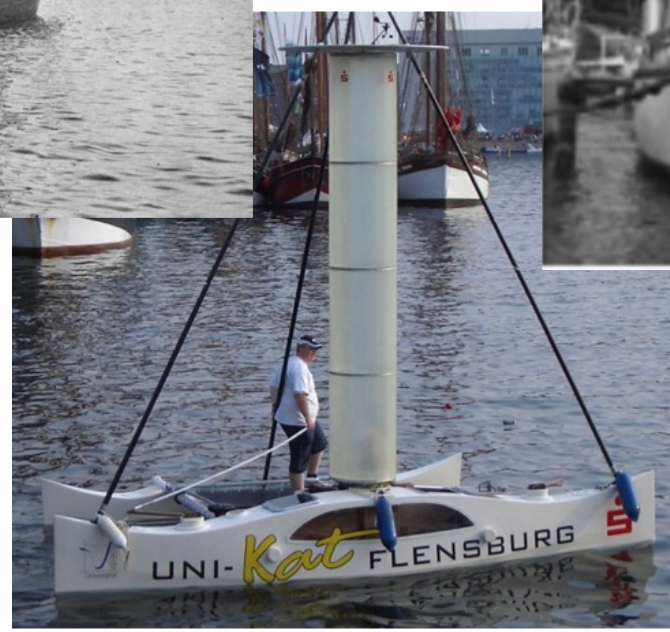
Yacht, Mondsee Austria

Force is linear in v, not v^2

➔ less sensitivity to gusts.

Force $90°$ to wind

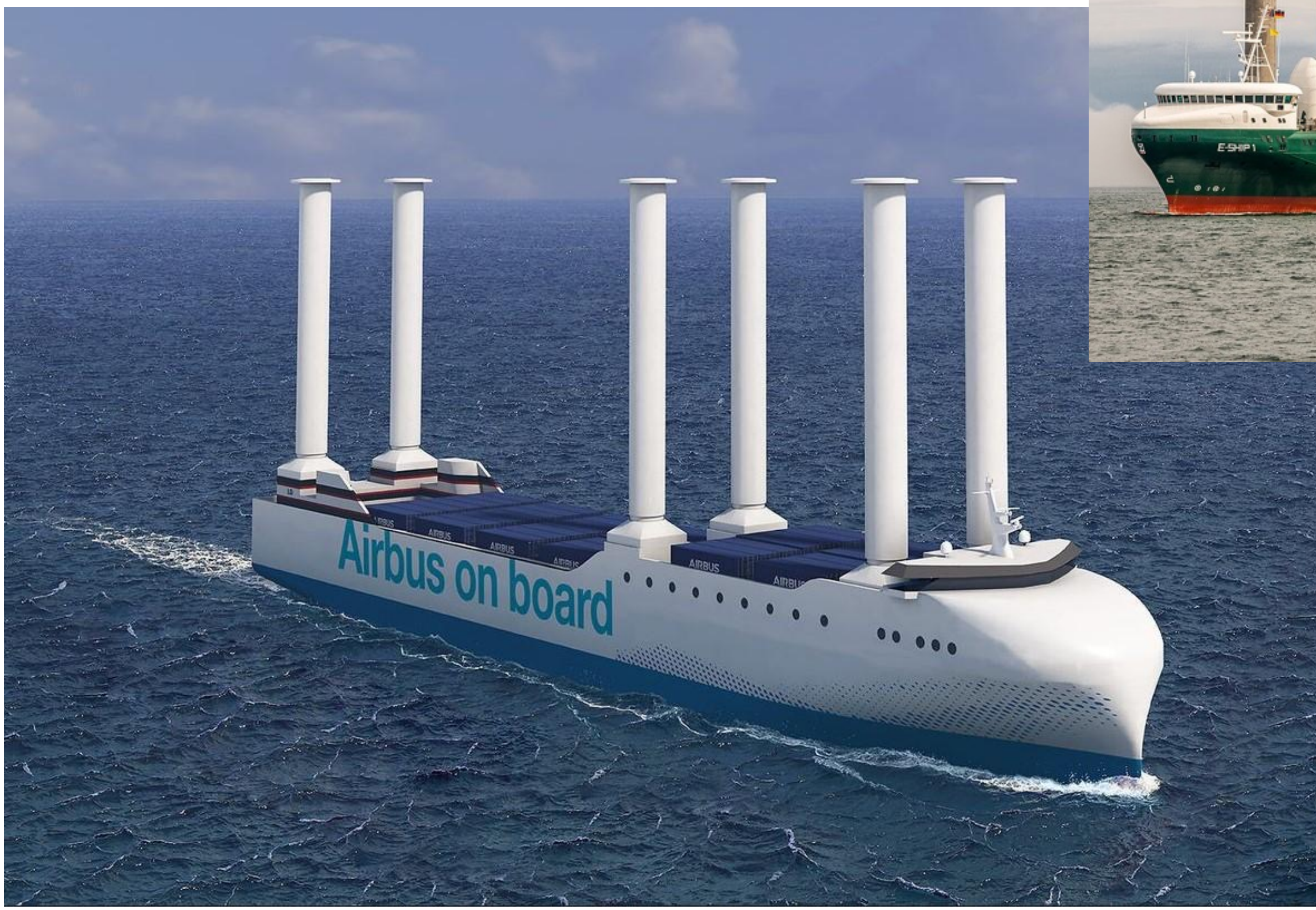
➔ great for beam reach.

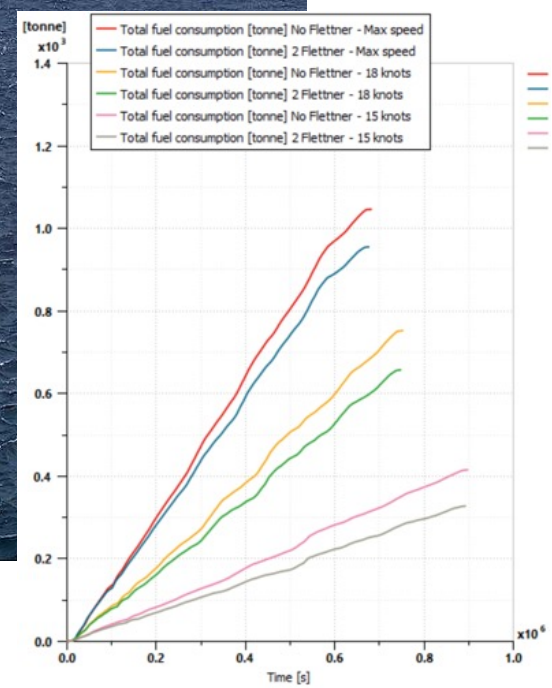


Catamaran Felensburg 2007

Cargo transport with Flettner Rotors

Energy savings in planned and existing cargo transport



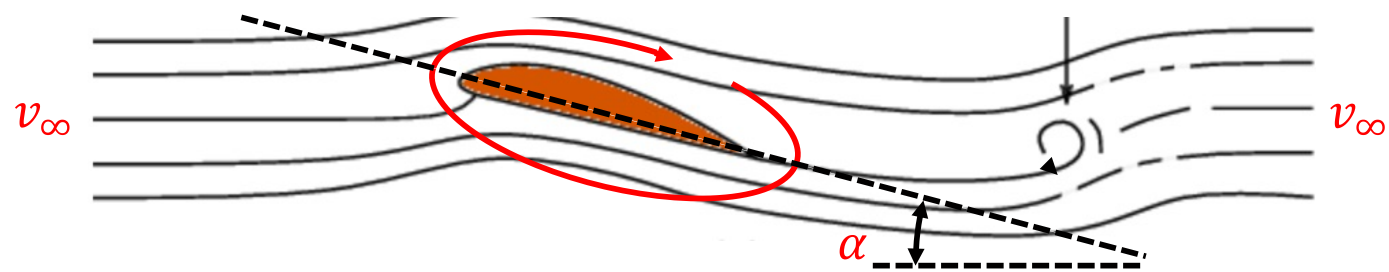


https://newatlas.com/transport/airbus-low-carbon-ocean-fleet-flettner-rotors/

Scaling of Lift and Circulation

The circulation is moves the circulation point to the sharp end of the **wing**, *sail, or keel*. For this Kutta condition) the circulation Γ is proportional to:

the velocity v_∞, the angle to the fluid α, and the width of the wing b



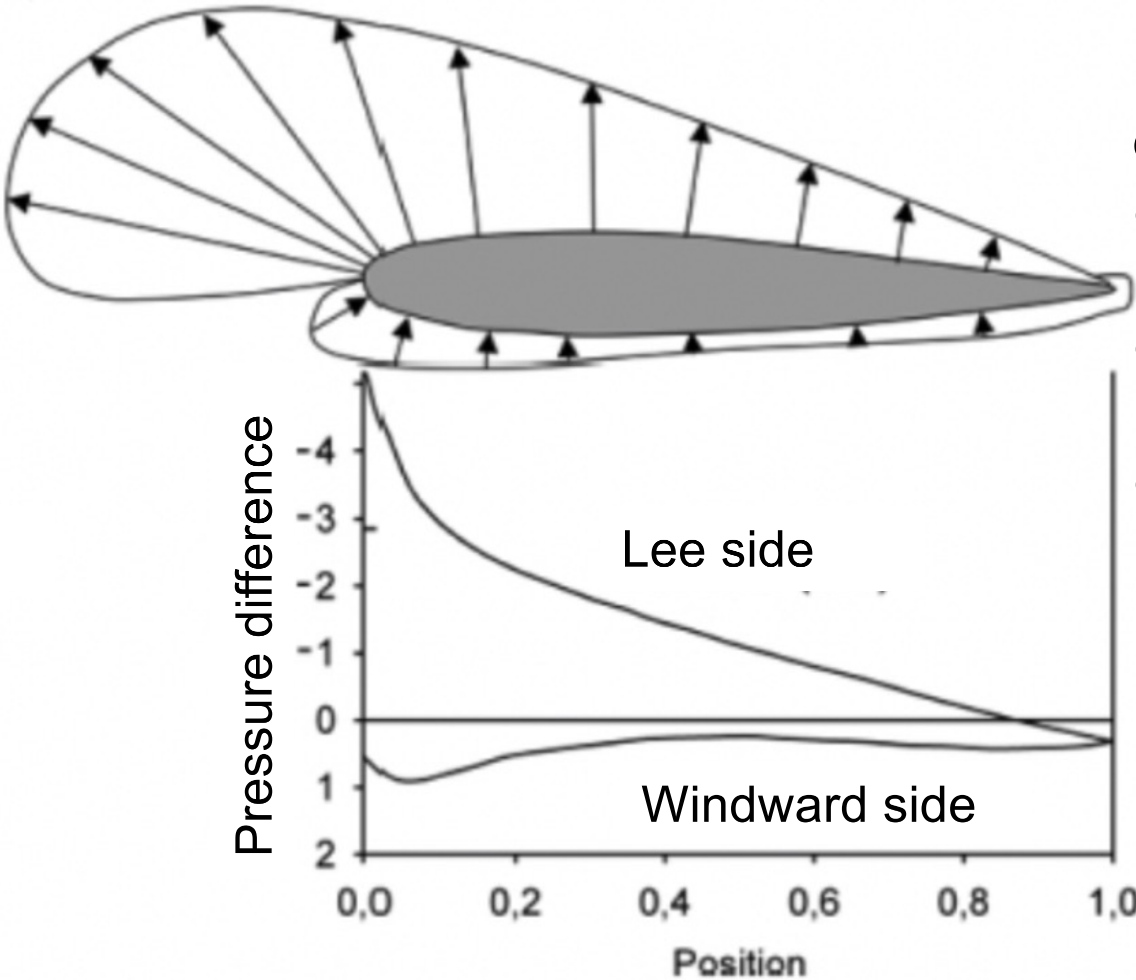
Also, the Lift is proportional to the length of the wing (area A over width b), if it has uniform cross section.

$$L \propto \rho \cdot v_\infty^2 \cdot A \cdot \alpha$$

- ➔ Sails are sensitive to gusts; twice the wind leads to 4 times the lift.
- ➔ Tripling wind does happen at Cayuga lake, e.g., two weeks ago ➔ Lift times 10 !
- ➔ If you feel you might want to **reef the sail**, you should immediately reef the sail.
- ➔ For largest lift, increase the sail angle, i.e., close the sail, until just before turbulence develops (as shown by tell tails). (However, drad may also increase)

Pressure on a Wing, Sail, or Keel

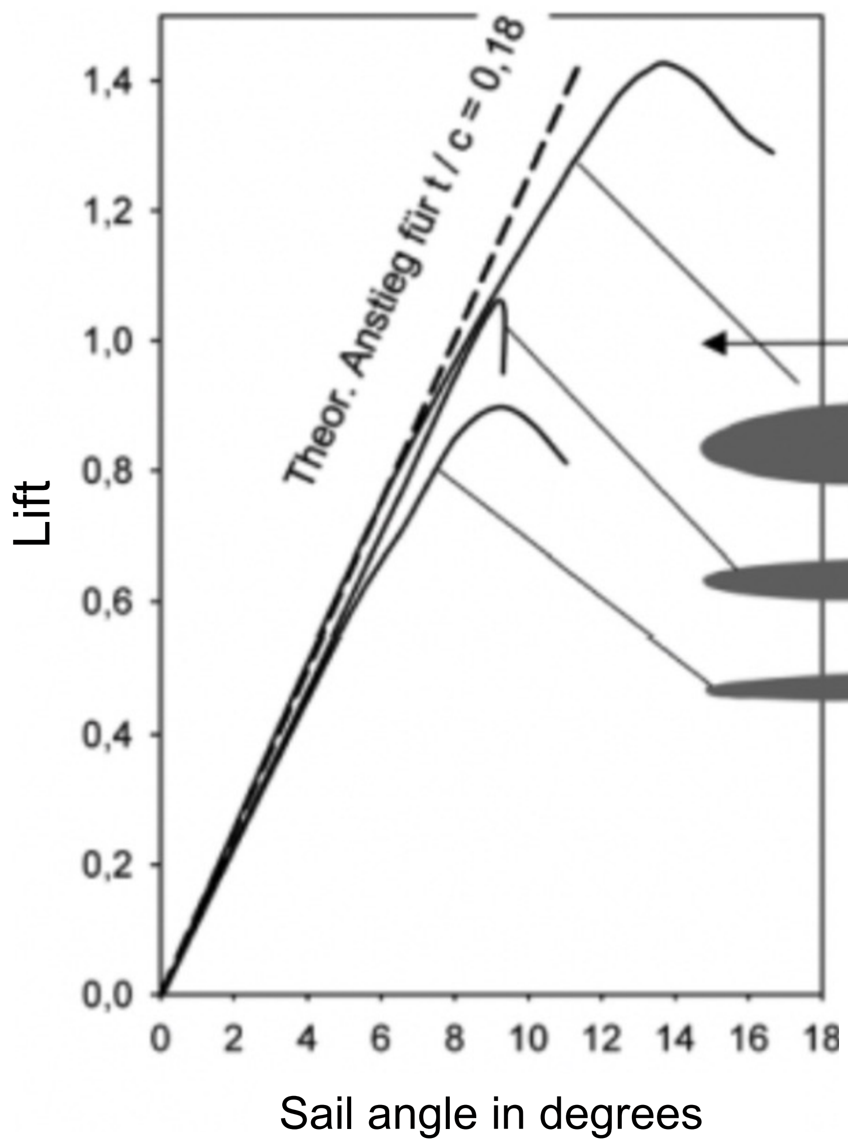
- The decrease in pressure on the lee side of the sail is much larger than the increase of pressure on the windward side.
- The decrease in pressure is largest toward the front of the sail.



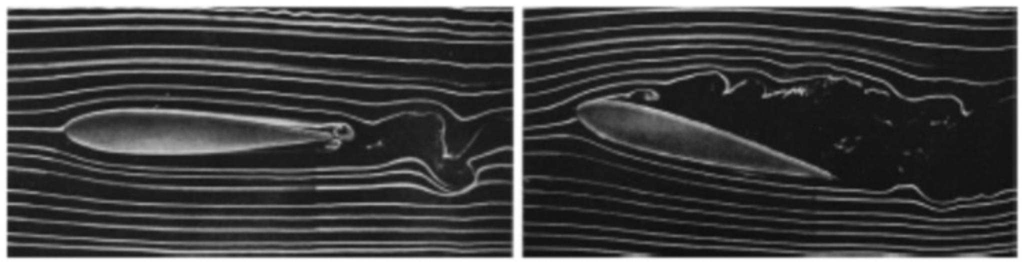
Different applications need different profiles:

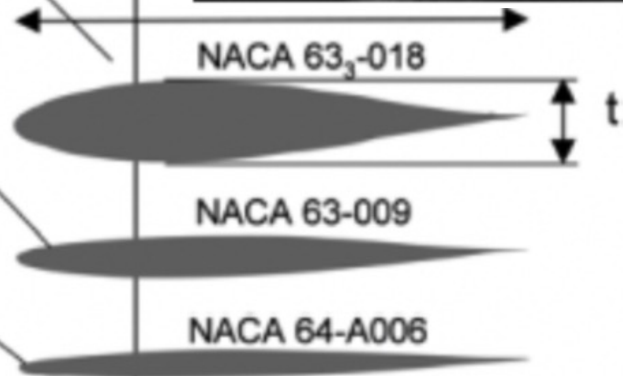
- The keel usually only sees small approach angles.
- The airplane needs to maintain lift most urgently.
- In a sail, approach angles and sail shape can be changed.

Pressure for different wing profiles



Breakdown of Kutta condition, separation of flow, breakdown of lift, stalling.





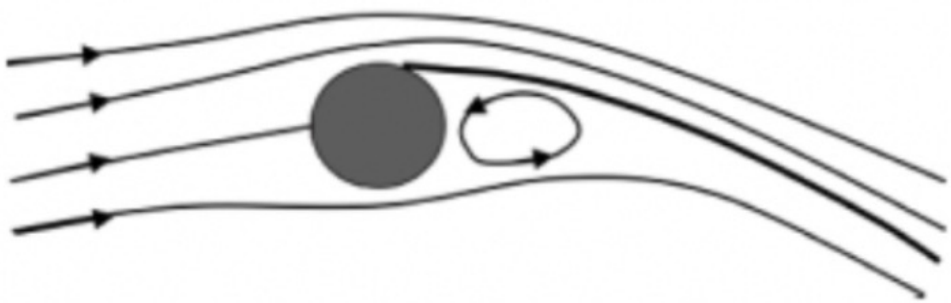
- ➔ Linear dependence on sail angle
- ➔ Maximum just before breakdown of lift
- ➔ Deeper wing allows for more lift
- ➔ Keels need only small angles ➔ narrow
- ➔ Rudder needs large angles ➔ deeper
- ➔ Airplanes must not stall ➔ deep wings

Communication of Sails with their Mast

A sail functions like a wing made of a bent sheet.

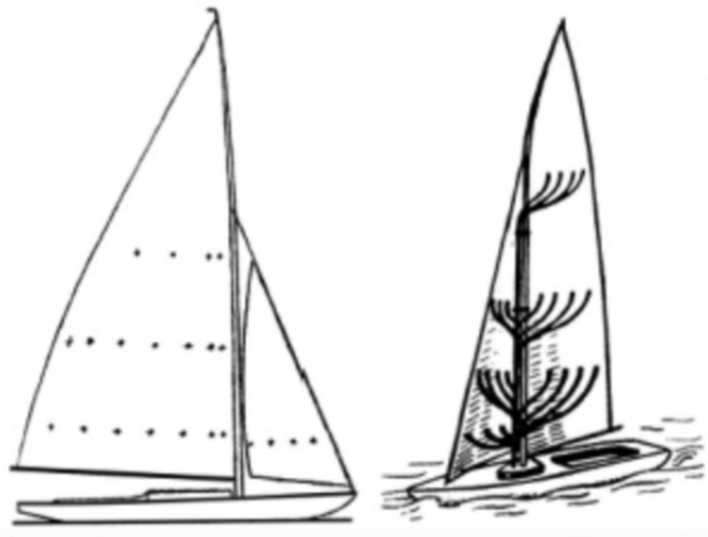
In modern elongated masts, turbulences between mast and sail are minimized.

Streamlined mast

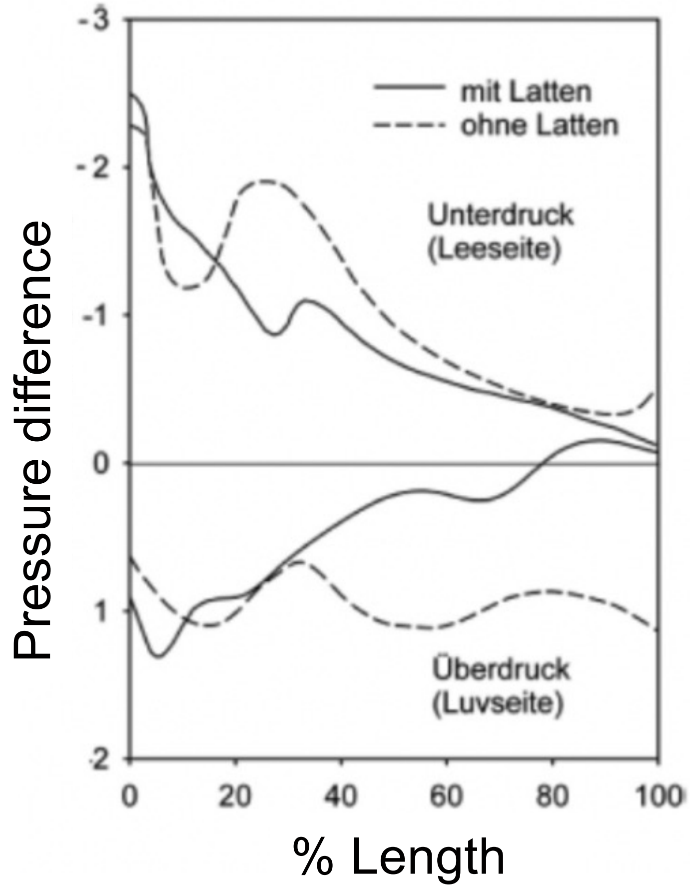


Older mast, e.g., for
Gaff rigged boats.

Maintaining the Sail Shape



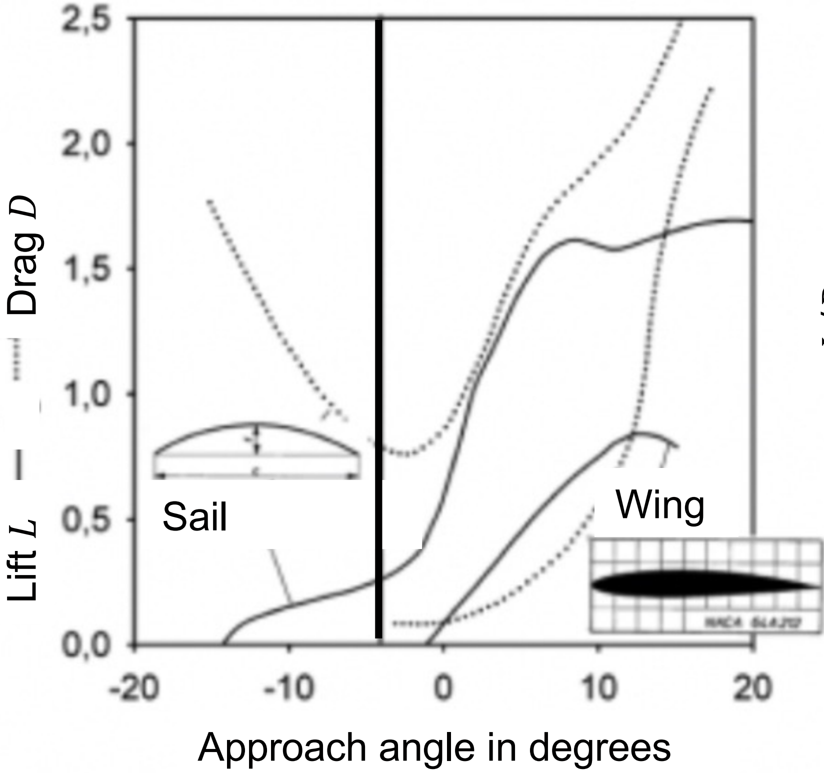
Measurement of the pressure 1915

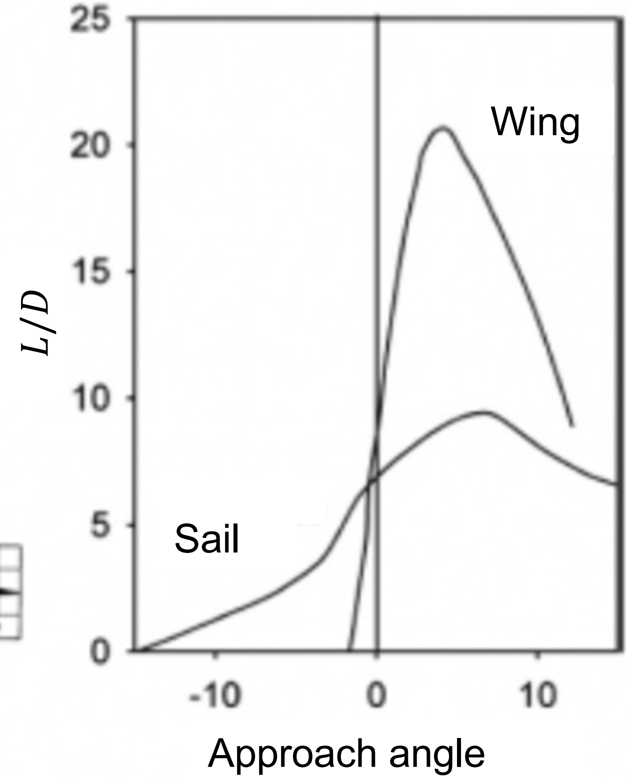


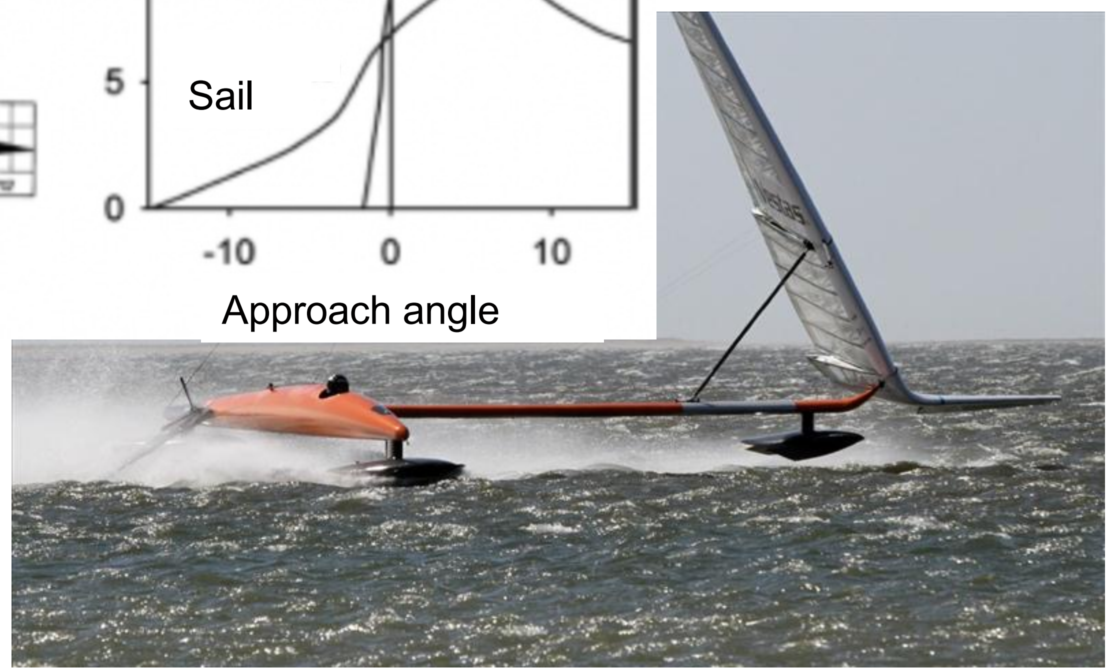
Battens maintain the sail shape at the leech, avoiding curling of the leech that produces pressure and drag

➔ Lift goes up, but drag can go up faster

The Best Sail Angle – Maximal Lift / Drag Ratio

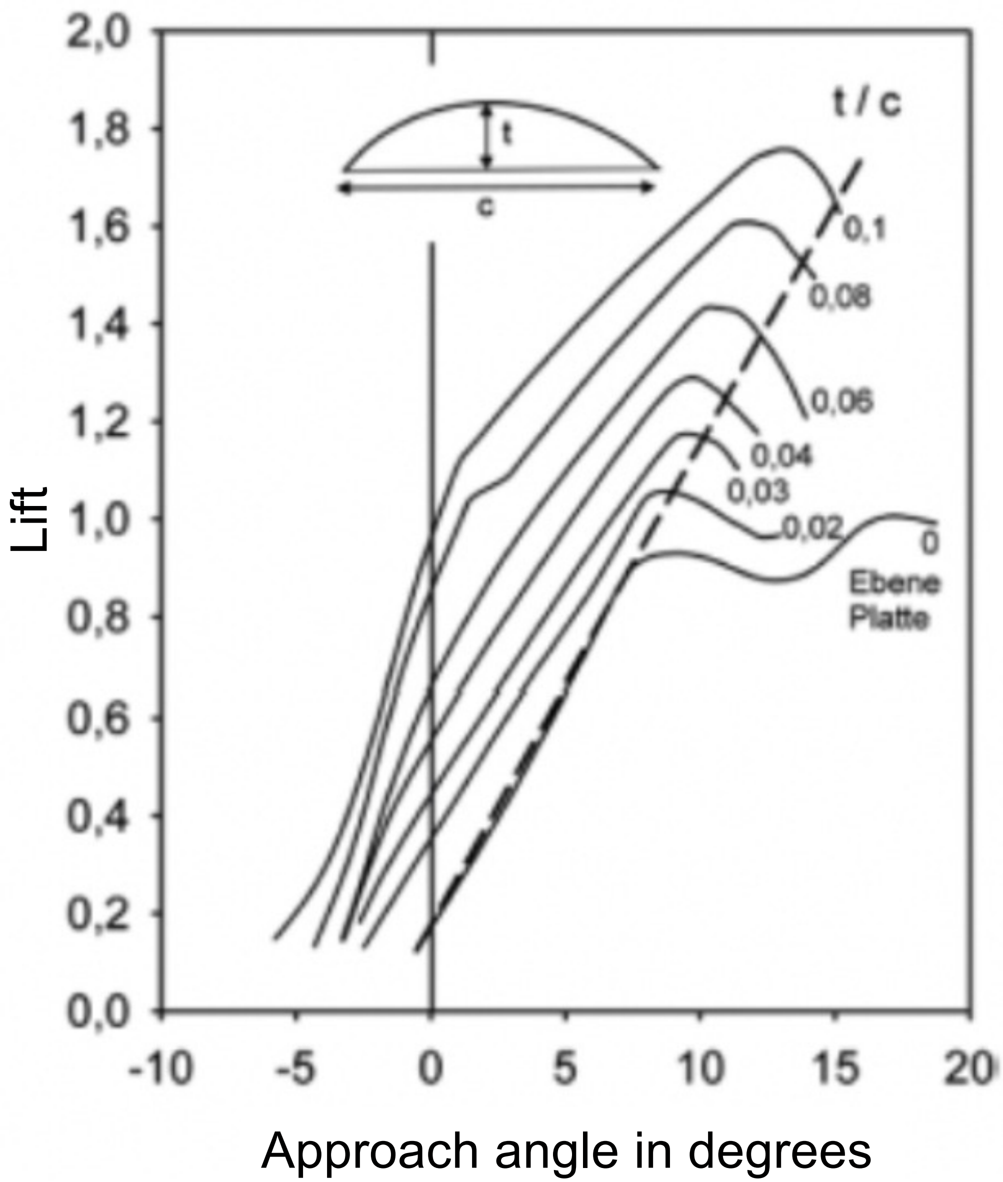






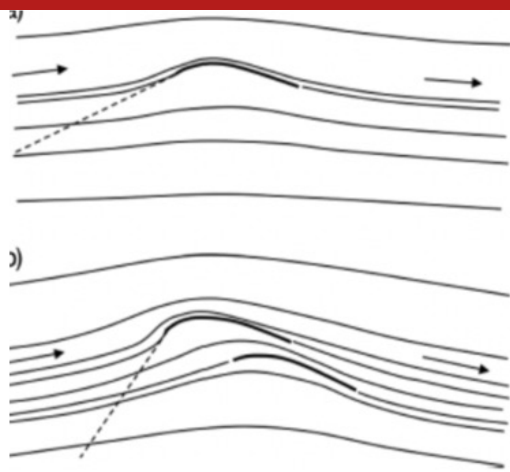
- The optimal point has the largest L/D
- The optimal angle is smaller than that with maximal lift
- The optimal angle has the wind approximately tangential to the luff.
- Only for large velocities, a solid wing shape has advantages over a sail

Lift for different Sail Shapes



- Like a wing, deeper sails can reach larger approach angles.
- Deeper sails produce more lift.
- Even negative approach angles can produce lift, as known from sheets on a laundry line.
- Too much lift can produce too much heel. It then helps to flatten the sail.

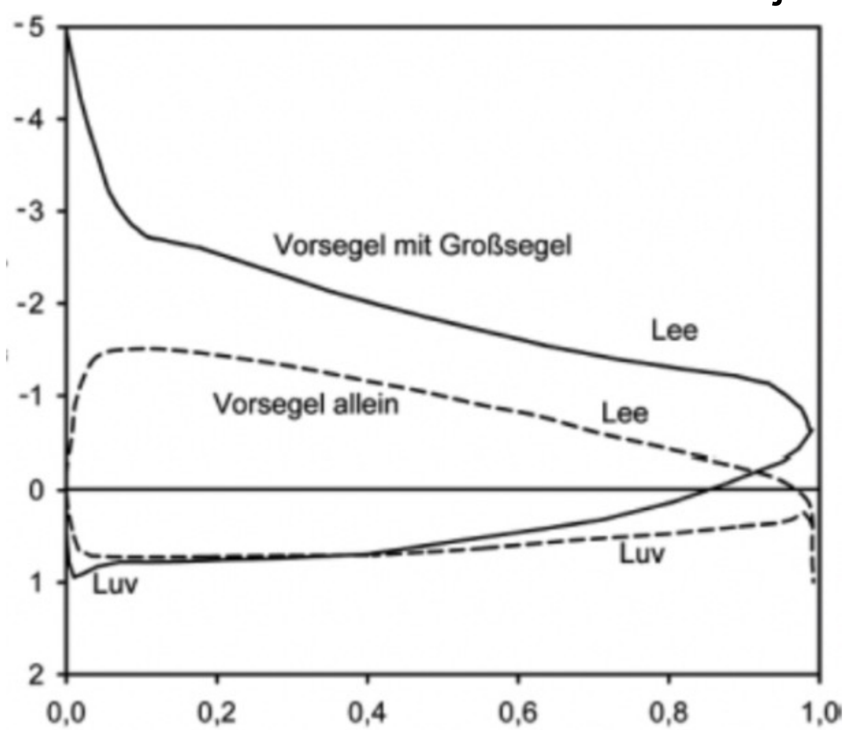
Communication between Jib and Main Sails

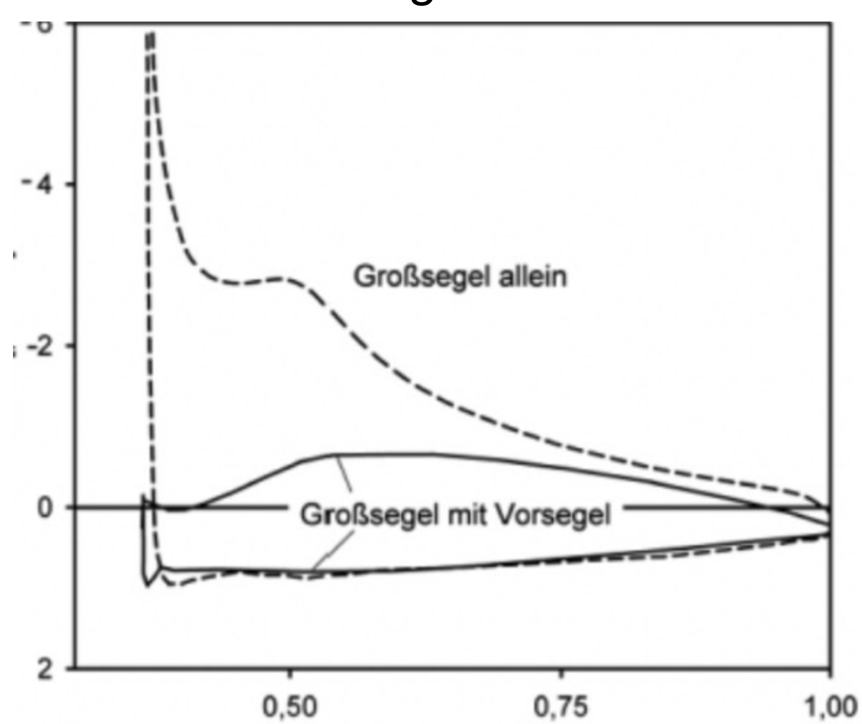


Wrong common explanation: The Jib does not funnel air with higher velocity over the main.

Correct: The jib rather reduces the lift on the main, but the combined long sail has larger lift at its front, i.e., at the jib.

The jib also reduces the damage from the mast.



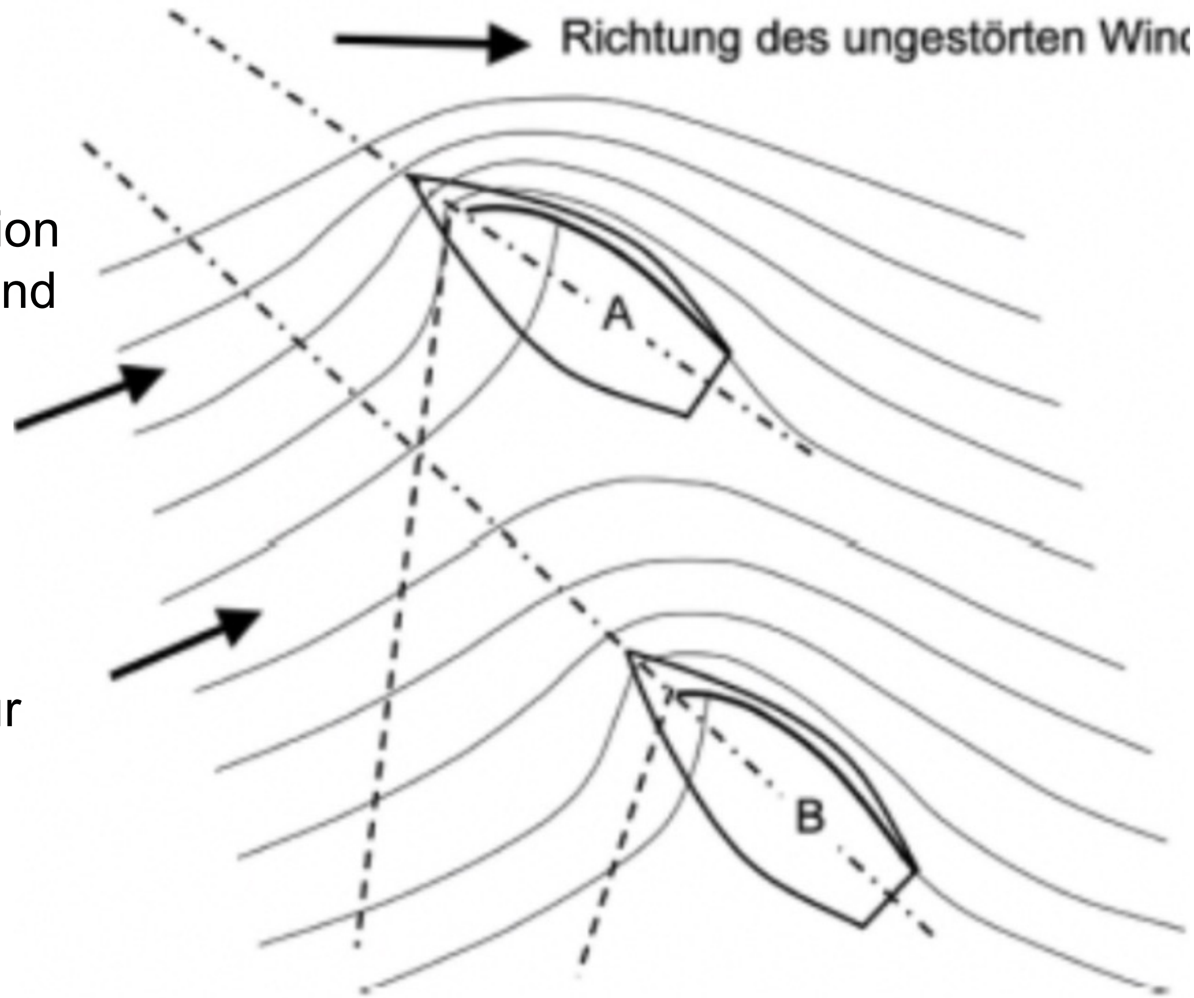


Interaction of between Sails of different Boats

The boat in the safe lee position can go steeper against the wind with better VMG.

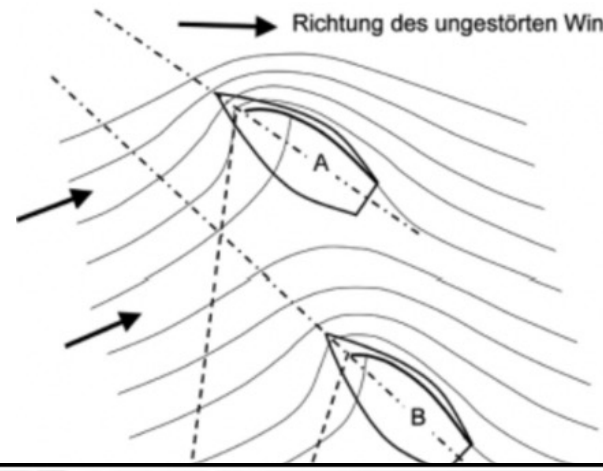
In bicycling, don't give wind shadow to your opponent.

In sailing, don't give lift to your opponent.



The save lee position takes lift from the opponent

More heel indicates the safe lee position, taking lift from the opponent.



No safe lee position in downwind sailing

- In downwind sailing one can give turbulence to one's opponent.
- The distance where the downwind boat can be influenced is about 4 mast heights.

Circulations does not change

Circulation is the sum of all $\vec{v}$ encountered on a closed loop

$$\Gamma = \oint \vec{v} \cdot ds = Sum\ over\ \vec{v} \cdot \Delta\vec{s}\ \ along\ the\ closed\ loop.$$

Force on a little cube of fluid:

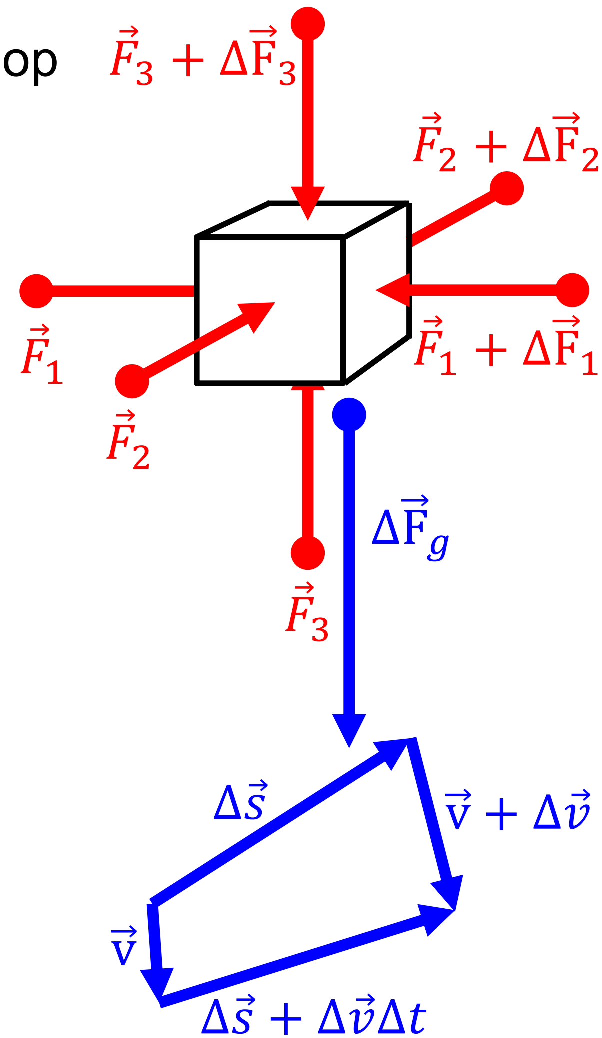
The sum of these forces and gravity move the fluid cube accelerating its velocity:

$$\Delta\vec{v} \propto \Delta\vec{F}_1 + \Delta\vec{F}_2 + \Delta\vec{F}_3 + \Delta\,\vec{F}_g.$$

As the closed loop moves with the fluid, circulation could change.

(a) because the forces on the moving cubes along the closed loop change the velocities, but the forces all sum up to 0, as all changes along a closed loop sum up to 0.

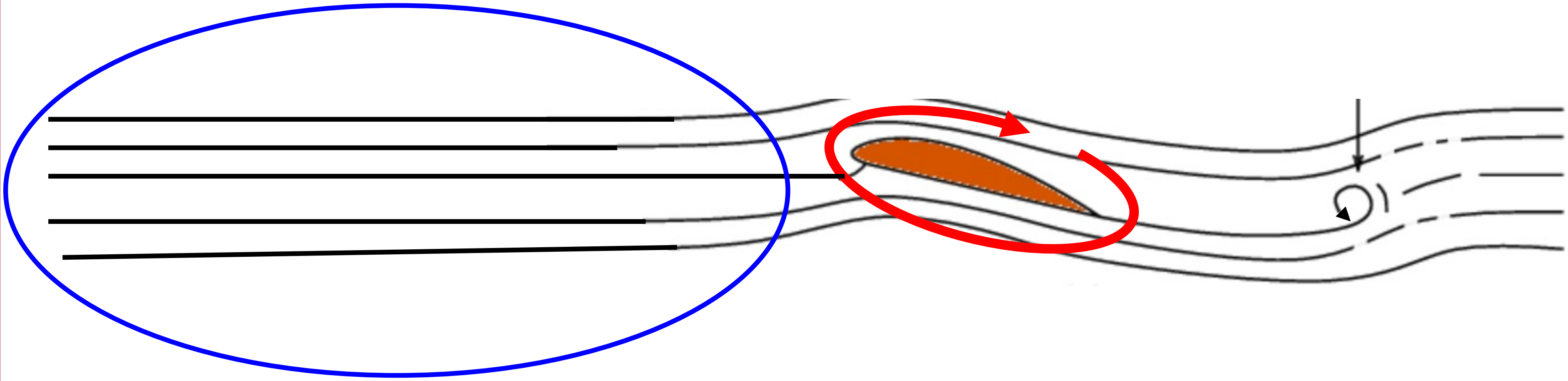
(b) because the path over which one sums changes during Δt as the cubes flow; $\Delta\vec{s}$ changes by $\Delta\vec{v}\Delta t$. The change to the circulation is then the sum over $\vec{v} \cdot \Delta\vec{v}\Delta t = \frac{1}{2}[(\vec{v}+\Delta\vec{v})^2 - \vec{v}^2]\Delta t = \Delta(\vec{v}^2)\,\Delta t$. But, again, in a close loop, all these changes also add up to 0.



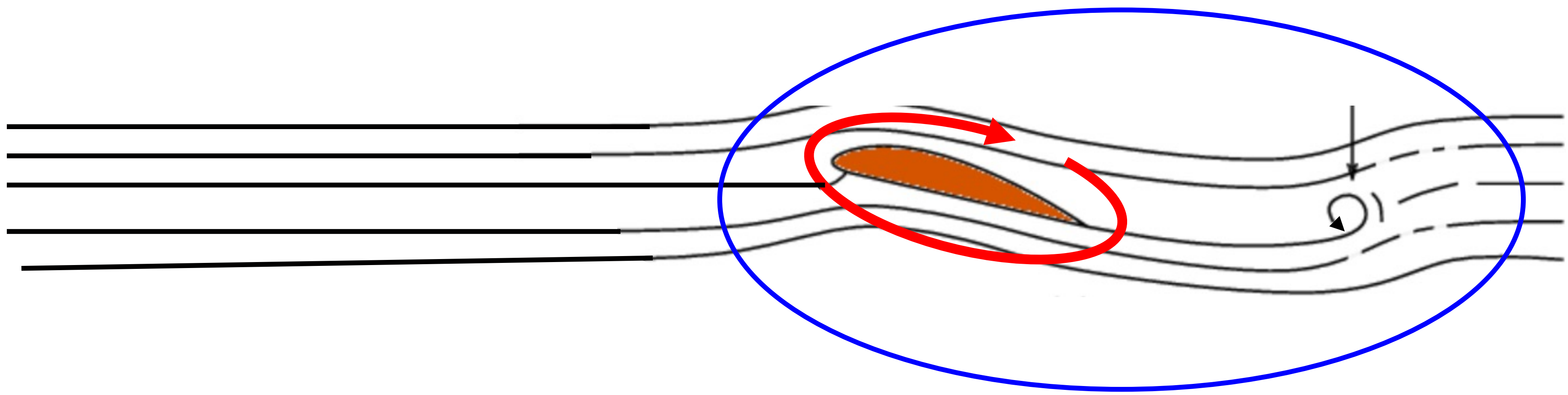
$\Delta\Gamma = 0$

while following the flow

$\Delta\Gamma = 0$ while Following the Flow



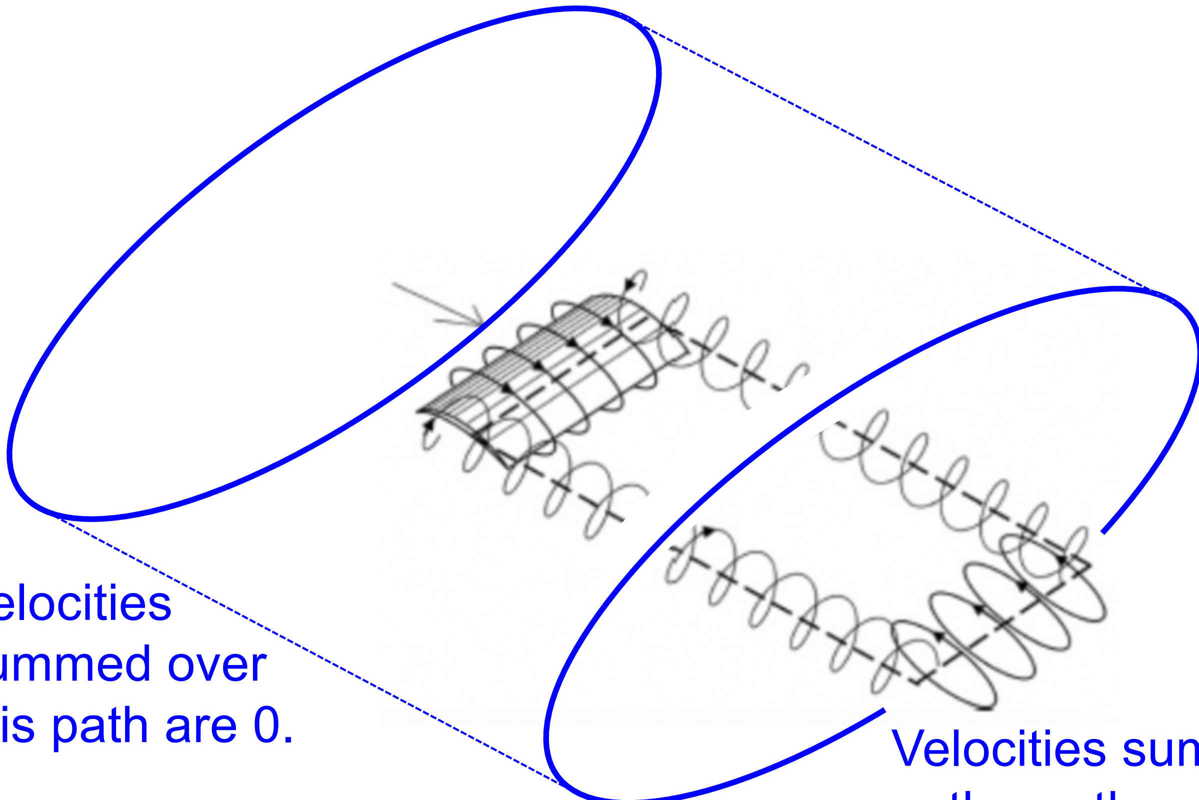
The velocities summed over this path are 0.



➔ The velocities summed over this path must also be 0.

Circulations don’t Start or End

In incompressible fluids, circulation or vertices don’t have a start or end point. They get created in loops or end at the boundary of the fluid.



Vertex loop
behind a wing

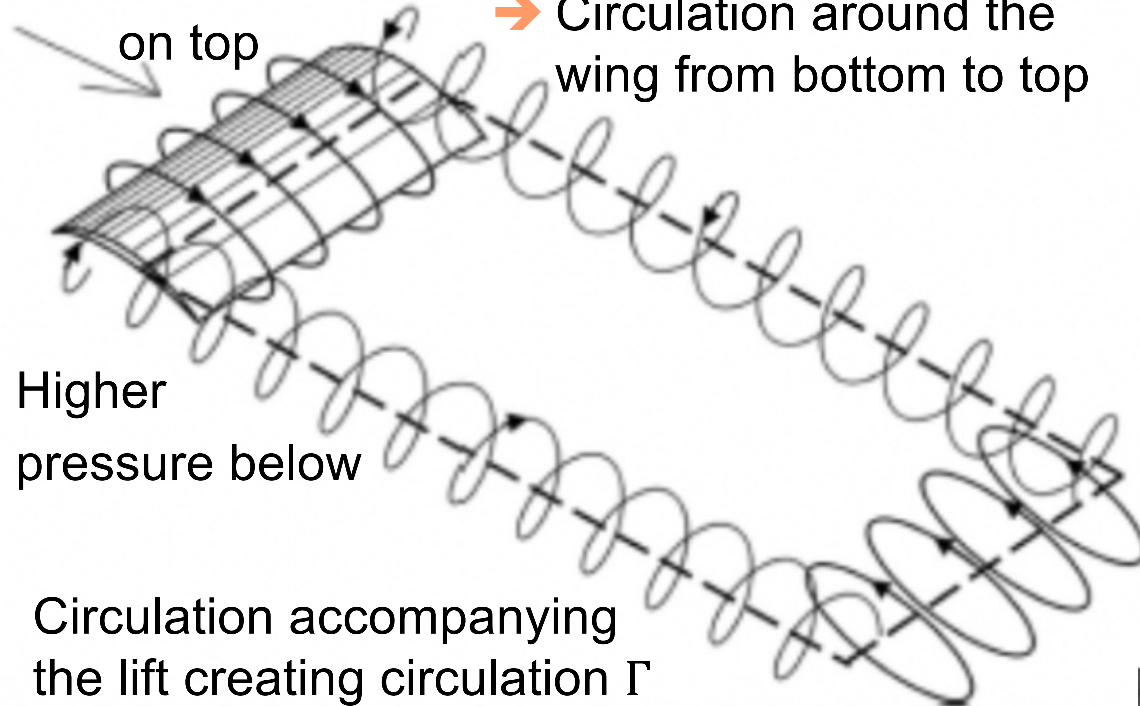
Velocities
summed over
this path are 0.

Velocities summed over this
path are therefore also 0.

Circulations trailing wings, sails, and keels

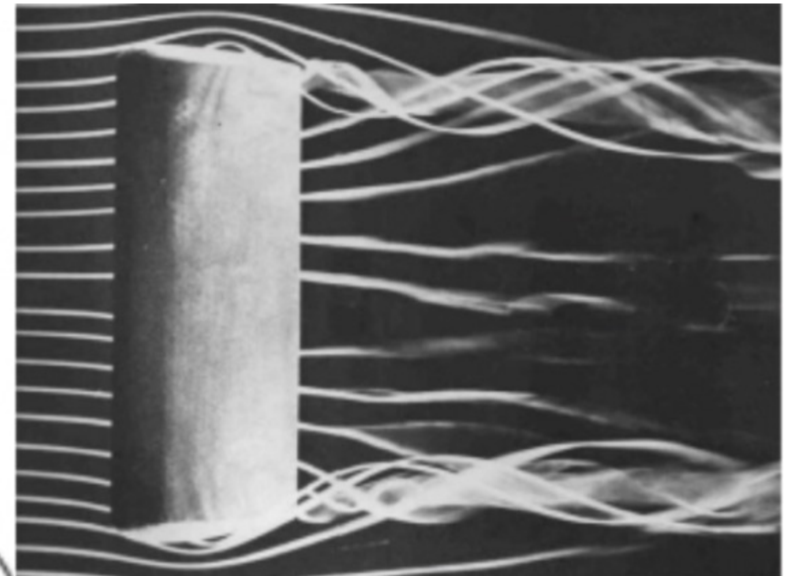
Lower pressure on top

→ Circulation around the wing from bottom to top

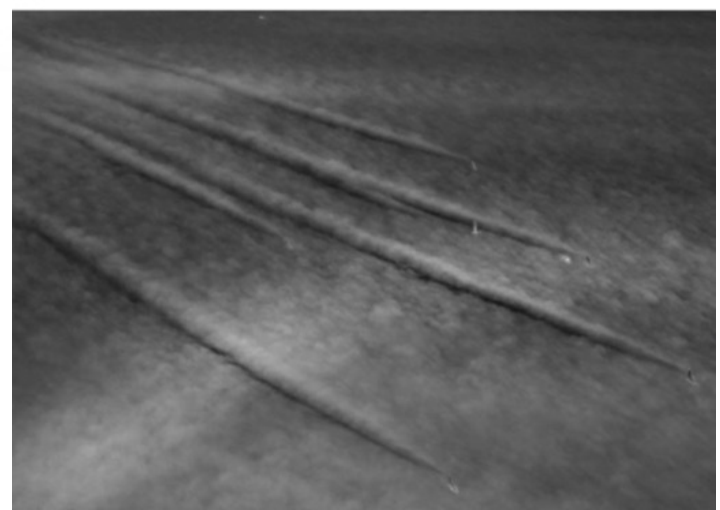


Higher pressure below

Circulation accompanying the lift creating circulation Γ around the wing.

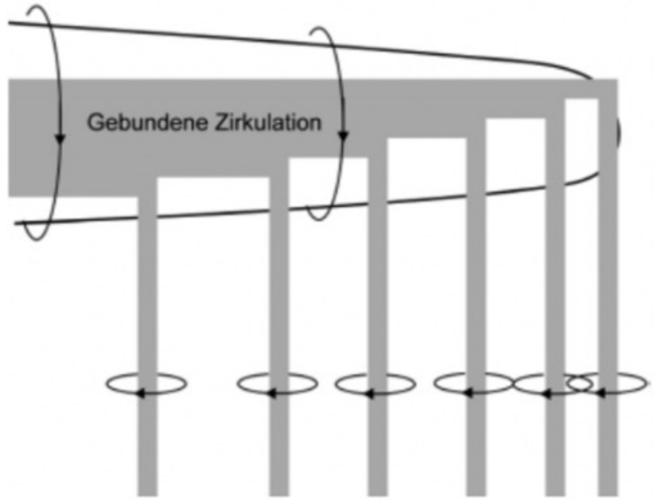


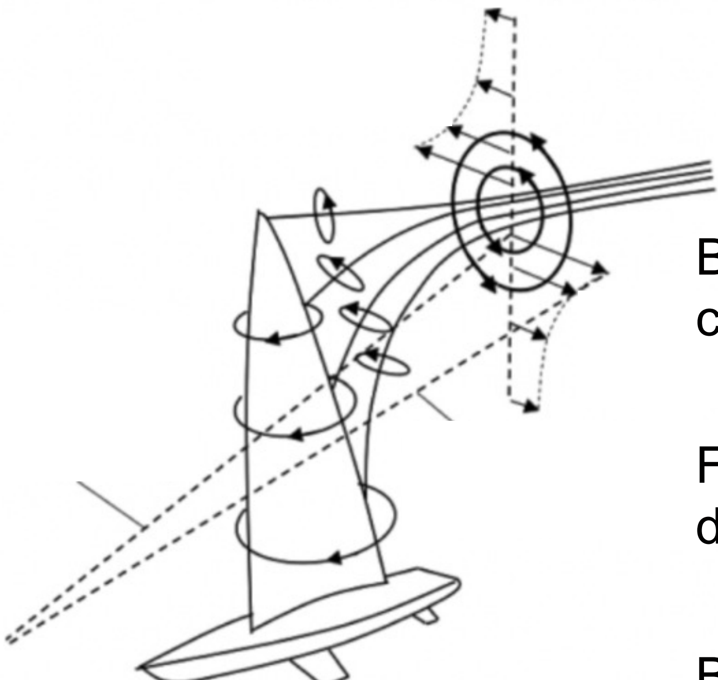
Circulations don't start or end inside an incompressible fluid



Circulation over the mast of each sail in an ocean race, shown in the fog.

Circulations around tapered wings





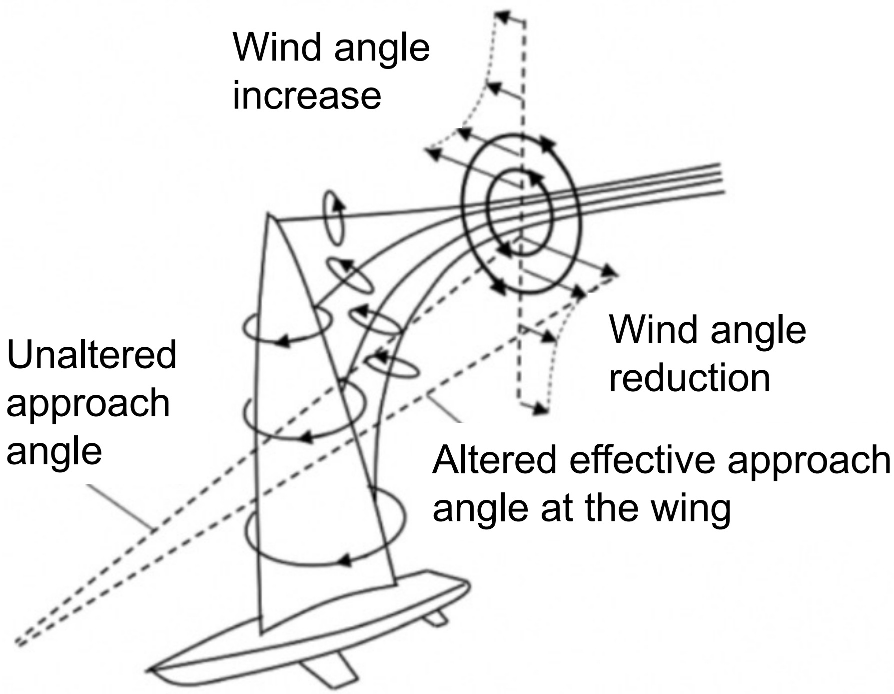
Wing circulation visualize by dye

Behind sails and tapered airplane wings a sheet of circulation combines into a vertex.

For planes the vertex remains for miles and can be dangerous for trailing airplanes.

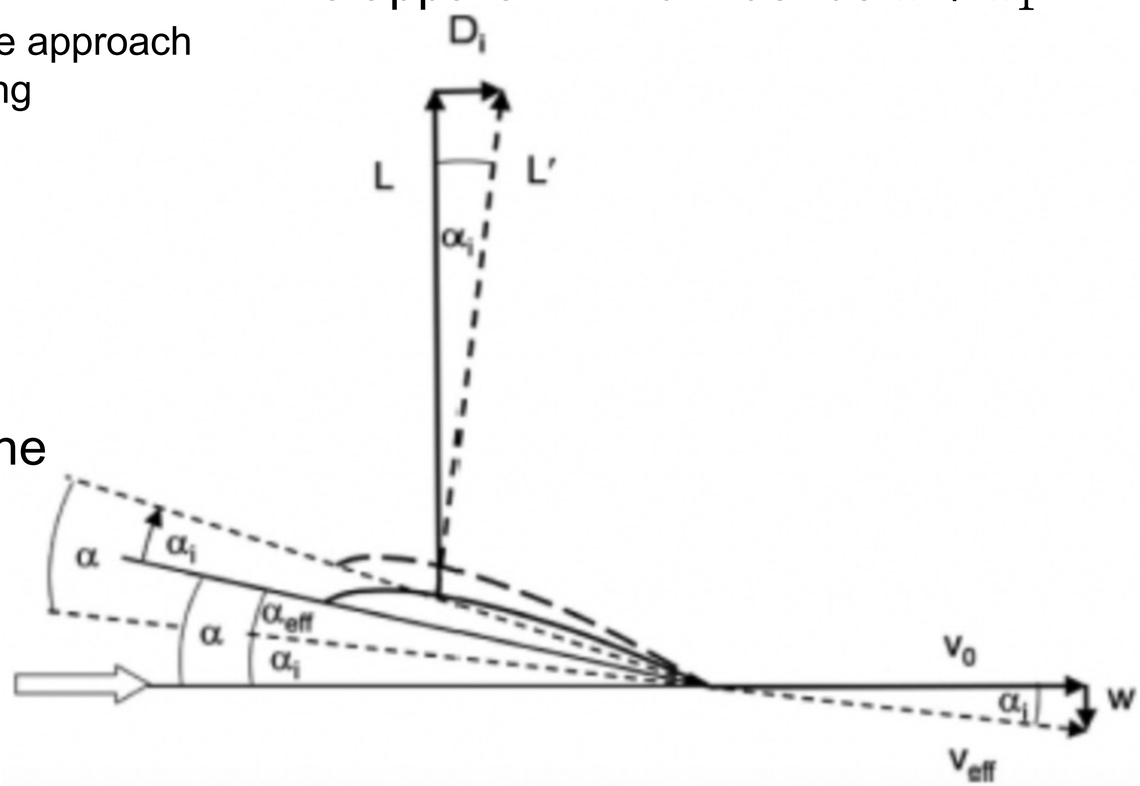
Birds use the upwind ahead by flying in V-formation.

Approach angle changed due to circulations



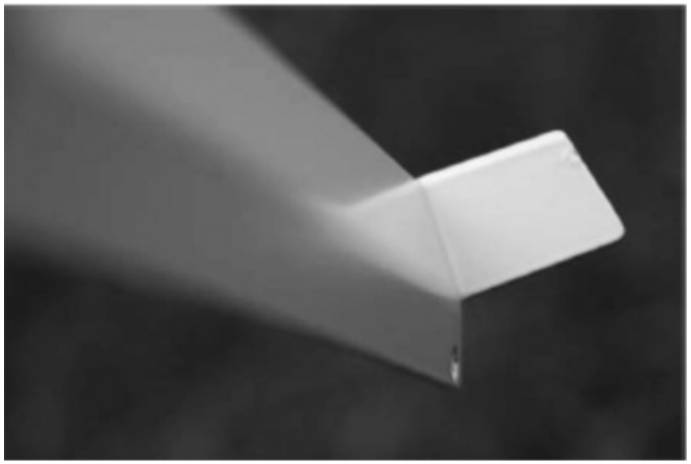
The circulation behind the sail changes the wind direction after the sail.

- ➔ The effective approach angel is decreased by an angle α_1.
- ➔ To have the approach α, the angle to the apparent wind must be $\alpha + \alpha_1$.



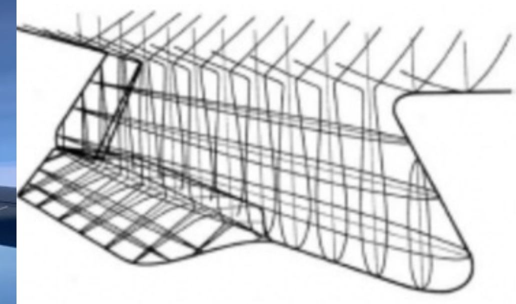
Interpretation: The boat must drift a little downwind to gain the energy it left in the circulation.

Winglets lead vertex further away from the wing



Winglet on a wind generator

Winglets on an airplane



Wing keel

America Cup Winnders

1851: Yacht club of NY

24 more times: Yacht club of NY

1883: Royal Perth Yacht club

With the first (and secret) wing keel

Wing keel of Australia II, America Cup 1983

Sail Shape for Uniform Wind Angle

The kinetic energy of a particle is $E_{kin} \propto v^2$.
(A car with twice the energy does 4 times the damage.)

The total sideways wind in the vertex is proportional to the sum of velocities.

For a given $W \propto \sum v_i$, the energy $E_{kin} \propto \sum v_i^2$ is smallest when all particles have the same velocity ➔ minimal drag.

A sail shape with the same sideways velocity everywhere along the leech minimizes this energy.

Additionally, the effective wind angle is then the same at all height up the sail.

It turn out that an **elliptical sail shape** approximately has this property, called elliptic loading of the circulation.

It is disturbed by twist in the sail.

Best Aspect Ratio of Wings, Sails, and Keels

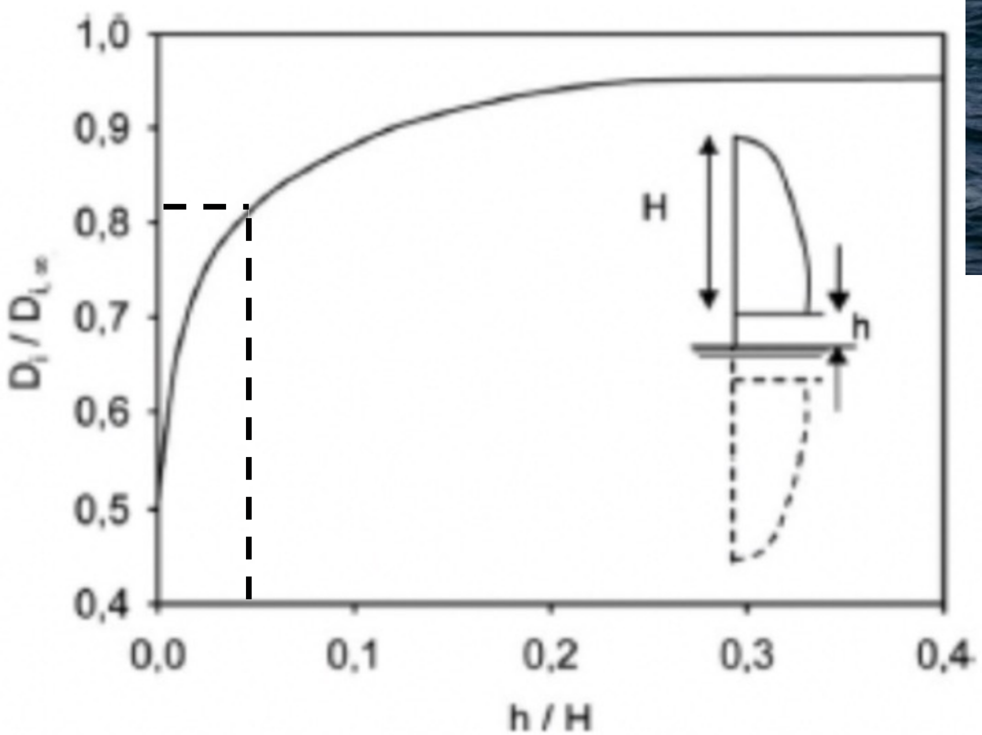
The circulation around the end of the wing is less relevant for longer wings and taller sails or keels.

- Optimized gliders have long an narrow wings.
- Racing keels are long and narrow
- Sails have gotten longer and narrower, if lateral stability can be maintained.

The Air-Water Interphase

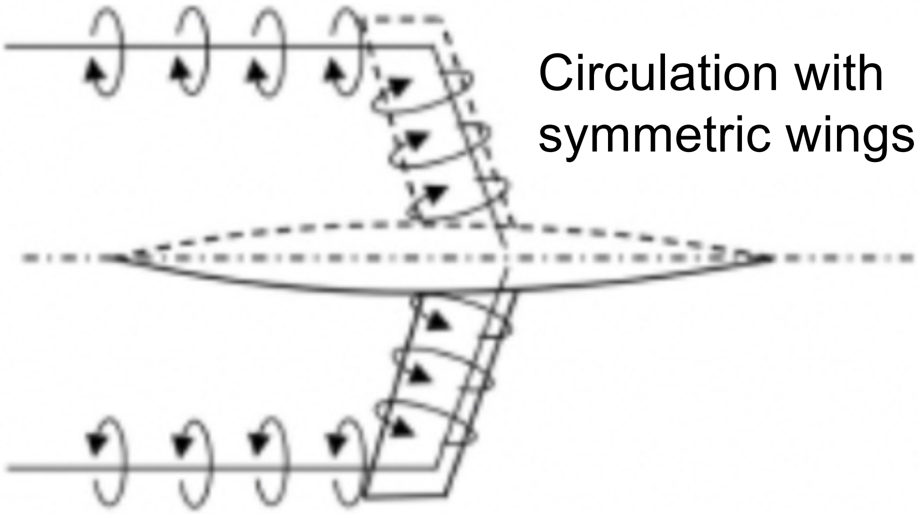
Circulation also exists around the foot (lower edge) of a sail, except when the sail goes down to the deck.

The Drag on the sail increases strongly with the gap under the sail, by 64% when the gap is 5% of the sail height.

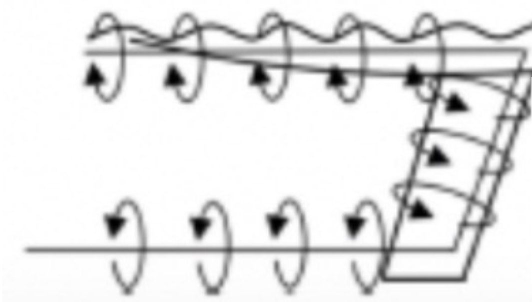


Circulation influencing the Water

The Sails



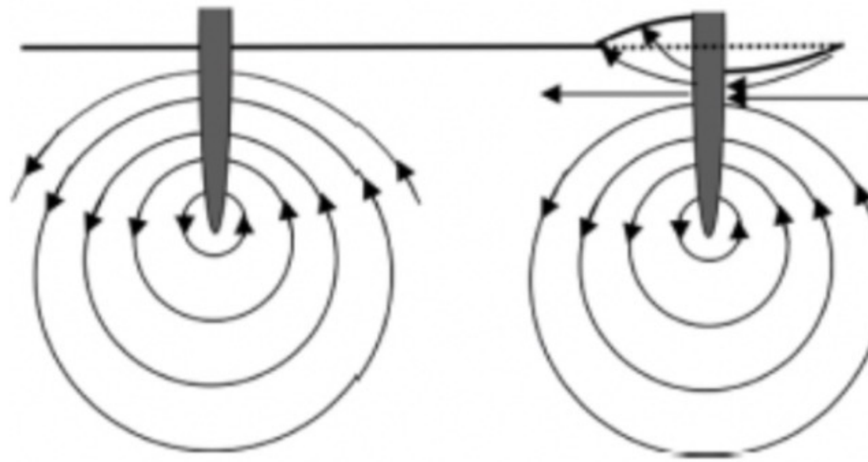
Circulation with symmetric wings



One wing is replaced by water.

Its circulation is taken over by water waves, increasing drag of the boat.

The Keel

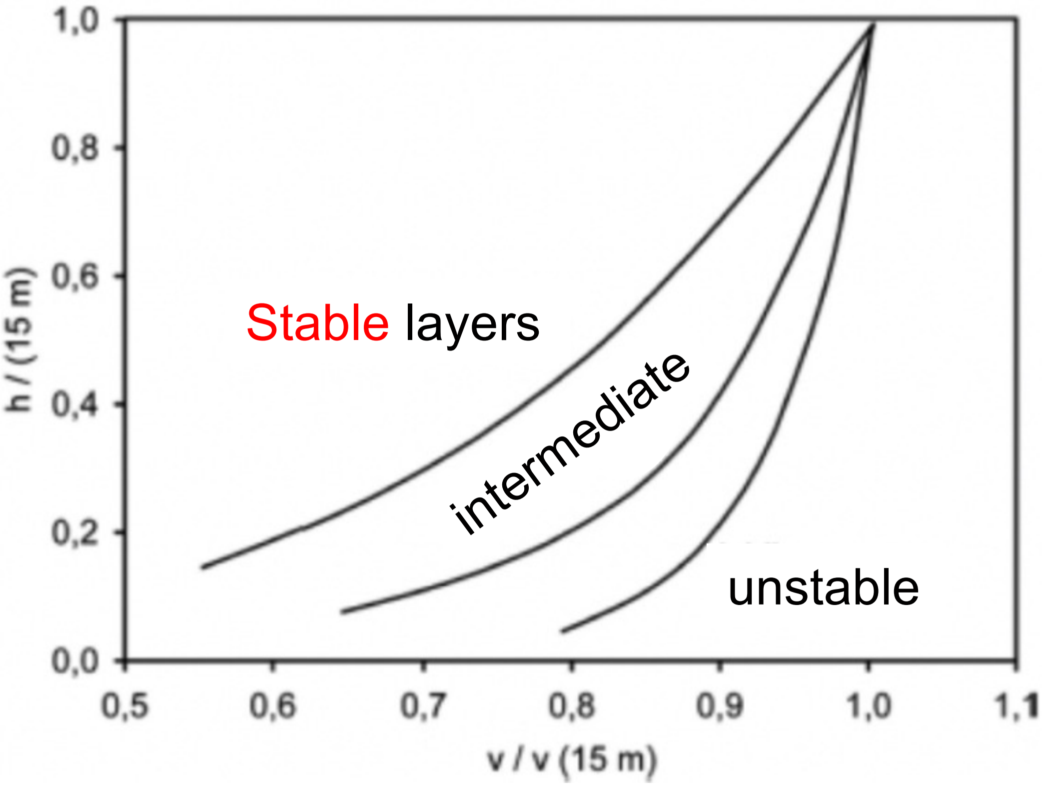


On the low-pressure side of the keel, water lowered, also inducing waves.

Sail Twist

Wind speeds tend to be lower on the water0 surface (and 0 directly on the water.

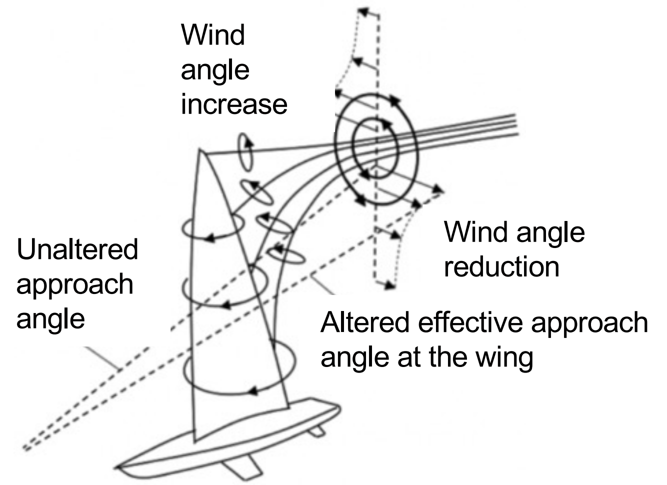
If the wind is a laminar flow with stable layers, the wind changes more slowly than for turbulences above the water.

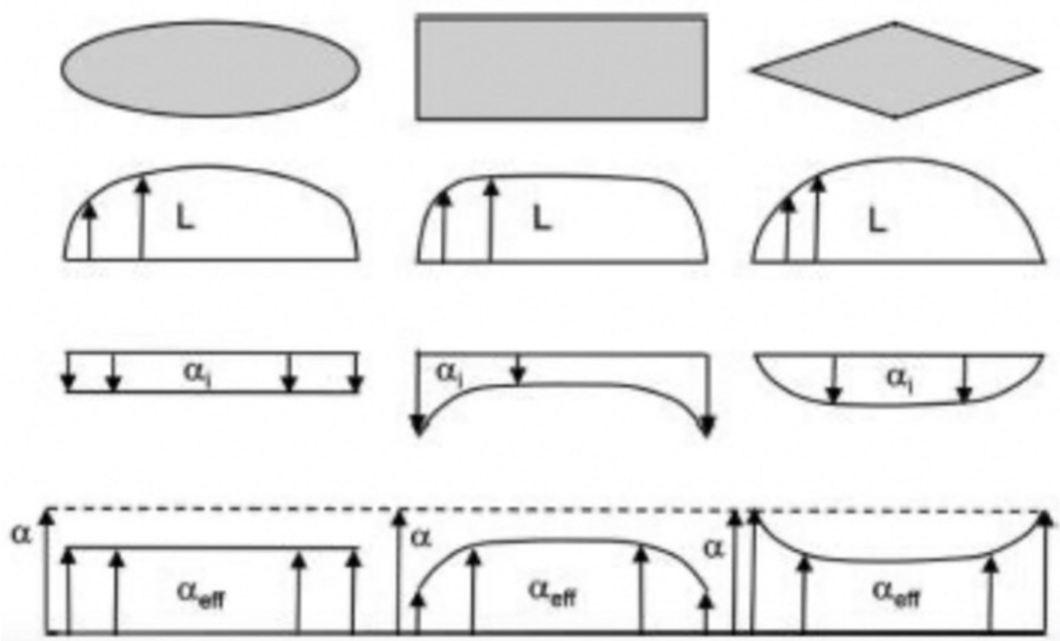


➔ The apparent wind is more forward for lower true wind ➔ closer low part of sail.

Sail Twist for Uniform Wind

For triangular sails (Bermuda rigging), Twist is useful to compensate for the decreasing induced angle form the trailing circulation.





Sail type

Change α_i in approach angle due to circulation

Required sail angle α.

➔ Twist for a triangular sail

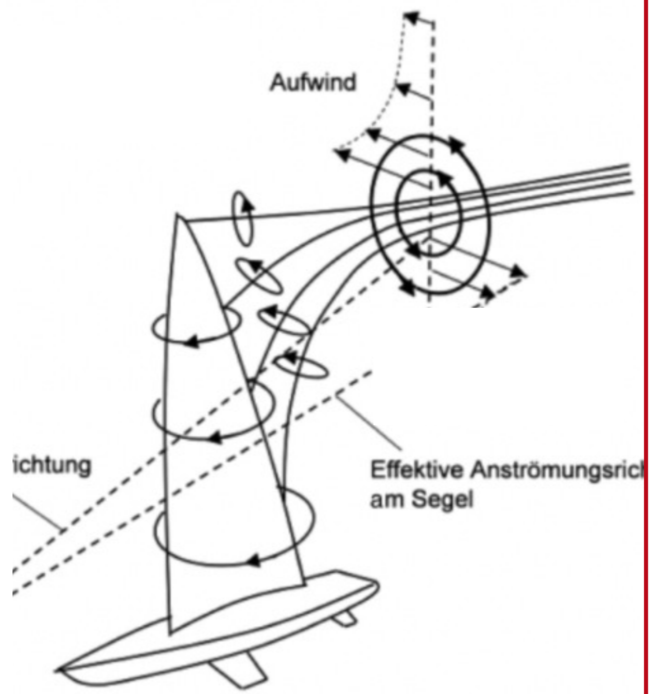
Limits to the Narrow-Wing Theory

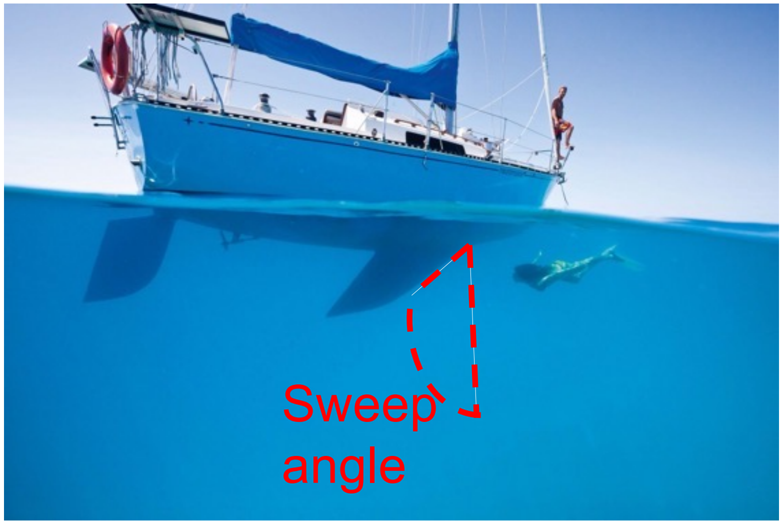
Statements so far related to long narrow sails or keels that crate lift by circulation around the wing, and where the flow around the top and bottoms were relatively small perturbations.

This does not apply when

- The keel is long and not deep.

- There is sweep to the keel.
- Strongly different flow at different parts.

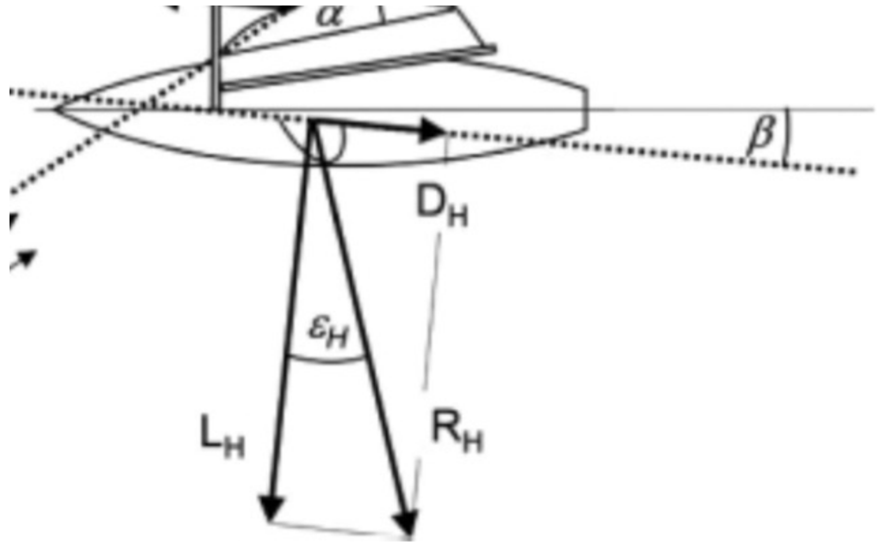


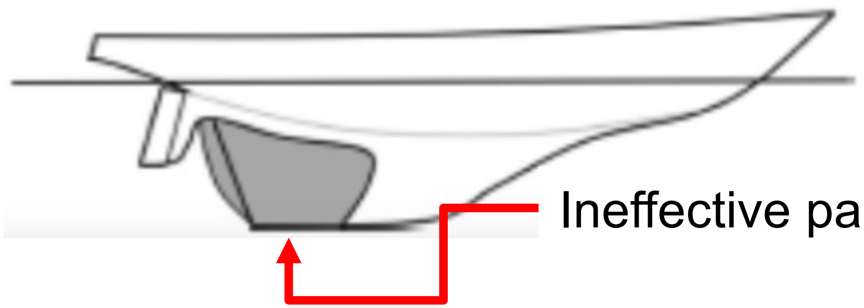


Long versus High Keels

As with a sail, the largest force is created in the front region of the keel.

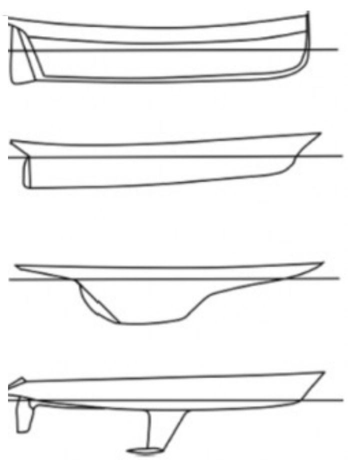
The back region of a long keel therefore does not contribute much to the lift force but increases the drag force.





Ineffective part of the keel

- Development to deeper and narrower keels.
- But they have less directional stability.



17th century

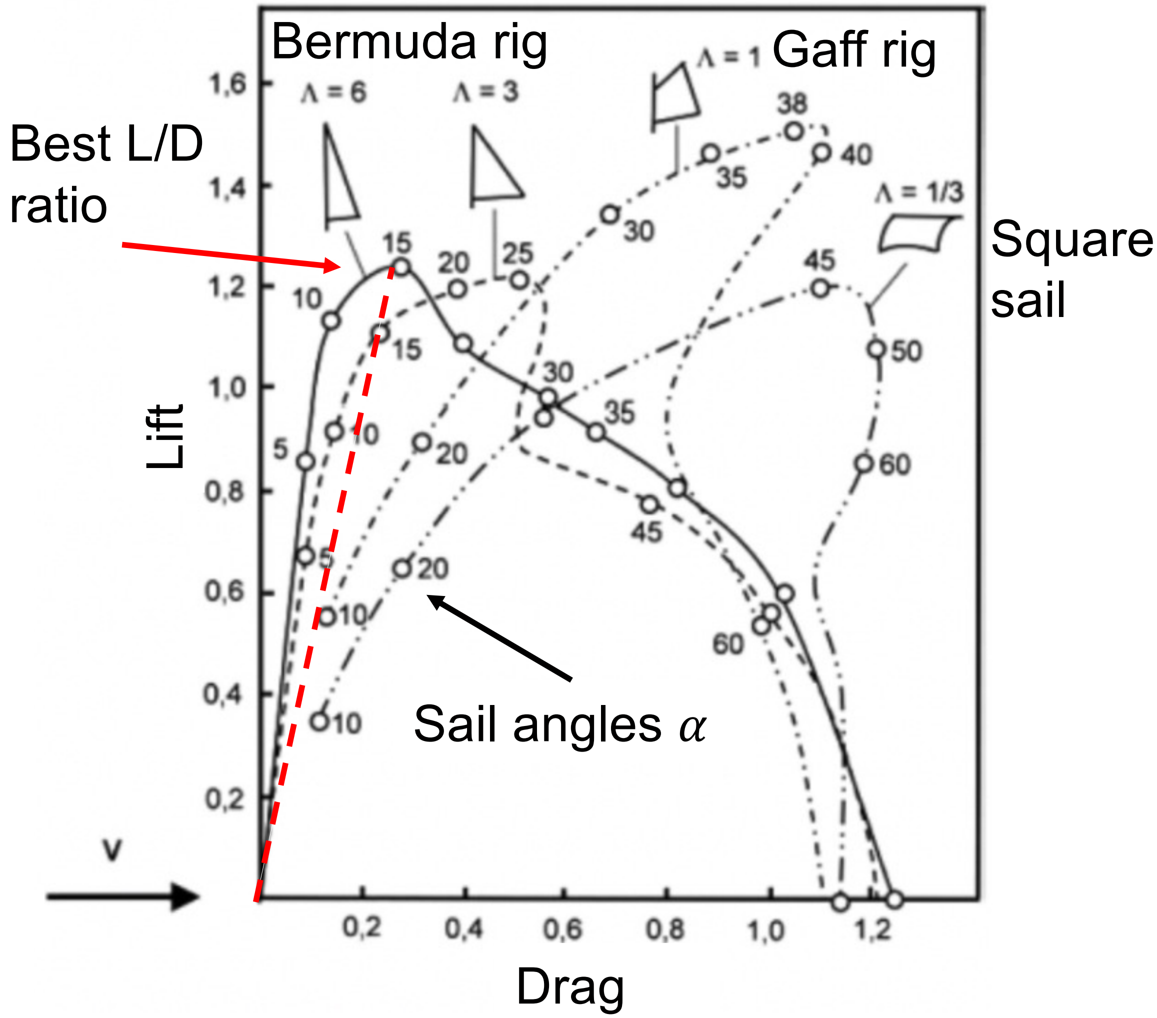
1851s Schooner America

1895s America Cup Defender

1985s Volvo Ocean Race

2020 Vendee Globe Race

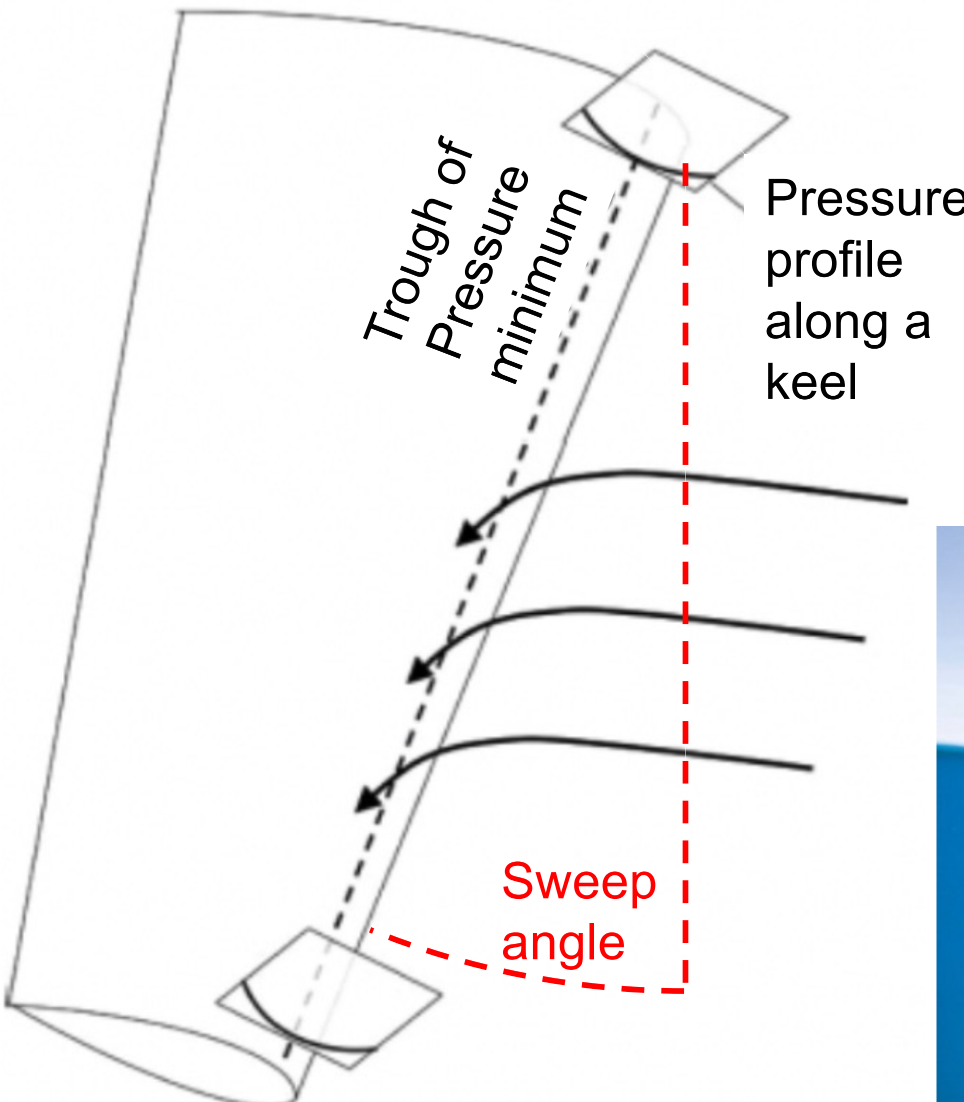
Long versus High Sails



Tall, narrow sails are better for close reach (upwind)

Round deep sails are better for broad reach (downwind)

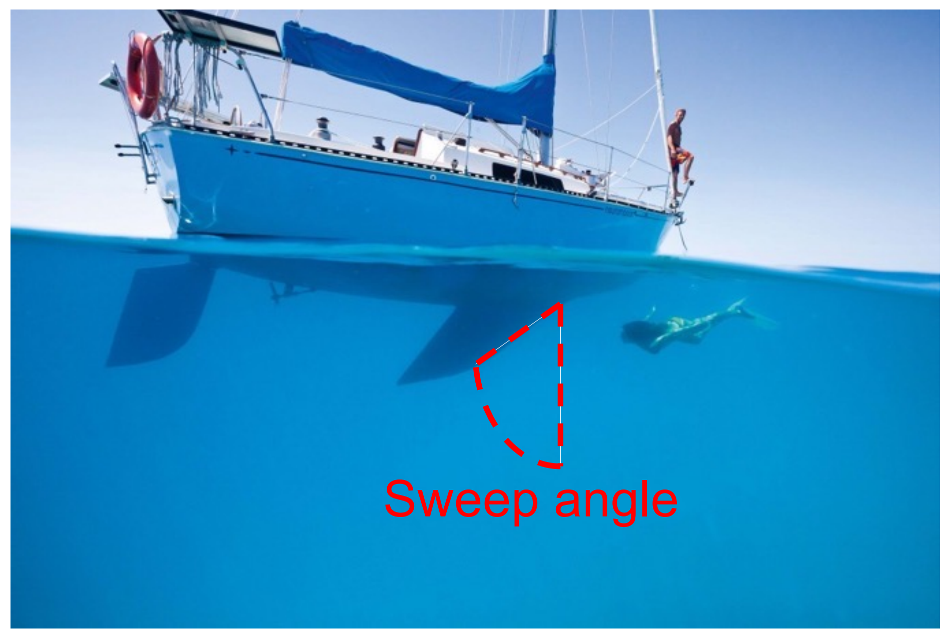
Sweep of Keels



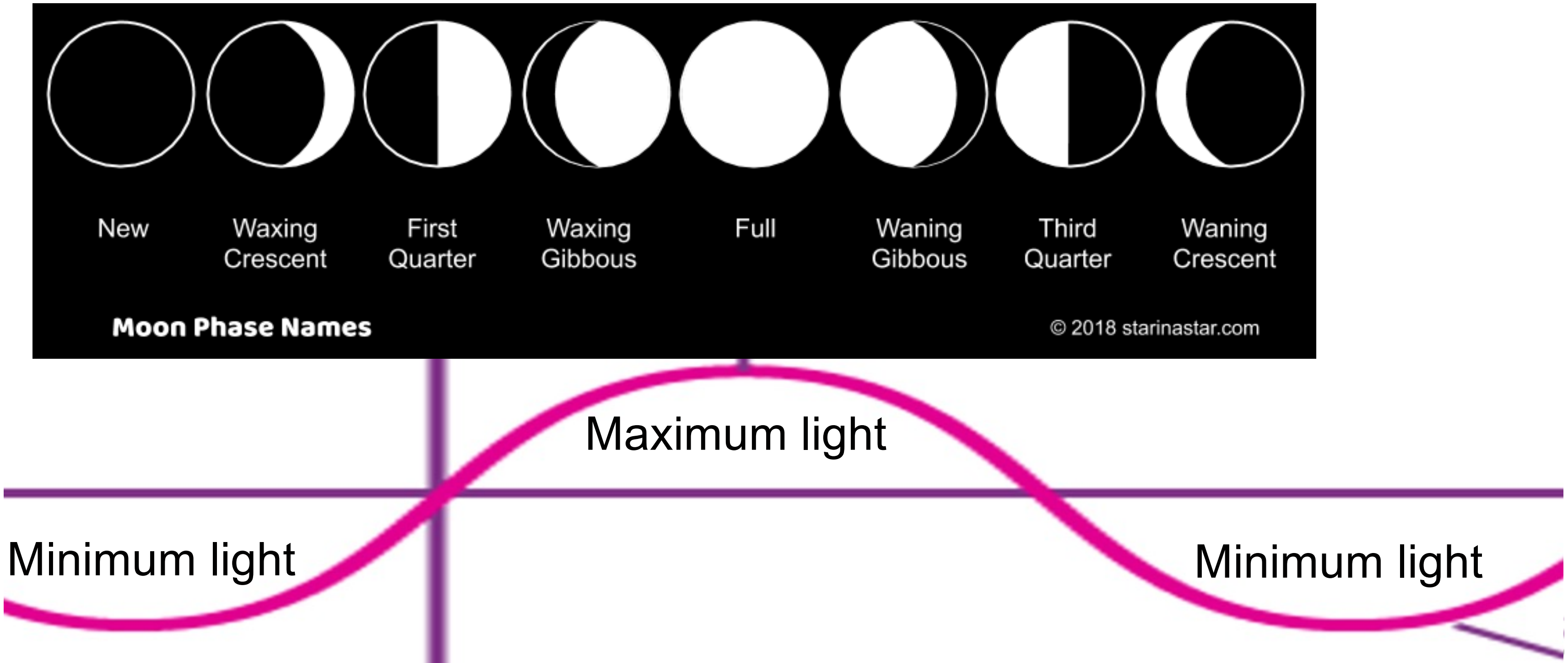
Positive sweep angles:

the fluid flows along the trough minimal pressure toward the bottom of the keel, making it more effective.

The top of the keel that is disturbed by waves becomes less important.

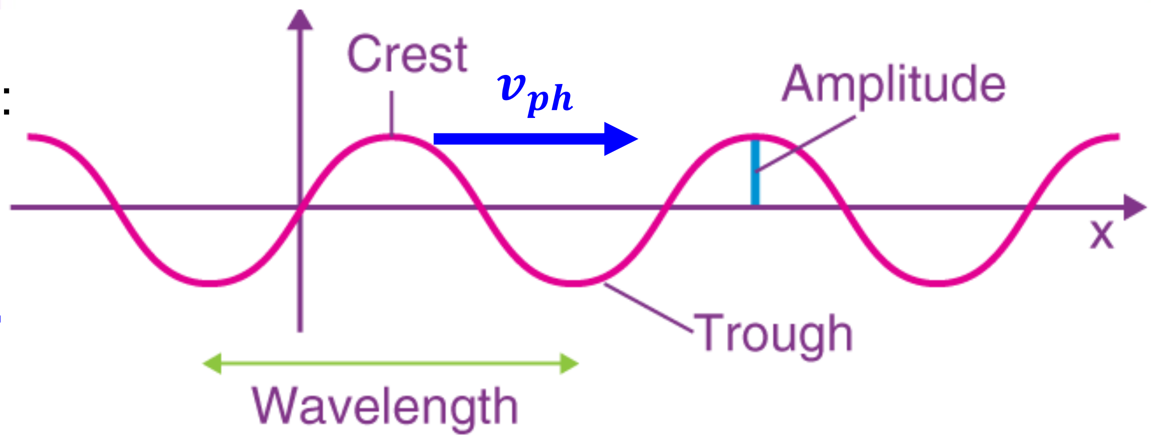


Phases of a Wave

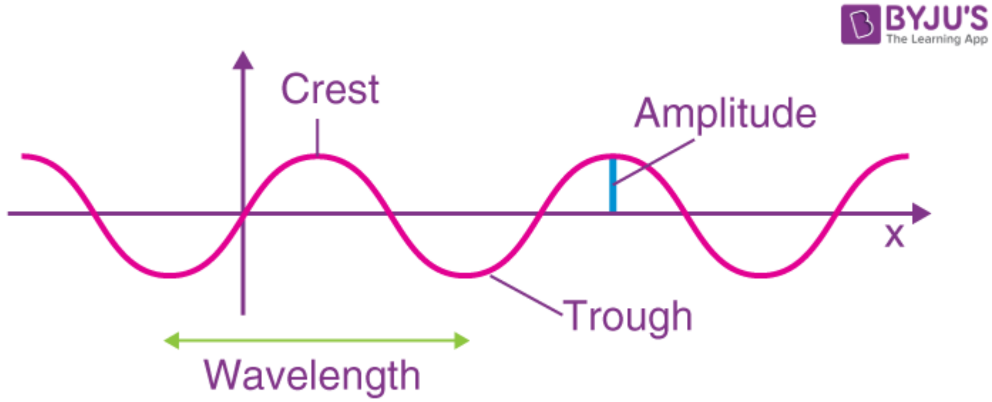


A water wave also has phases:

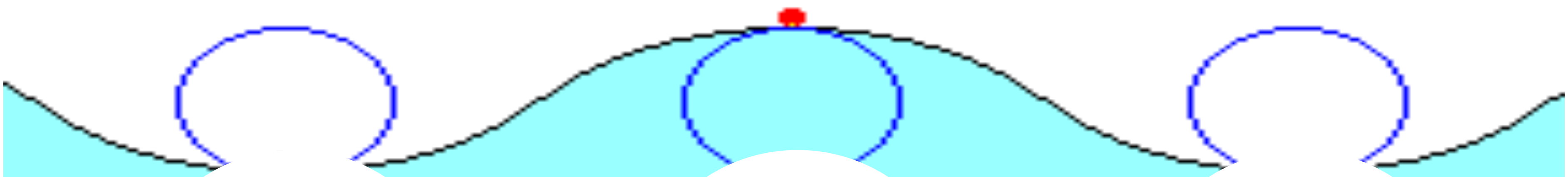
The velocity of the crest is called the **phase velocity** v_{ph}.



The Air-Water Interphase - Waves



If the water does not flow, water particles must move as much to the right as they move to the left. And waves make particles on the surface move up as much as down.



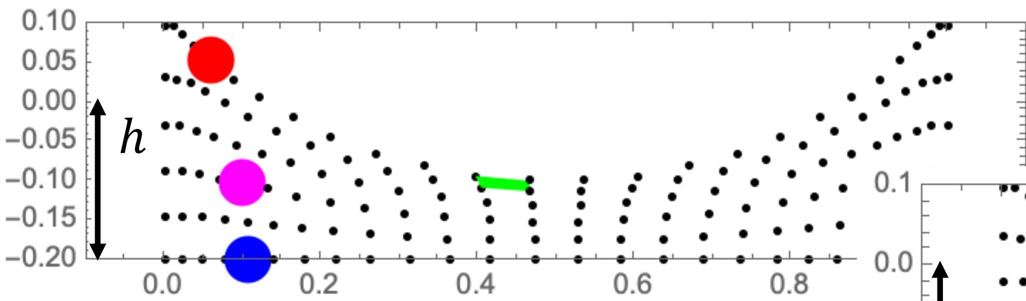
Periodic motion of a pater particle on the surface of a wave

While transporting water from the wave crests to the trough, all particles oscillate back and for the and up and down periodically on elliptical trajectories.

Particle Motion in Water Waves

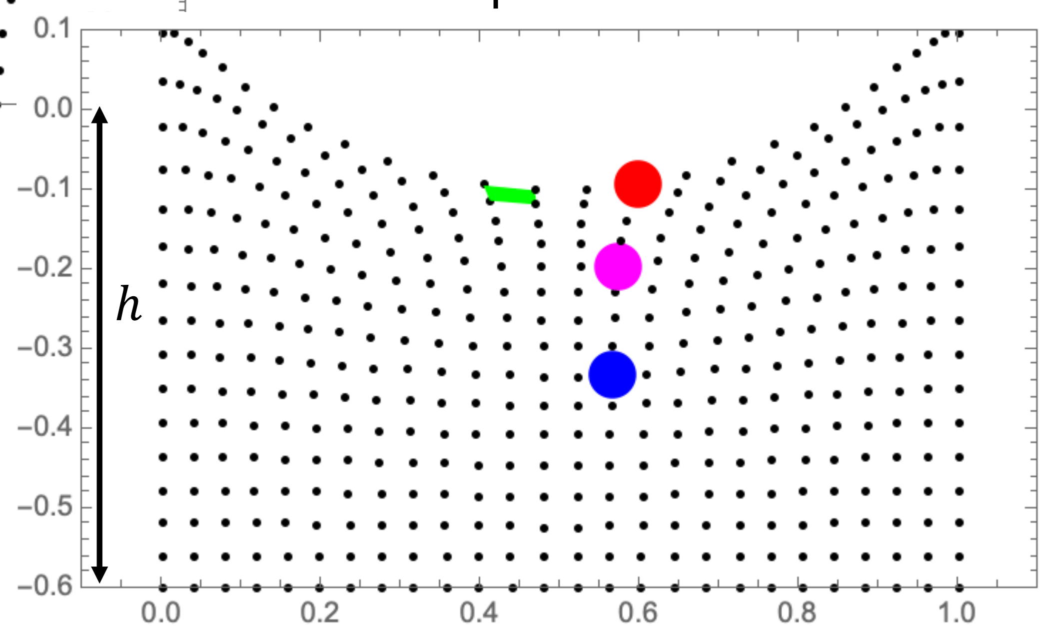
While transporting water from the wave crests to the trough, all particles oscillate back and for the and up and down periodically.

- At the same time, the water cannot be compressed anywhere. (**Green** shape keeps the same area)
- And at the sea floor, there cannot be any vertical motion
- If there is no circulation, formulas for the particle motion can be derived.



Shallow-water waves

Deep-water waves



➔ The horizontal oscillation diminishes with $cosh\big(k(h+z)\big)$.

The vertical oscillation diminishes with $sinh\big(k(h+z)\big)$.

Deep and Shallow-Water Waves

Assuming harmonic motion:

A particle initially at x_0, z_0 is at $\mathrm{x}_0 + A_x(z_0) sin(\omega\, t - k\, x_0)$ and $z_0 + A_z(z_0) cos(\omega\, t - k\, x_0)$.

It's velocity is

$\omega A_x(z_0) cos(\omega\, t - k\, x_0)$ and

$-\omega A_z(z_0) sin(\omega\, t - k\, x_0)$.

To 1st order in small amplitudes:

$$v_x = \omega A_x(z) cos(\omega\, t - k\, x)$$
$$v_z = -\omega A_z(z) sin(\omega\, t - k\, x)$$

Uncompressible means $\partial_x v_x + \partial_z v_z = 0$ or $kA_x(z) = \partial_z A_z$

Circulation free means $\partial_x v_z = \partial_z v_x$ or $kA_z(z)= \partial_z A_x(z)$

➔ $\partial_z^2 A_z(z)= k^2 A_z(z)$ with $A_z(0) = A$ and $A_z(-h) = 0$

➔ $A_z(z) = A \frac{sinh(k(h+z))}{sinh(k\, h)}$ and $A_x(z) = A \frac{cosh(k(h+z))}{sinh(k\, h)}$.

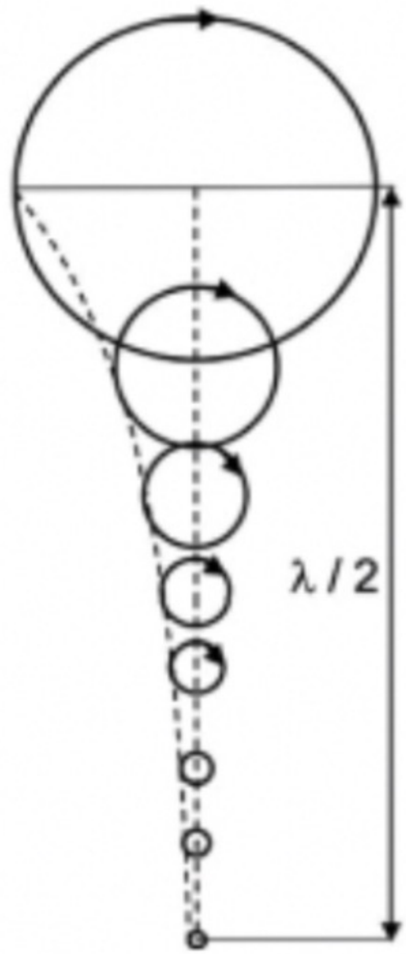
➔ For $k\, h \gg 1$, $A_z(z) = \mathrm{A_x(z)}$, i.e., circular motion.

Deep and Shallow-Water Waves

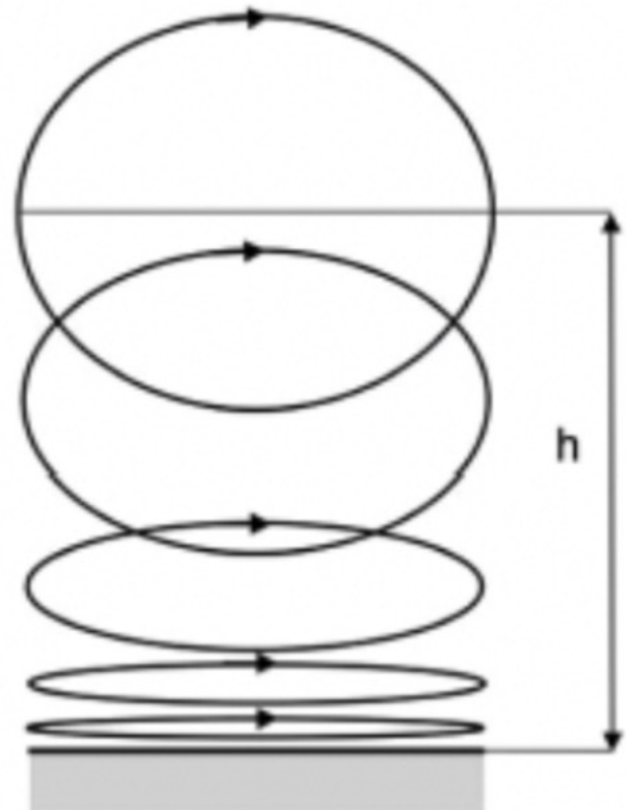
The wave structure becomes independent of the water depth when it is deeper than half the wavelength.

- Waves in Cayuga lake are deep-water waves. They don't change with depth, except directly at shore.
- Several km long tsunami waves in the ocean are shallow water waves. Their characteristics change with depth.

Deep-water waves

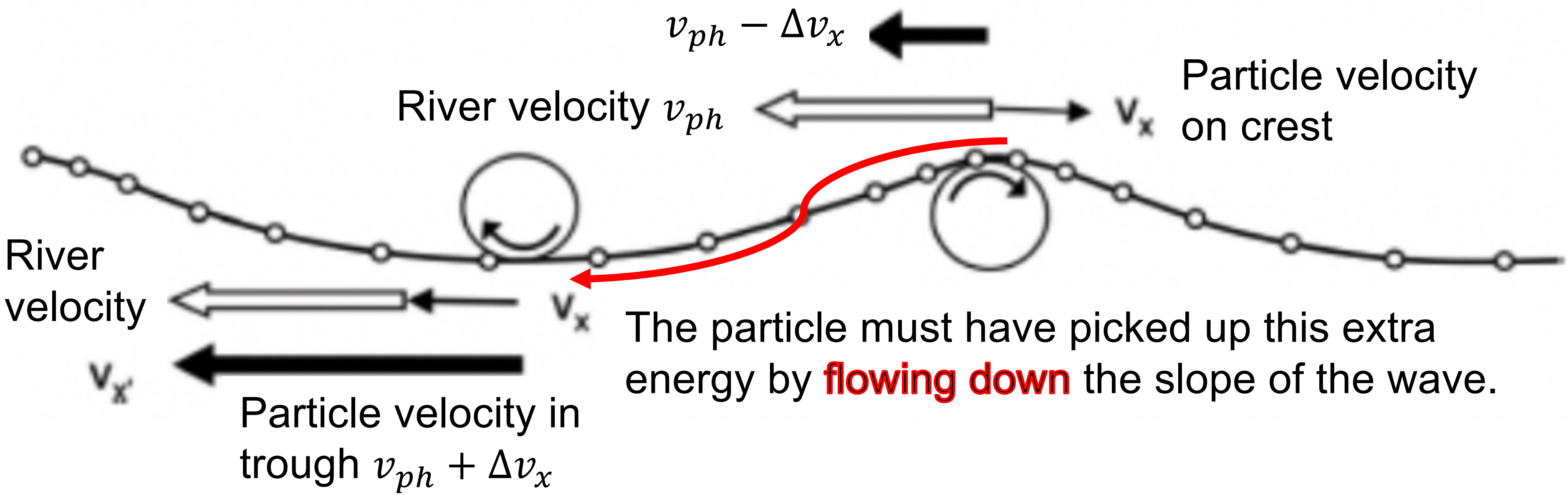


Shallow-water waves



Stationary Waves in a River

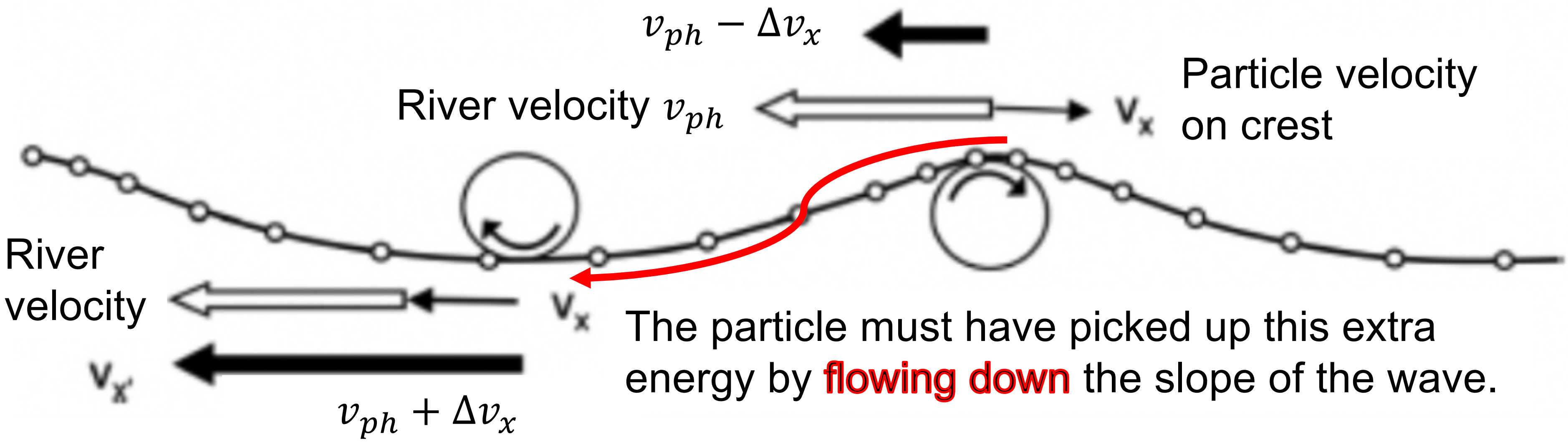
If a river flows at the same speed as the wave, it's phase velocity v_{ph}, but in opposite direction, the wave pattern is stationary, and water particles flow up and down through that pattern.



A general wave could have any wavelength and any oscillation frequency. But energy conservation for a water particle describes how much its velocity is changed by flowing down a hill under gravitational acceleration g.

The velocity Δv_x depends on the oscillation frequency, and having it change just by the right amount while flowing down a quarter wavelength means that for any wavelength only velocity v_{ph} is possible.

Stationary Waves in a River



The particle must have picked up this extra energy by **flowing down** the slope of the wave.

Energy conservation for a water mass m describes how much its velocity changes by flowing down a hill under gravitational acceleration g.

$$\frac{1}{2}m(v_{ph} - \Delta v_x)^2 + mg\Delta z = \frac{1}{2}m\left(v_{ph} + \Delta v_x\right)^2$$

For small amplitudes: $2\, v_{ph} \cdot \Delta v_x = g\Delta z$

$2\, v_{ph} \cdot \omega \cosh(kh) = g \sinh(kh)$ with $\mathrm{v_{ph}} = \frac{\omega}{k}$

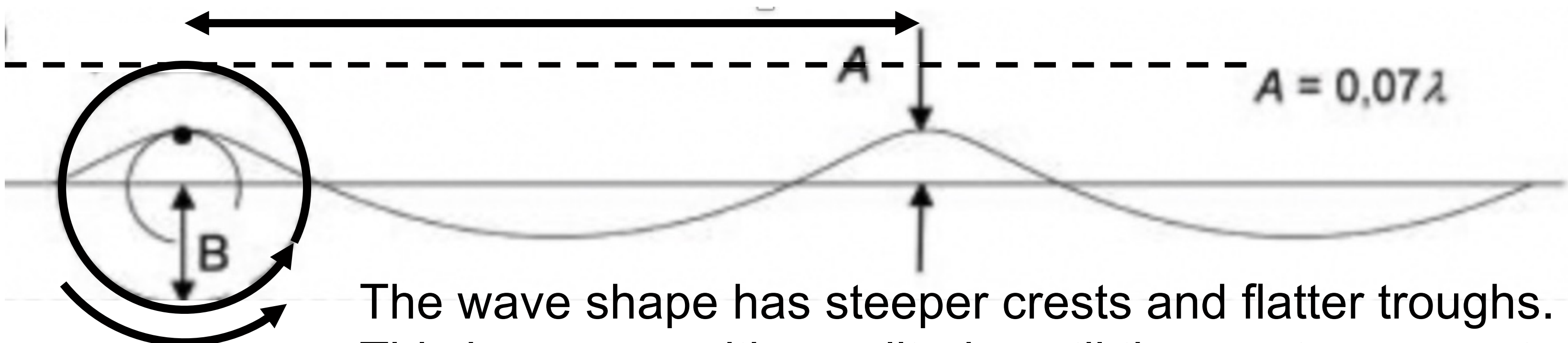
➔ $\omega = \sqrt{k\ g \tanh(kh)}$ and Phase velocity: $\mathrm{v_{ph}} = \sqrt{\frac{g}{k}\tanh(kh)}$

Each wavelength has its own velocity!

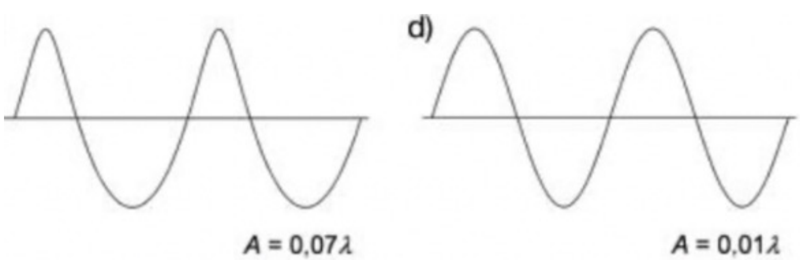
Deep-Water waves

For very large depth, $h \gg \lambda$: $v_{ph} = \sqrt{\frac{g\,\lambda}{2\pi}} \approx 1.3\text{kn}\sqrt{\lambda[ft]}$

Wave shape parametrized by x_0: $x = x_0 - A\,sin(k\,x_0)$, $z = A\,cos(k\,x_0)$.



The wave shape has steeper crests and flatter troughs. This increases with amplitude until the crests are so steep that the wind catches them am produces white caps.



Largest waves:

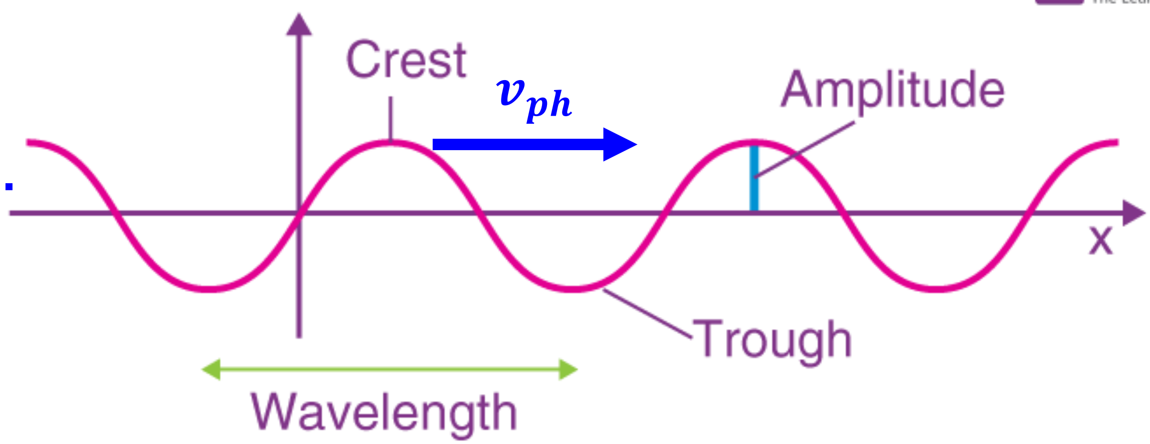
a) $\frac{\Delta x}{\Delta x_0} > 0 \rightarrow A < \frac{\lambda}{2\pi}$

b) Vertical acceleration: $\ddot{z} = A\omega^2 - g = Ak - 1 < 0$

c) In reality: $A < \frac{\lambda}{14}$

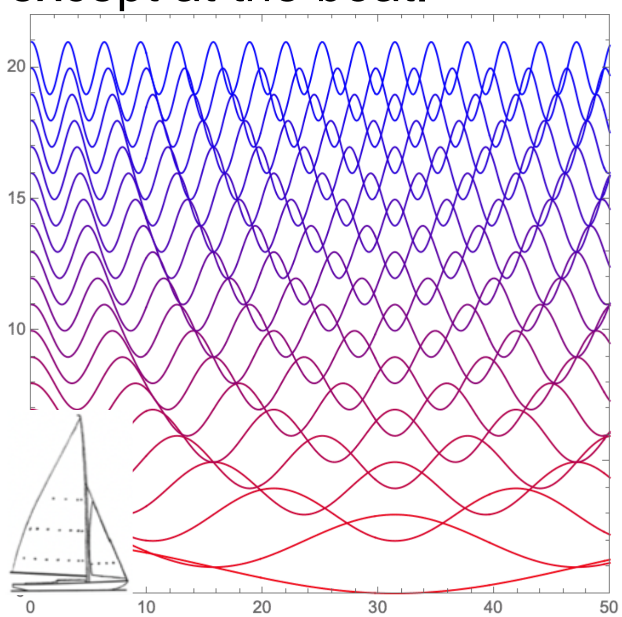
Group Velocity for Deep-Water Waves

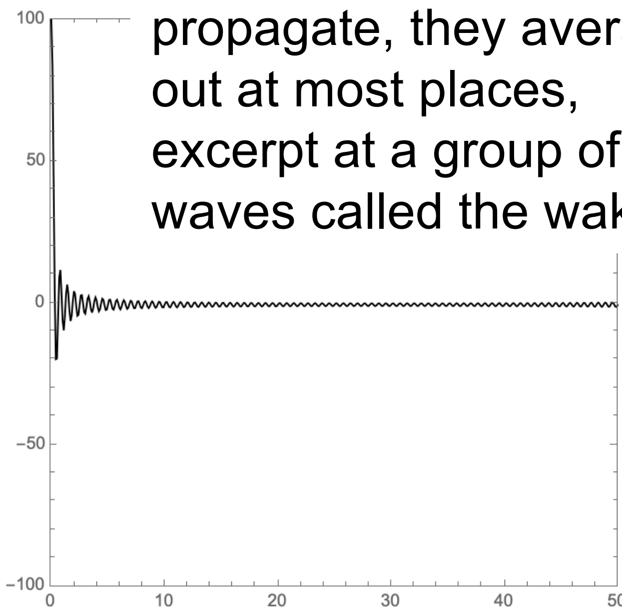
A water wave also has phases:



The velocity of the crest is called the **phase velocity** v_{ph}.

A boat excites waves of many wavelength. Initially their sum averages out everywhere, except at the boat.





When all these waves propagate, they average out at most places, excerpt at a group of waves called the wake.

For deep-water waves, $v_{gr} = \frac{1}{2} v_{ph}$

(from $v_{gr} = \mathrm{d}\omega/dk$)

➔ Observe on the lake how small waves pass from the back to the front of a wake pattern.

Shallow-Water waves

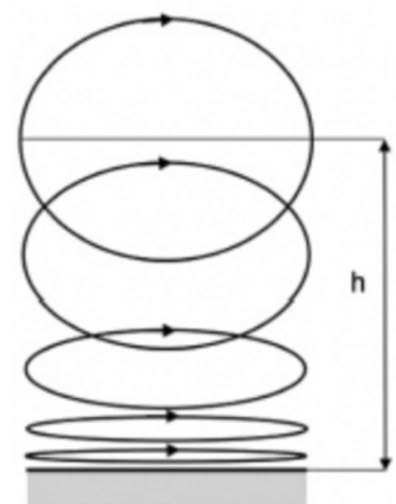
For small depth, $h \ll \lambda$:

$$v_{ph} = \sqrt{gh} \approx 3.3\text{kn}\sqrt{h[ft]}$$

1) All wavelength have the same velocity. ➔ There is no dispersion of waves with different wavelength.

2) At smaller depth, wave becomes slower. ➔ Waves get larger at the beach while they pile up.

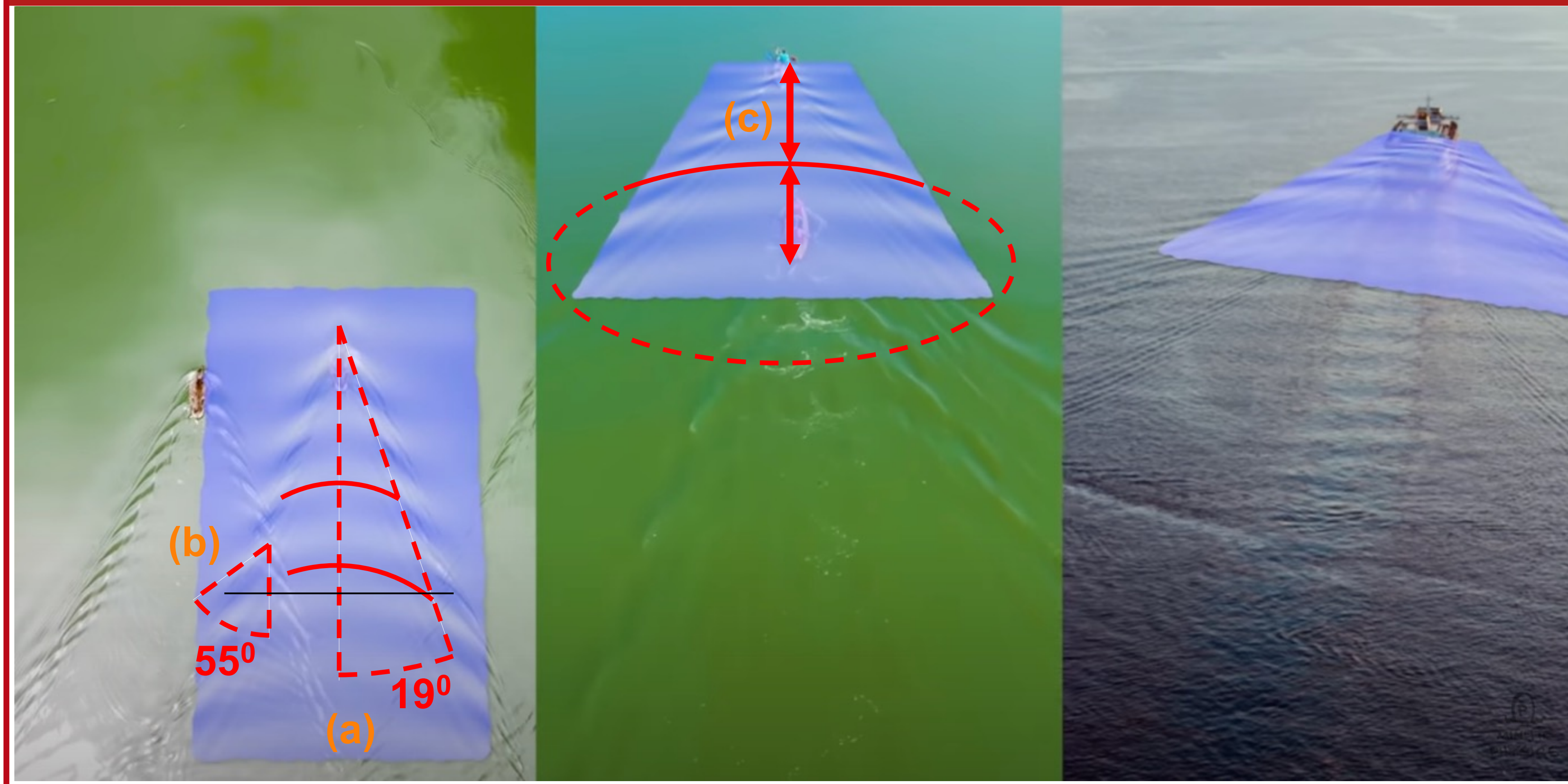
3) Tsunamis are most devastating at shallow costs, because of this pileup, and because their particles mostly move horizontally.



4) Tsunami waves are always shallow-water waves, even in the ocean. They have several 100km wavelength, but the oceans are on average around 4km deep.

5) At 4km depth, their speed is $\sqrt{gh} = 430\,\frac{mi}{h}$, 15h from Chile to Hawaii.

Wakes of Deep-Water Waves



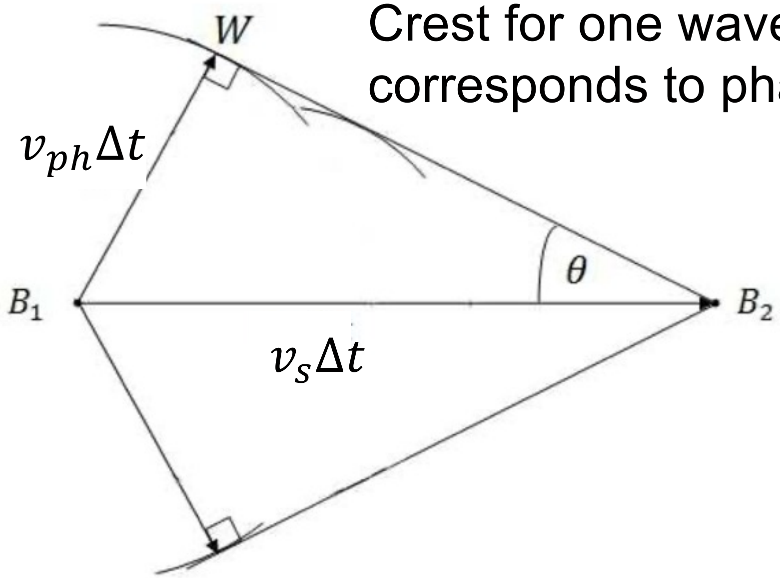
The wake of boats in deep water look very similar, independent of boat and speed.

a) There is a concave wave system behind the boat, with 19^0 transverse extend.

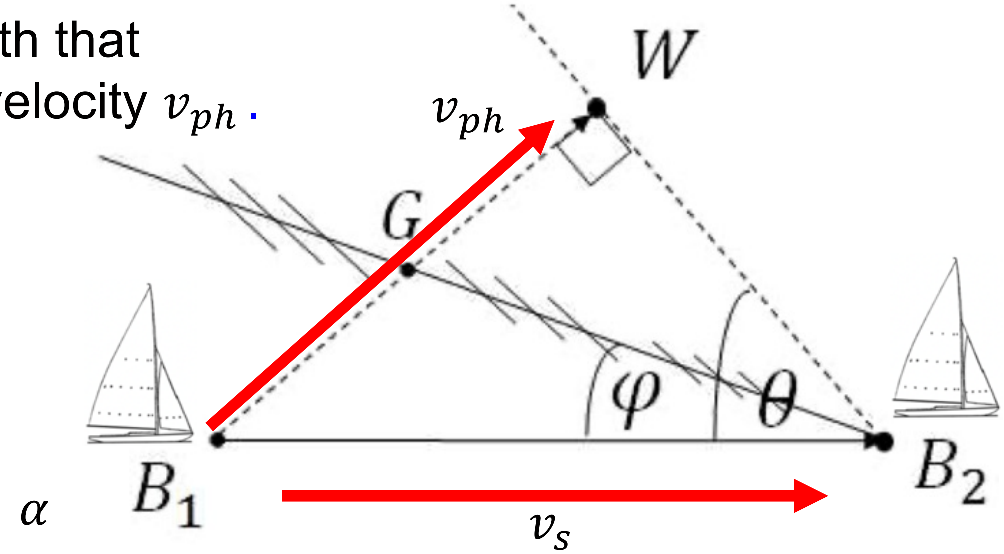
b) There is a divergent system with an angle of 35 degrees.

c) The concave waves have a radius equal to their distance from the boat.

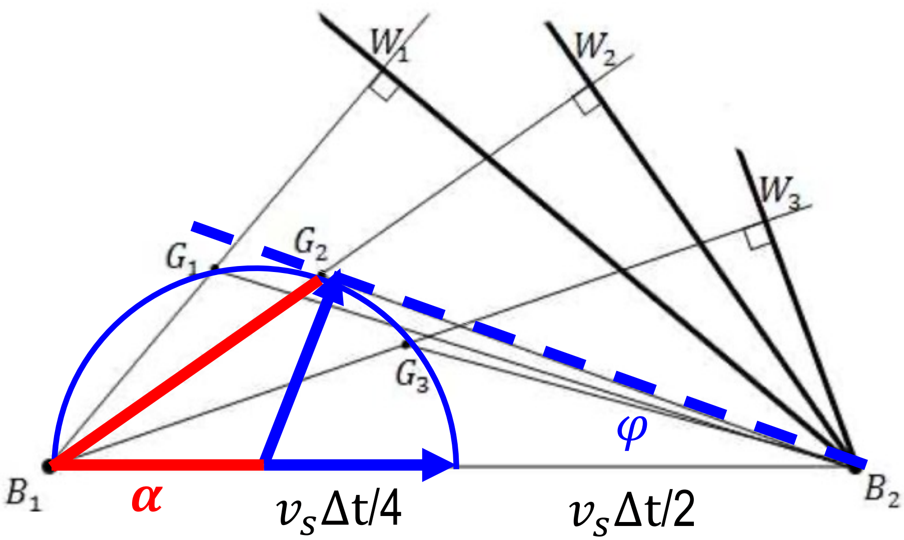
Waves behind a boat



Crest for one wavelength that corresponds to phase velocity v_{ph}.



Because different wavelengths are created, the main feature travels with the group velocity, i.e., with $v_{gh} = \frac{1}{2} v_{ph}$.



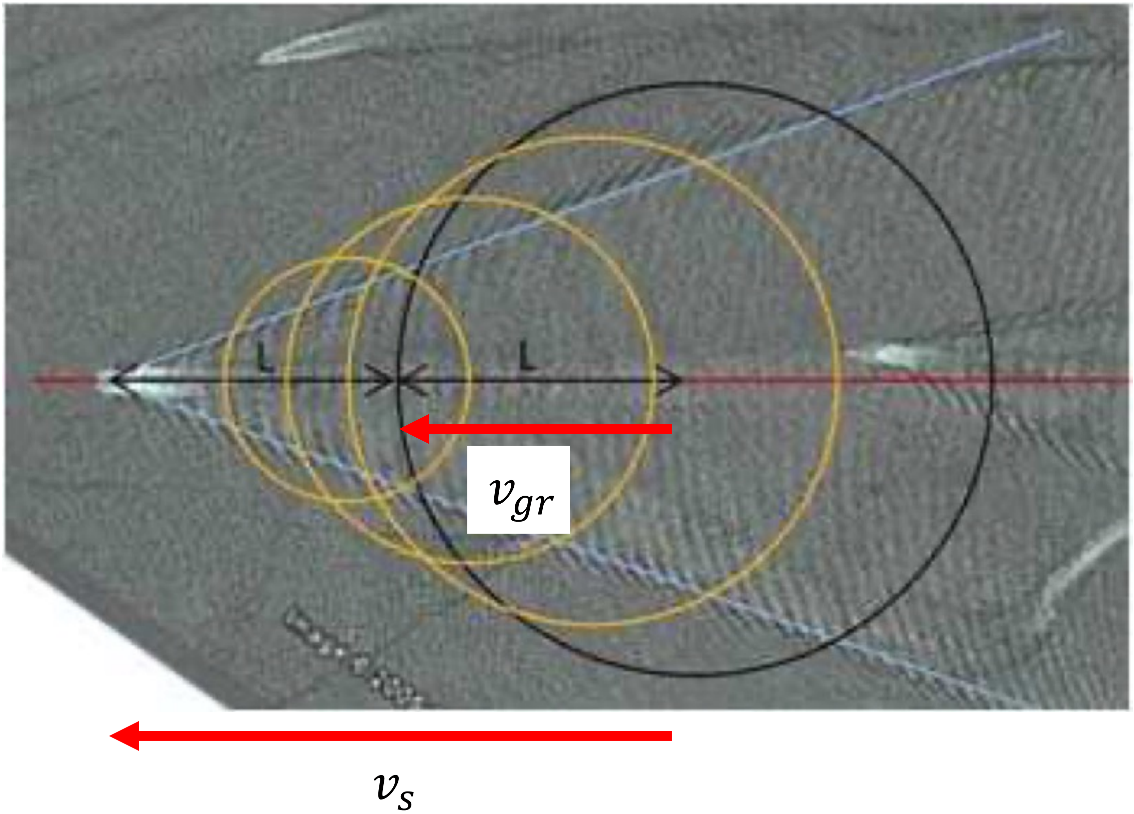
All these main features occur most densely packed at G_2:

$$\varphi = Arcsin\left(\frac{1}{3}\right) = 19°$$

The waves there propagate in toward W_2:

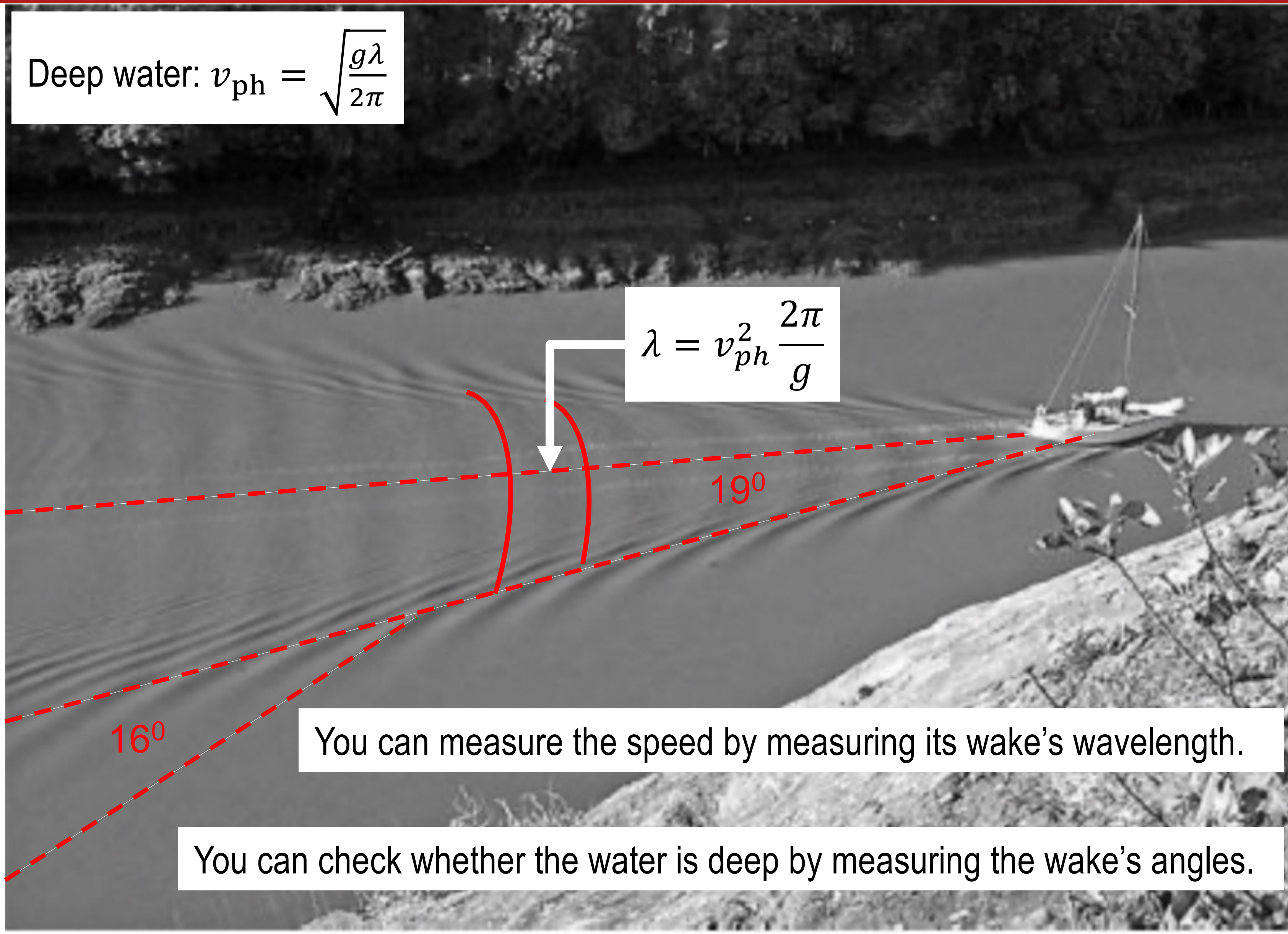
$$\theta = 90° - \alpha = 45° - \frac{\varphi}{2} = 55°$$

Waves behind a boat



- For the waves that keep up with the boat $v_{ph} = v_s$
- There are waves with a spread of frequencies emitted ➔ $v_{gr} = \frac{1}{2}\, v_s$.
- Therefore, the dominant wave features have a radius that is ½ the distance of their origin to the boat.

Wavelength in Deep-Water Wakes



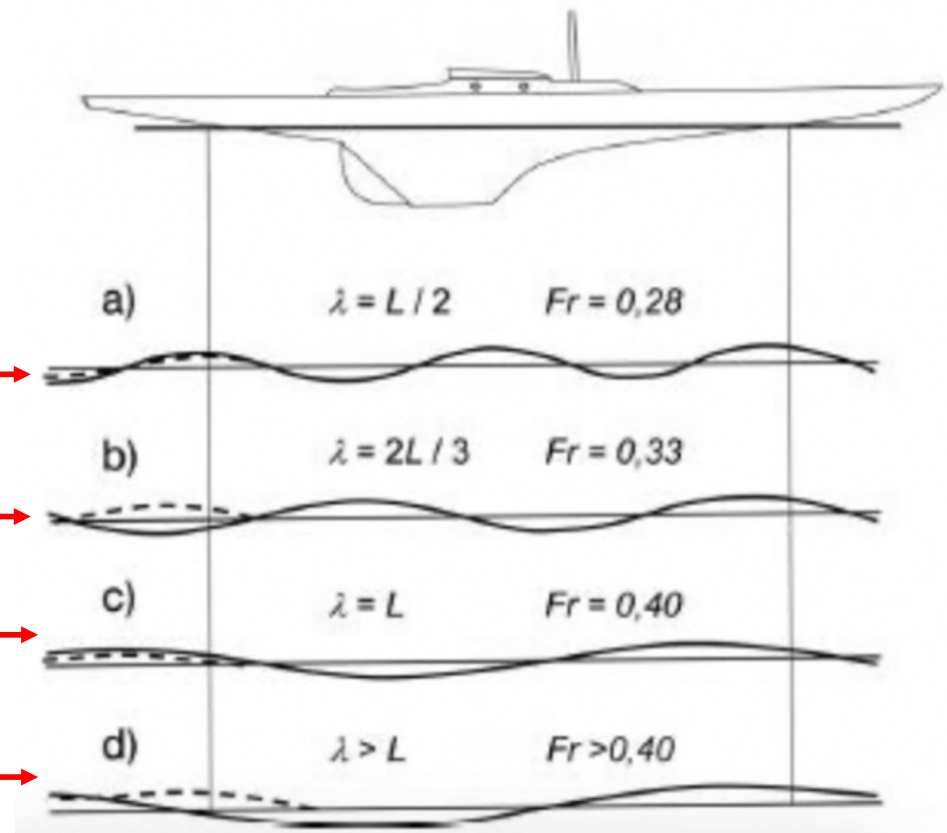
Wave Resistance and Hull Speed

The dominant creation of waves comes from the bow and the stern. It costs energy and creates the wave resistance.

The wave resistance is always a bit larger when the wave from the bow and the stern add to each other, and less when they subtract from each other.

Bow and stern waves add

Bow and stern waves subtract



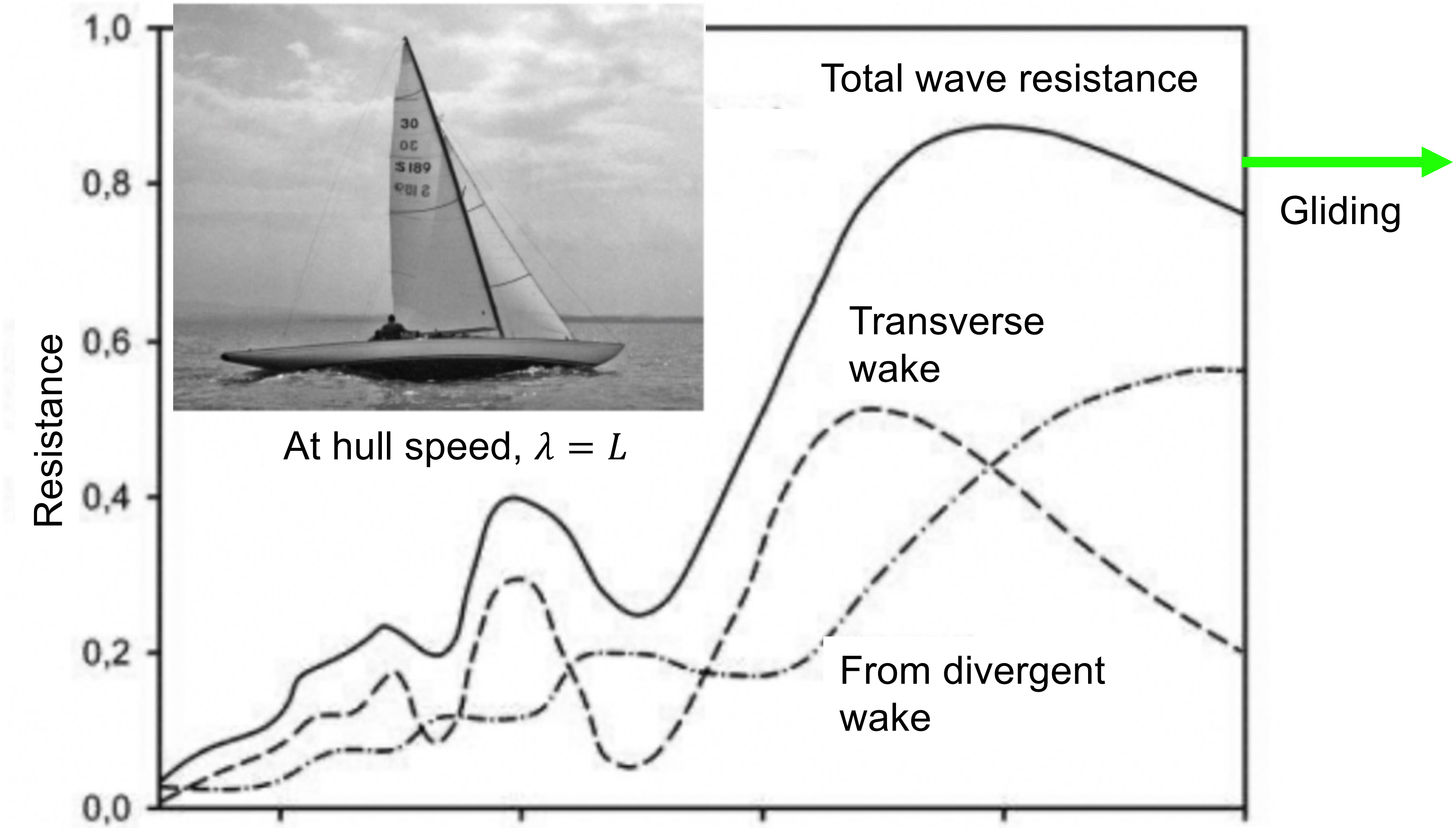
When the boat length L becomes bigger than λ, the boat must sail uphill against the wave

➔ limiting speed = hull speed

➔ $v_{hull} = \sqrt{\frac{gL}{2\pi}} = 1.3\text{kn}\sqrt{L[ft]}$

Froude Number $Fr = 0.4\, v/v_{hull}$

Wave Resistance and Hull Speed



Velocity

$\lambda = 0.25\,L$, $v = v_{hull}/2$

$\lambda = 0.56\,L$, $v = v_{hull}3/4$

$\lambda = L$, $v = v_{hull}$

$\lambda = 1.56\,L$, $v = v_{hull}5/4$

$\lambda = 2.25\,L$, $v = v_{hull}\,3/2$

How much Power does a Yacht Produce?

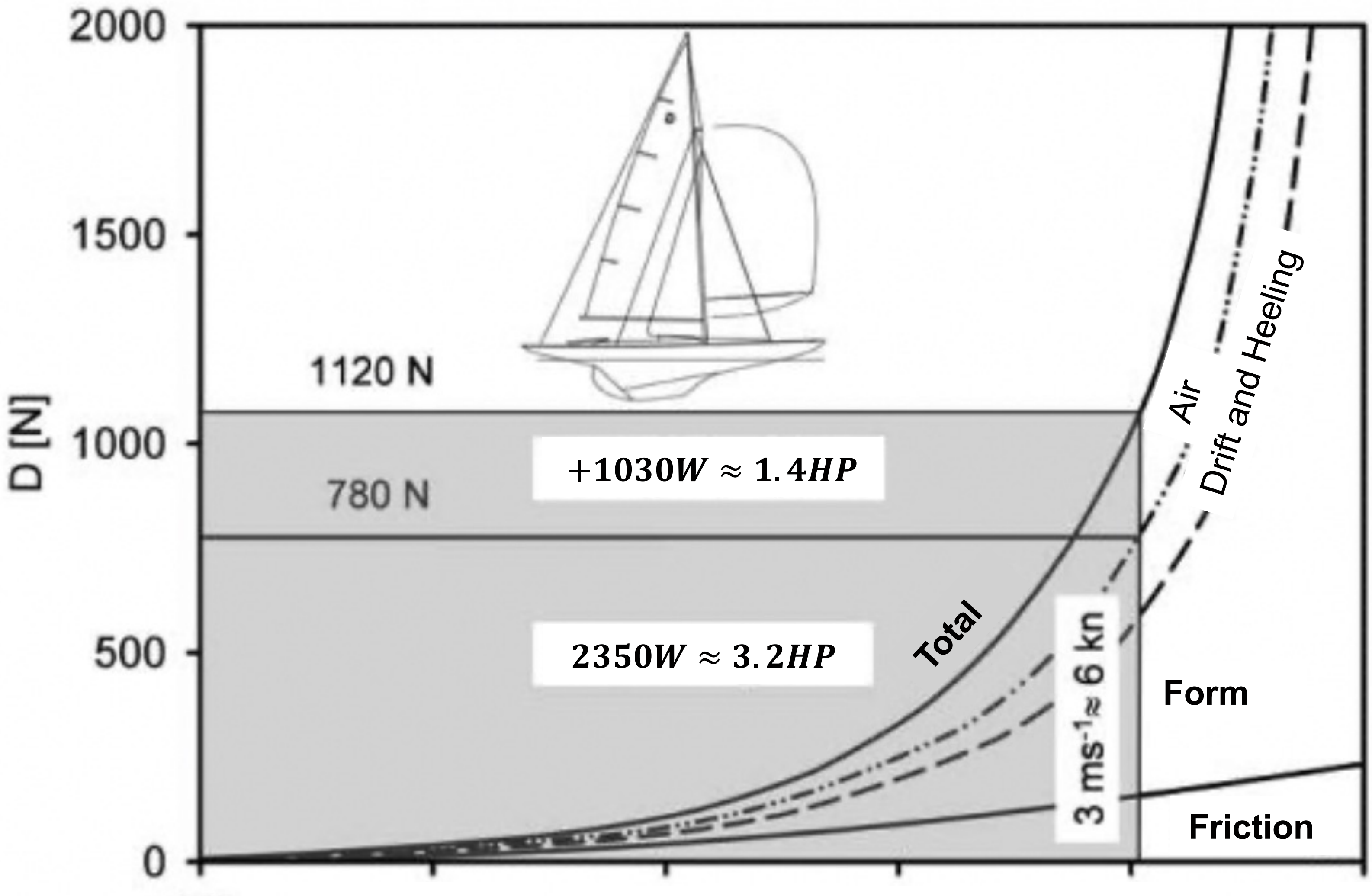
To move the boat, the sails must provide power to overcome the following effects:

1. Friction of water along the boat.
2. Increase of water friction because of sideways drift.
3. Increase of water friction because of heeling.
4. Friction in air from sail, mast, rigging, cabin, and hull, and circulation.
5. Form resistance, e.g., energy put into waves and eddies behind keel.

How does the power of sailing yachts compare to motorboats?

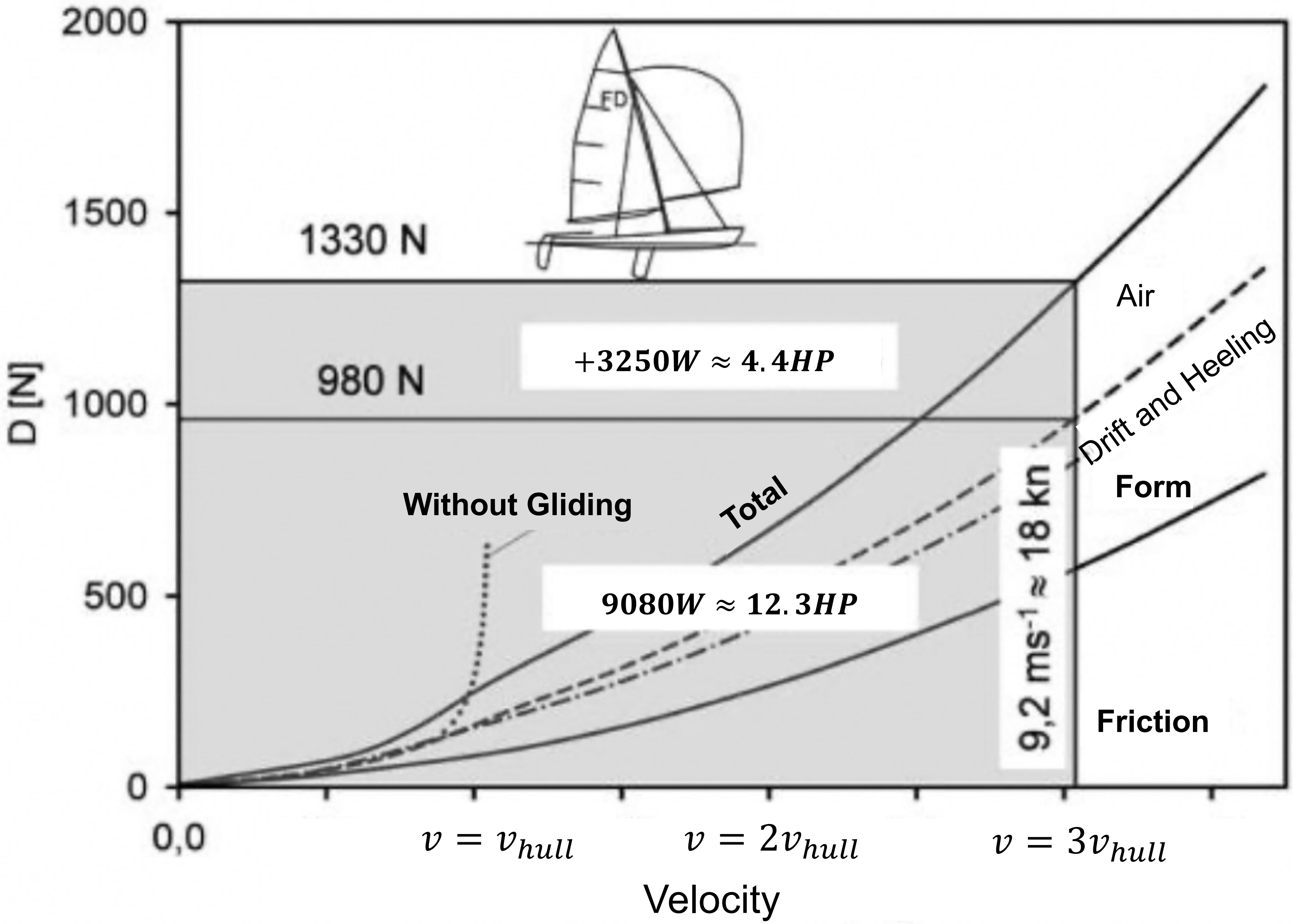
Example: a 350hp engine.

How much Power does a Yacht Produce?



	$\lambda = 0.13\ L$	$\lambda = 0.25\ L$	$\lambda = 0.65L$	$\lambda = L$	$\lambda = 1.56\ L$
Velocity	$v = v_{hull}/4$	$v = v_{hull}/2$	$v = v_{hull}3/4$	$v = v_{hull}$	$v = v_{hull}5/4$

Power for Gliding



Review: The Reynolds Number

Result: If the ratio of viscosity forces (from forces between molecules) and kinetic force (from moving masses) are the same, then the flow pattern is the same.

This ratio is the Reynold's number and scales like: $R = \frac{\rho}{\mu} \cdot v \cdot size$

$size$: a characteristic length, e.g., length of the sail.

v: speed of the fluid

ρ: density

μ: viscosity

Because $F_{mass} \propto \rho \cdot v^2$, $F_{friction} \propto \mu \cdot v/size$

$\frac{\mu}{\rho}$ is 15 times larger in air than in water.

➔ A hull model in air produces the same flow pattern as in water, if the air velocity is 15 times larger.

Flow model of an airplane wing

Review: Wave Resistance and Hull Speed

The dominant creation of waves comes from the bow and the stern. It costs energy and creates the wave resistance.

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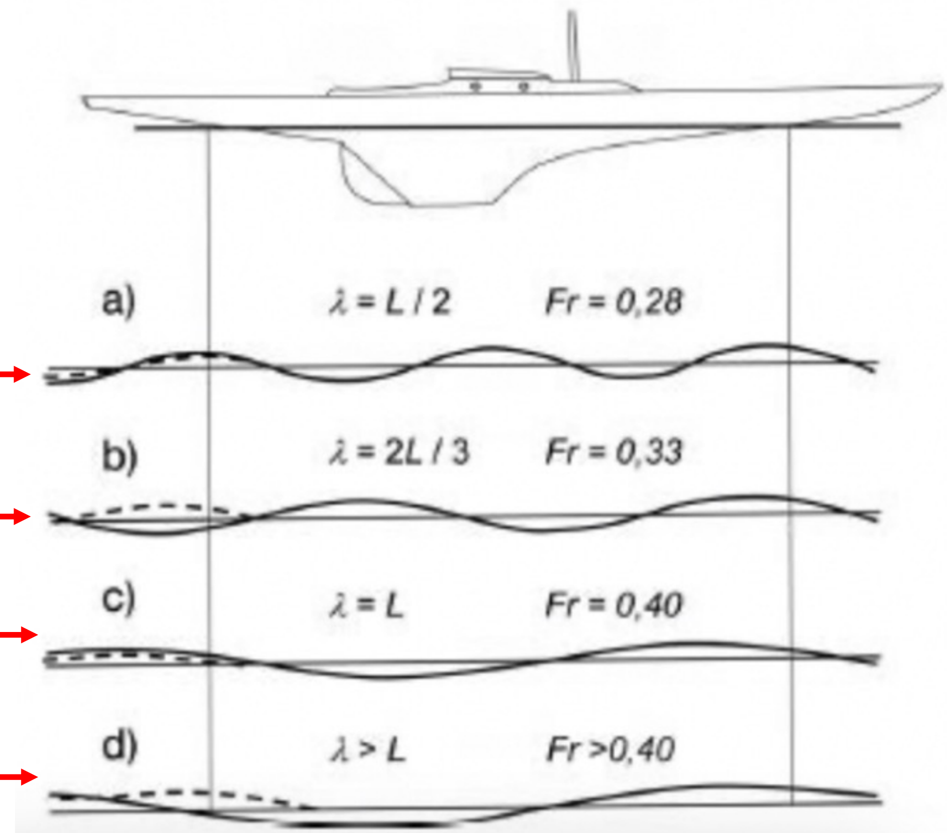

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➔ limiting speed = hull speed

➔ $v_{hull} = \sqrt{\frac{gL}{2\pi}} = 1.3\text{kn}\sqrt{L[ft]}$

Froude Number $Fr = 0.4\ v/v_{hull}$

Optimizing boat shapes on smaller models

Review Reynold's number: $R = \frac{\rho_{density}}{\mu_{viscosity}} \cdot v \cdot size$

Review Froude number: $Fr = \frac{v}{v_{hull}} = v/\sqrt{\frac{gL}{2\pi}}$

1) A smaller boat model has the same flow pattern in air, if it has the same Reynold's number in air, i.e., if $v_{model} = v_{boat} \frac{size_{boat}}{size_{model}}$

2) A smaller boat model has the same flow pattern in water, if it has the same Reynold's number in water, i.e., if $v_{model} = v_{boat} \frac{size_{boat}}{size_{model}}$

3) A smaller boat model has the same wave pattern, if it has the same Froude Number, i.e., if $v_{model} = v_{boat}\sqrt{\frac{size_{model}}{size_{ship}}}$

- There is no velocity where simultaneously the same flow pattern and the same wave pattern can be modeled in smaller size.

Small models augmented by friction calculations

Laminar flow: Friction force is proportional to velocity * size: $F \propto v \cdot size$

Turbulent flow: Friction force $F \propto v^2 \cdot area \propto (v \cdot size)^2$

➔ A model with the same Reynold's number has the same friction force.

A boat's wake pushes the boat back with the wave-resistance force. If the model has the same wave pattern as the boat, it experiences the same wave slopes.

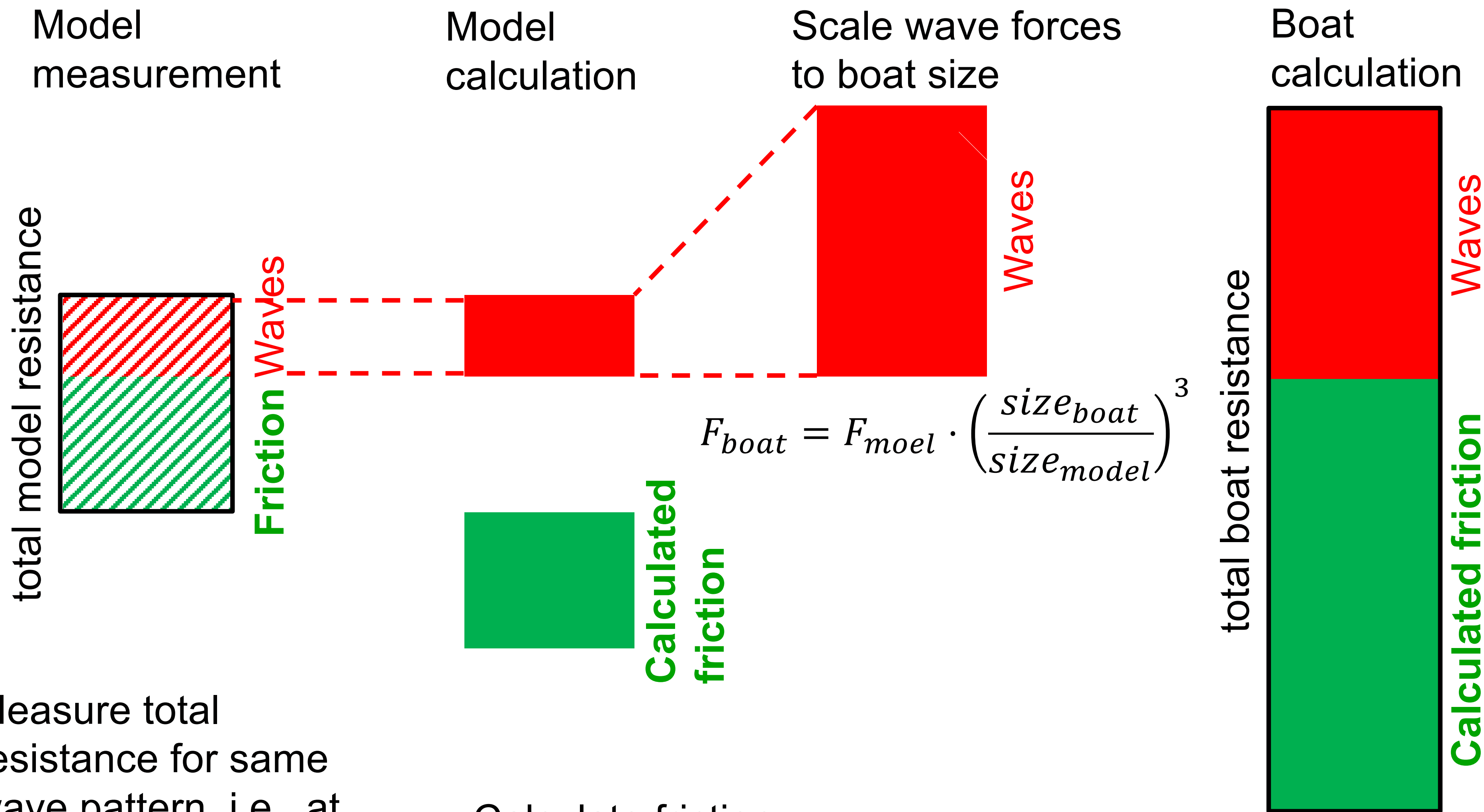
➔ A model with the same Froude number has a wave resistance force

$$F_{boat} = F_{model} \cdot \left(\frac{size_{boat}}{size_{model}}\right)^3$$

Experience: it is harder to compute the wave pattern than the friction forces, especially for laminar flow.

Strategy: Use model to measure the wave resistance and calculate the friction.

Strategy for using model to optimize boats



$$F_{boat} = F_{moel} \cdot \left(\frac{size_{boat}}{size_{model}}\right)^3$$

Measure total resistance for same wave pattern, i.e., at

$$v_{model} = v_{boat}\sqrt{\frac{size_{model}}{size_{boat}}}$$

Calculate friction of the model and subtract

Calculate friction of the boat and add

The Ability to Carry Sail and its Scaling with Size

If all parts of a boat are scaled in size, the sail-carry-number stays unchanged:

$$\mathrm{SCN}^3 = \frac{Sail\ Area^3}{Volume^2} \propto \frac{size^6}{size^6}$$

If one plots the SCN for many keel boats, one sees that larger boats are not just scaled up in size, but that larger boats tent to have larger SCN, i.e., sails are scaled up more than the rest of the boat.

Why can larger boats carry more than the scaled sail?

The heeling torque from a sail is approximately $T_{sail} \propto area \cdot hight \propto size_{sail}^3$

The righting torque from the keel is approximately $T_{keel} \propto dept \cdot mass \propto size_{hull}^4$

➔ The keel of a larger boat can therefore compensate more than a scaled up sail.

➔ In similar wind conditions, the same heel angle occurs if $size_{sail}^3/size_{hull}^4$ for two boats is the same.

Increased sail carrying capacity for larger boats

➔ In similar wind conditions, the same heel angle occurs if $size_{sail}^3/size_{hull}^4$ for two boats is the same, i.e. when sail area $\propto size_{hull}^{8/3} = size_{hull}^{2.7}$

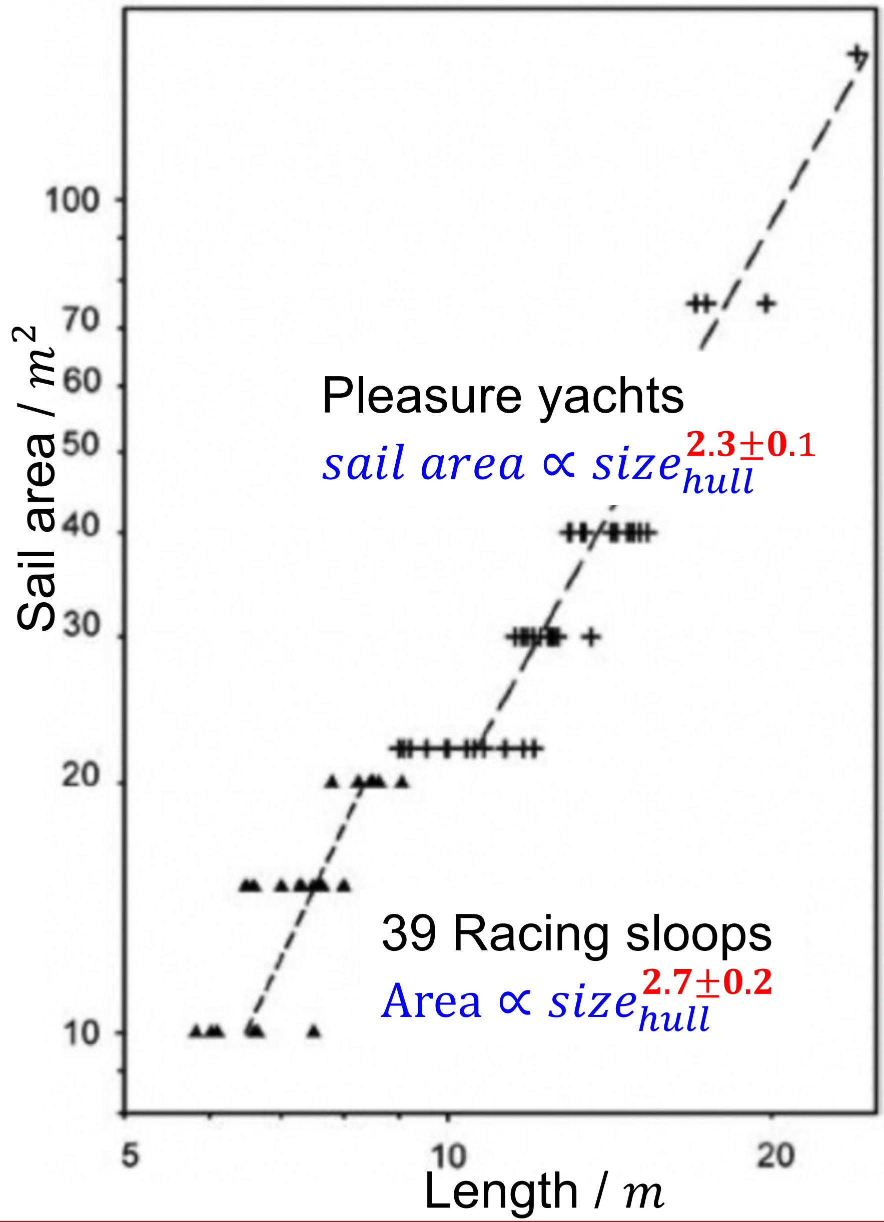
Larger pleasure yachts might scale their sail area less than with 8/3 because

(a) To of a large sail sees higher winds.

(b) Are meant for larger sees.

(c) Are meant for higher wind speeds.

Note: The slope in a double logarythimic plot indicates a power law: $y = a \cdot x^{\alpha}$



Wave resistance for different shapes

Several quantities are used to describe rough shape features:

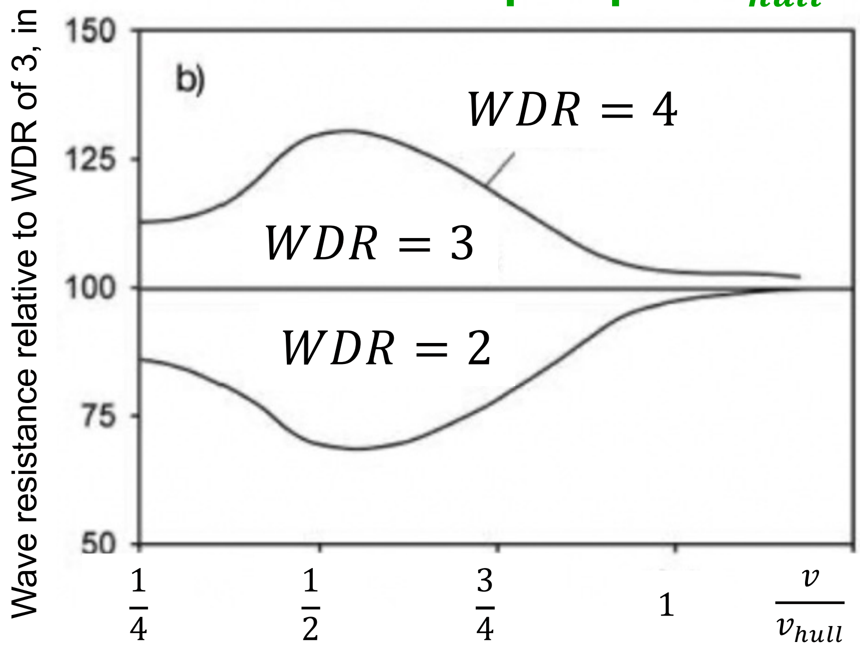
Width of water line = W_{WL}, Length of WL= L_{WL}, Volume displaced water = ∇

Hull depth (without keel) = D_{hull}

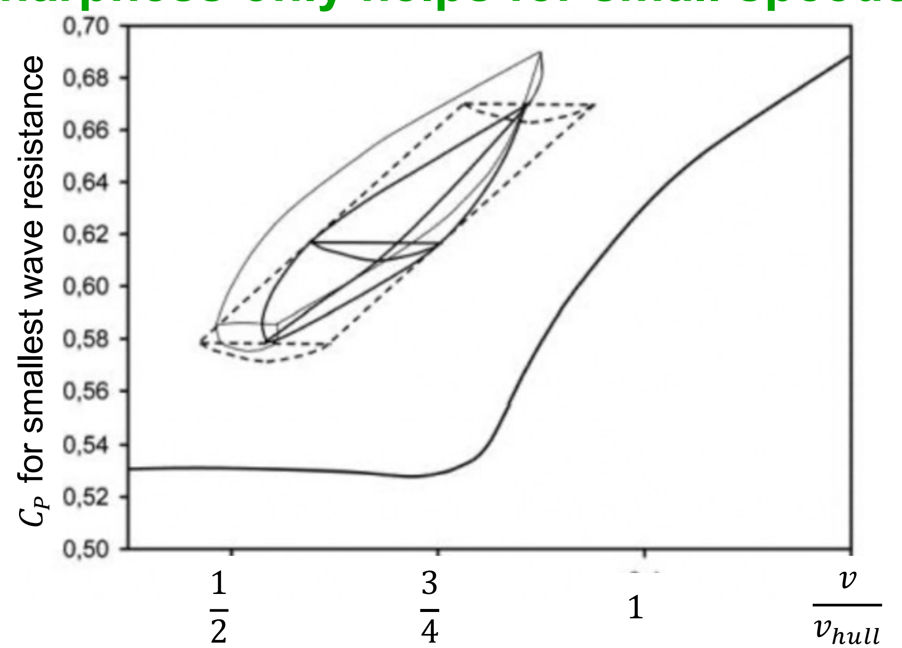
A) Width-Depth-Ratio $WDR = \frac{W_{WL}}{D_{hull}}$, small for deep narrow hulls (like catamarans)

B) Prismatic Coefficient $C_P = \frac{\nabla}{cylinder\ section\ volume}$ is small for sharp boats.

Small DWR helps up to v_{hull}



Sharpness only helps for small speeds



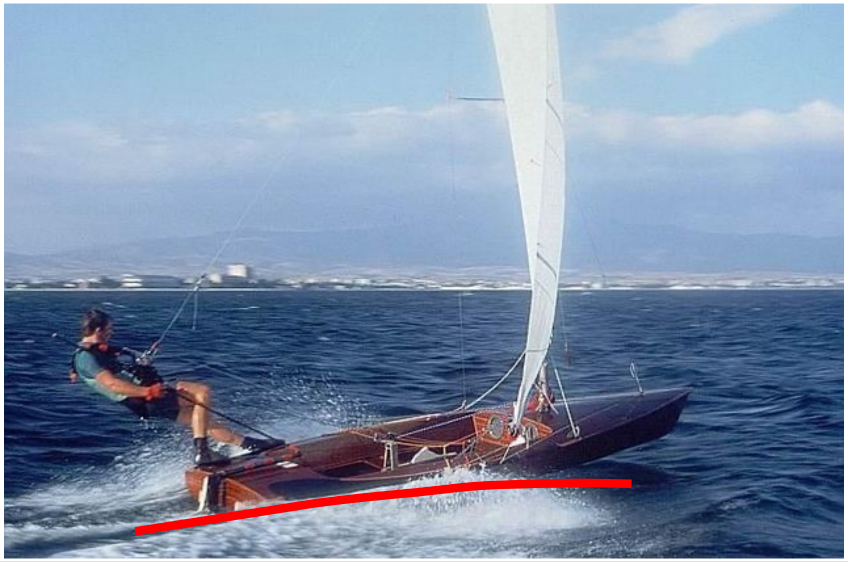
Gliding

Above hull speed, the wavelength of the bow wave becomes longer than the boat length so that the stern gets lowered and the boat sails uphill against its own bow wave.

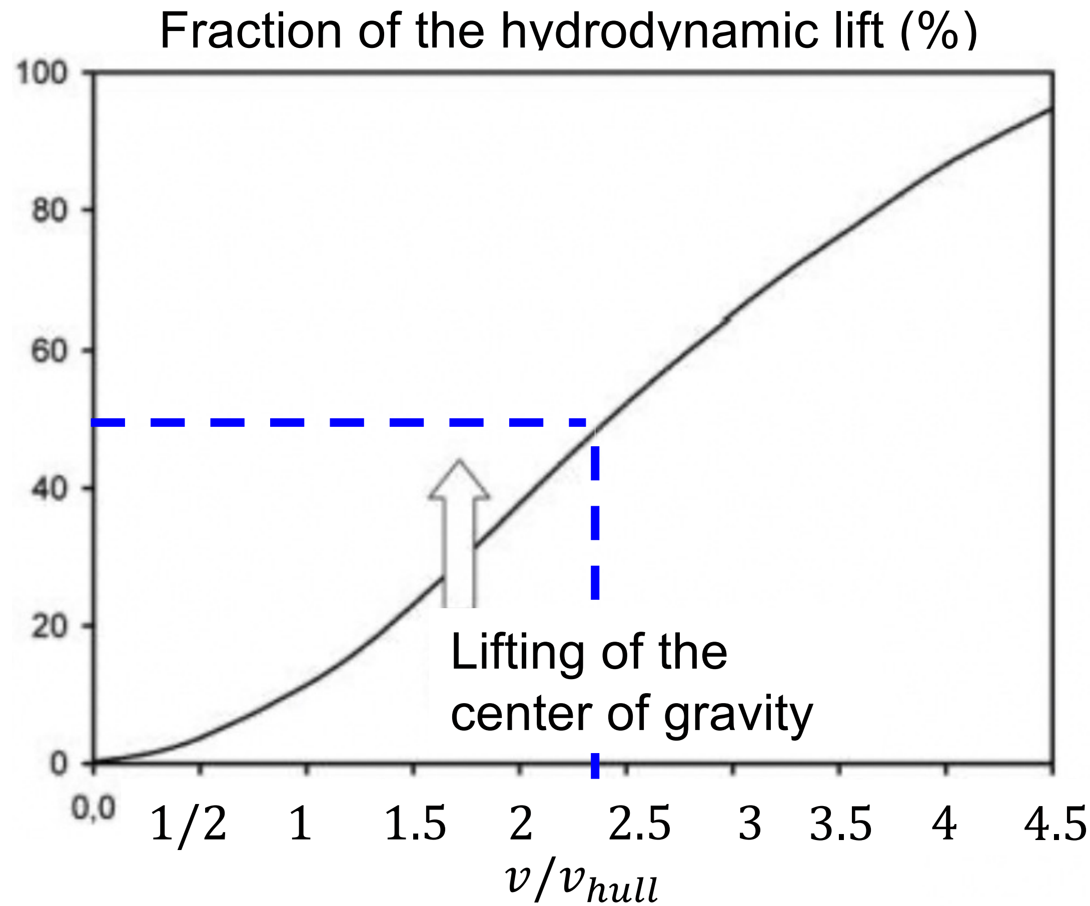
Once the boat is light enough to further accelerate two things happen:

(a) The wavelength gets so long that boat gets closer to the top of its bow wave where its slope decreases, and the wave resistance becomes less.

(b) The motion against the uphill water produces a vertical pressure force, the hydrodynamic lift, so that less water is displaced than the weight of the boat. The height of the bow wave therefore decreases.

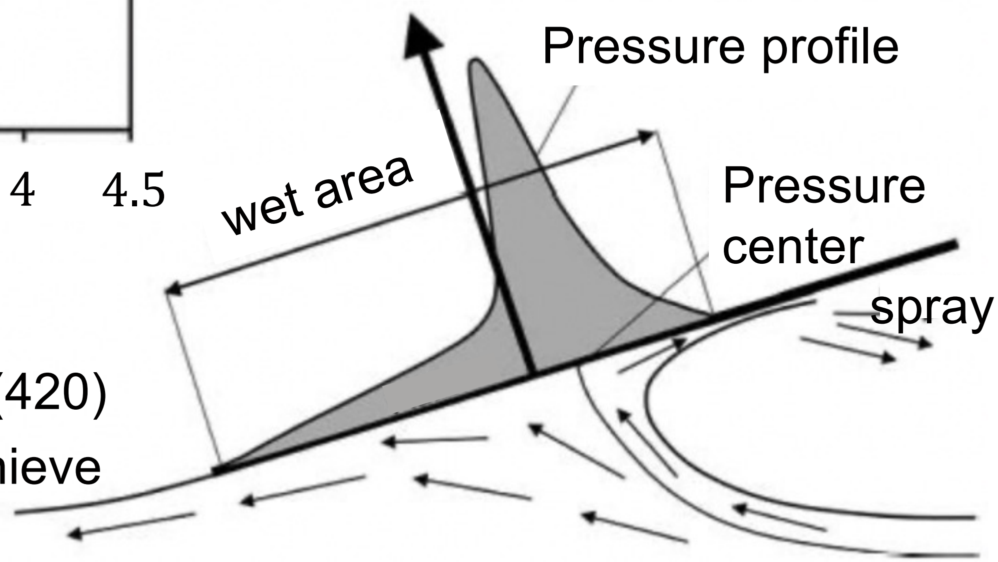


Gliding



Waterskiing

At 2 t0 2.5 times the hull speed, the water displacement is only half the boat weight.



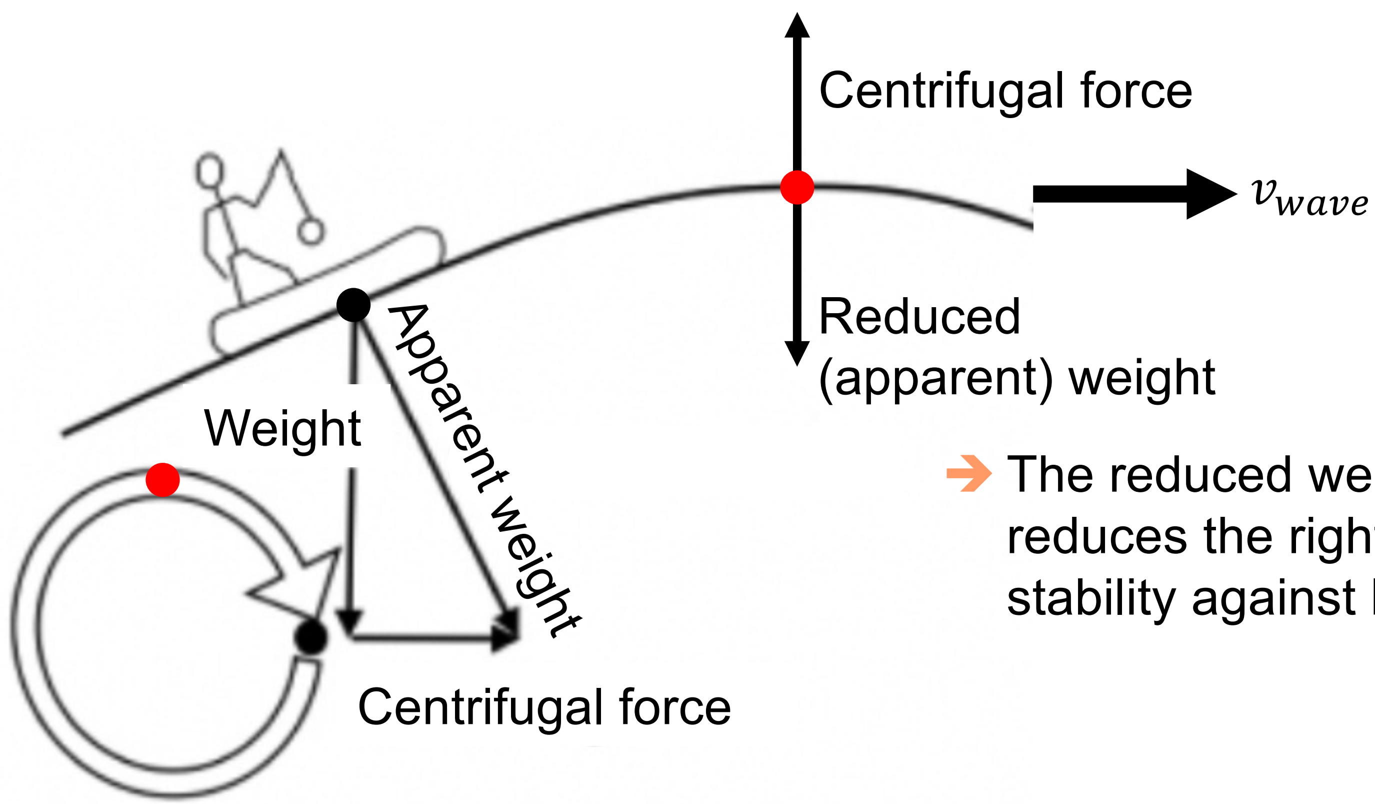
Good for gliding hulls are light and have:

a) Wide and flat hulls, with sudden end (420)

b) Changeable weight distribution to achieve best glide angle of about 7°.

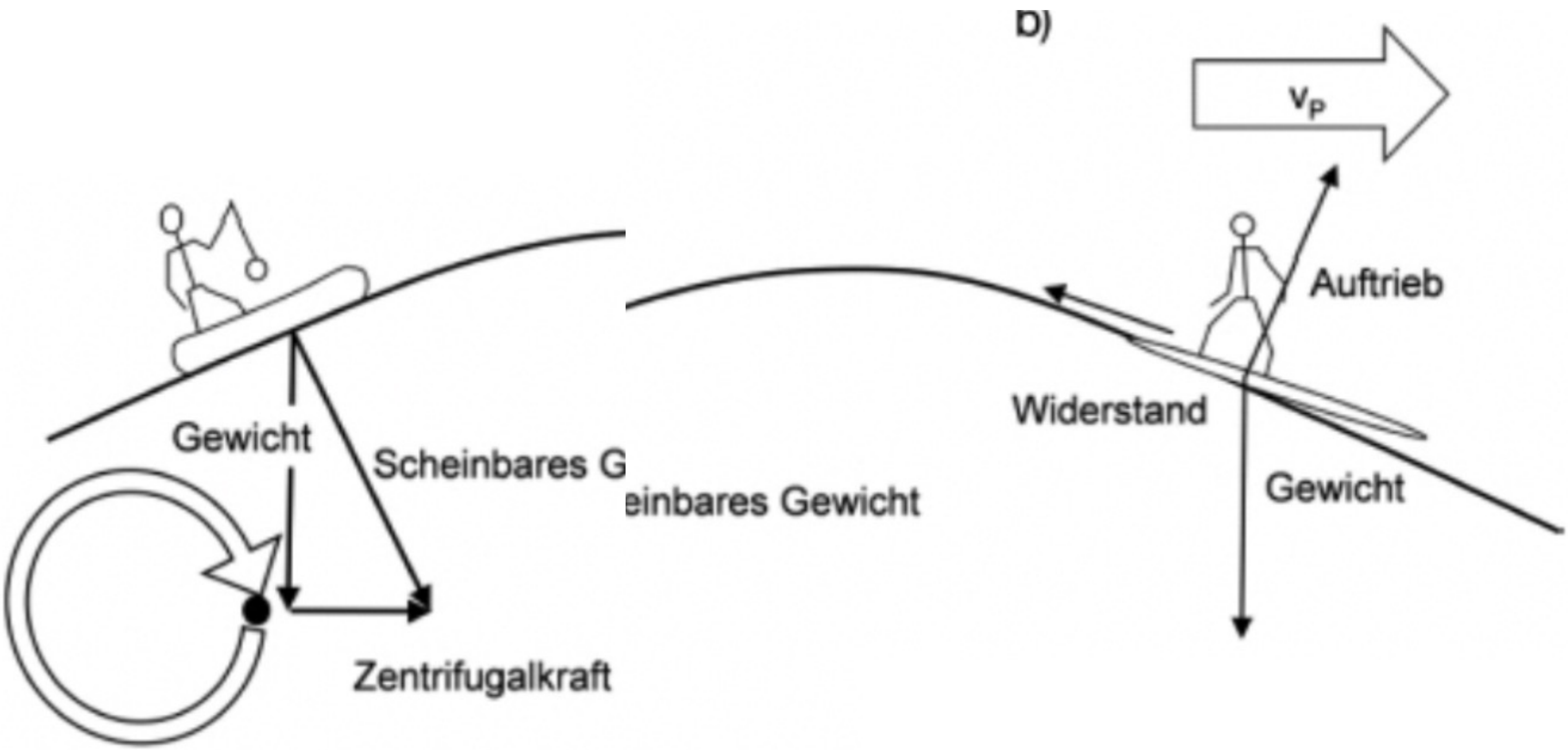
Can waves be good for speed ?

Motion together with water particles on a circular orbit, e.g., a rubber dinghy. The centrifugal force changes the apparent gravity.



→ The reduced weight at the wave crest reduces the righting torque, reducing stability against heeling.

Can waves be good for speed ?

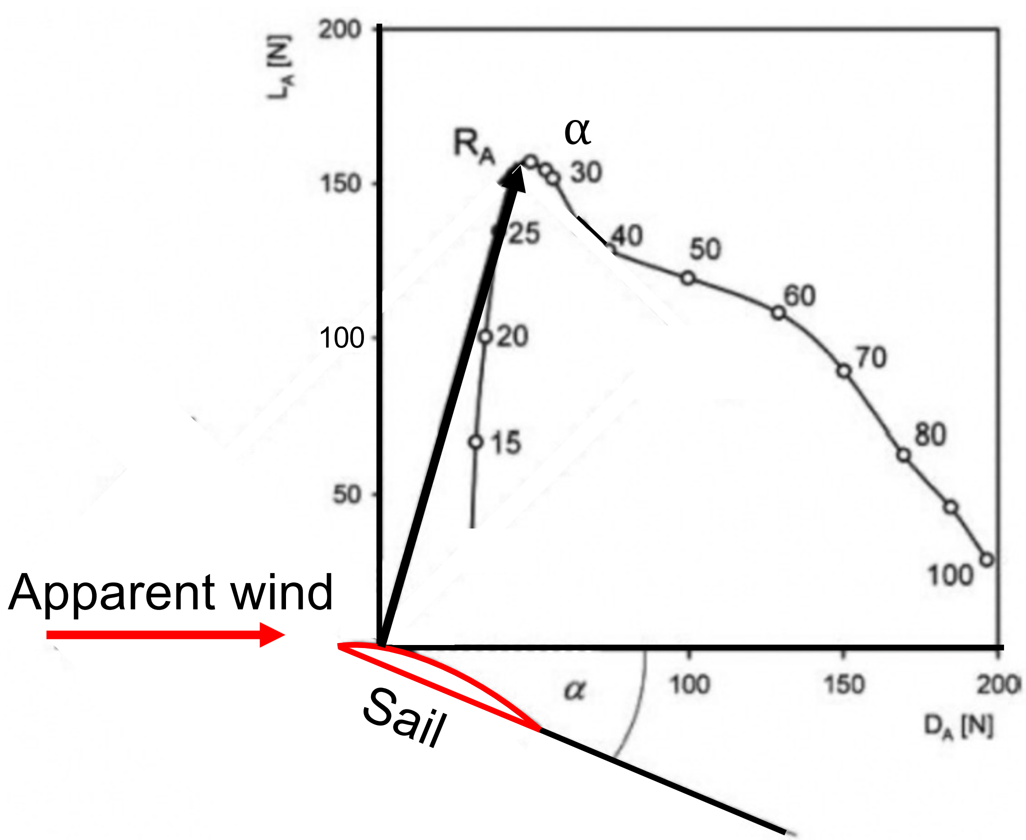


Good for gliding hulls are light and have:

a) Wide and flat hulls, with sudden end (420)
b) Changeable weight distribution to achieve best glide angle of about 7°.

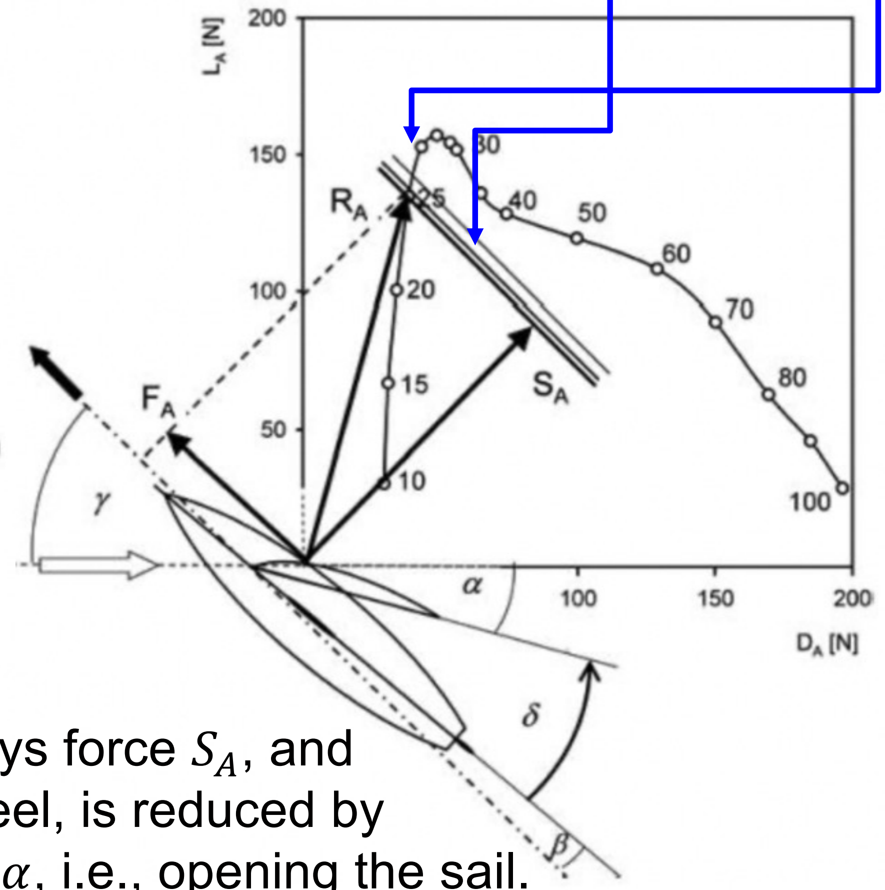
The Drag-Lift Polar Diagram in Air

Lift L_A and Drag D_A of only the sail



With a boat, it also yields forward and sideways forces.

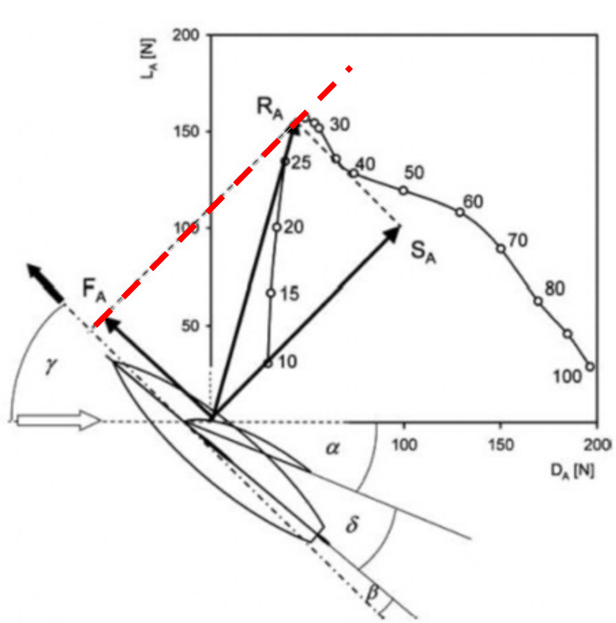
- α for the largest forward force
- Restrictions to the sideways force to limit heel



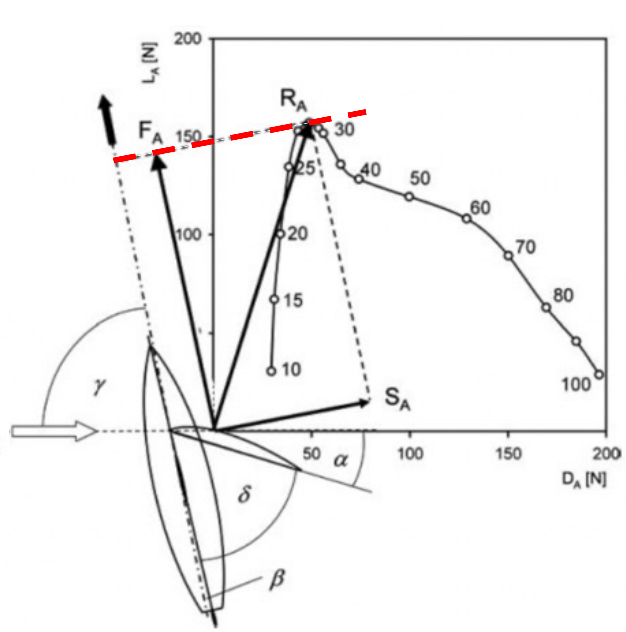
The sideways force S_A, and therefore heel, is reduced by decreasing α, i.e., opening the sail.

Peak Velocities from the Drag-Lift Polar Diagram

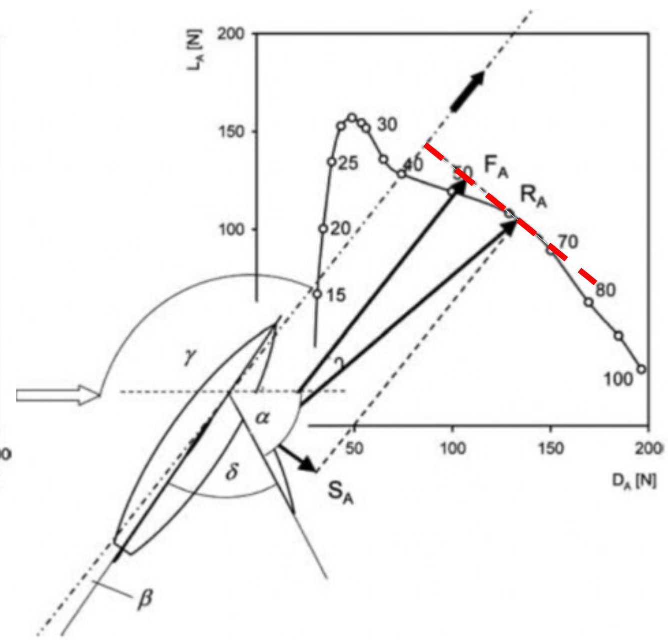
Close Haul



Close reach



Broad reach



Tangents for the peak velocities Haul

➔ Conclusion: Usually, the optimal sail angle is rather similar (and around 30^0) for all directions, except for a broad reach of more than about 135^{0}.

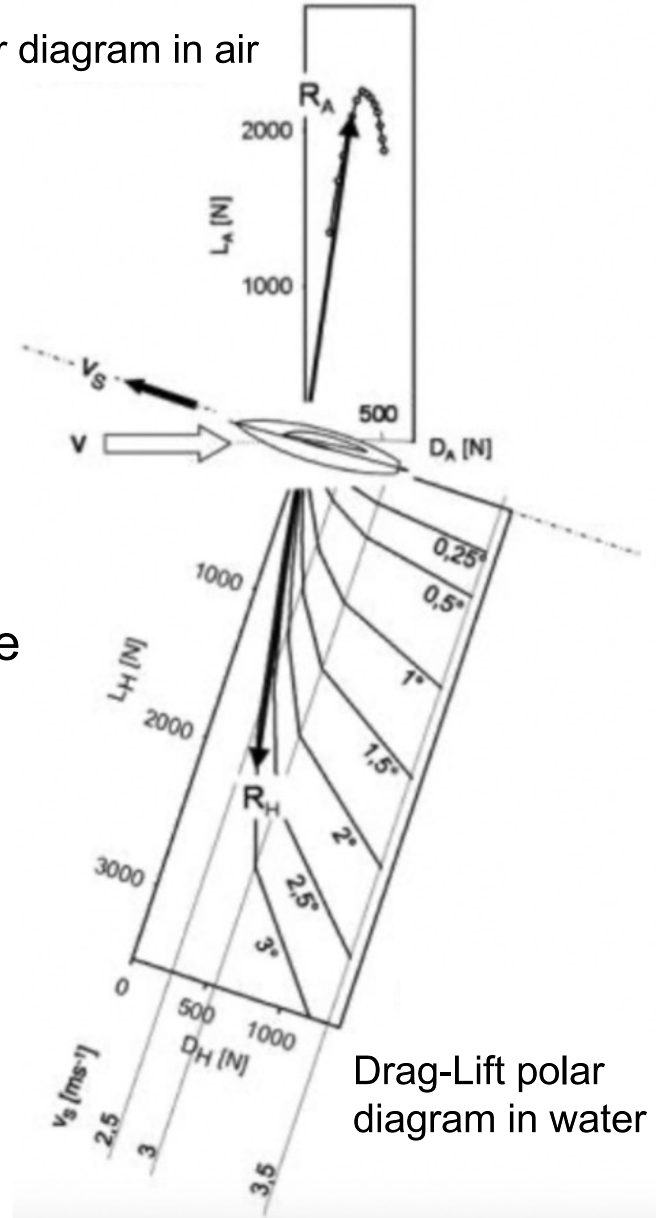
The Drag-Lift Polar Diagram in Water

Drag-Lift polar diagram in air

a) Sail angle α and apparent wind v determine the forward force F_A and the sideways force S_A in air.

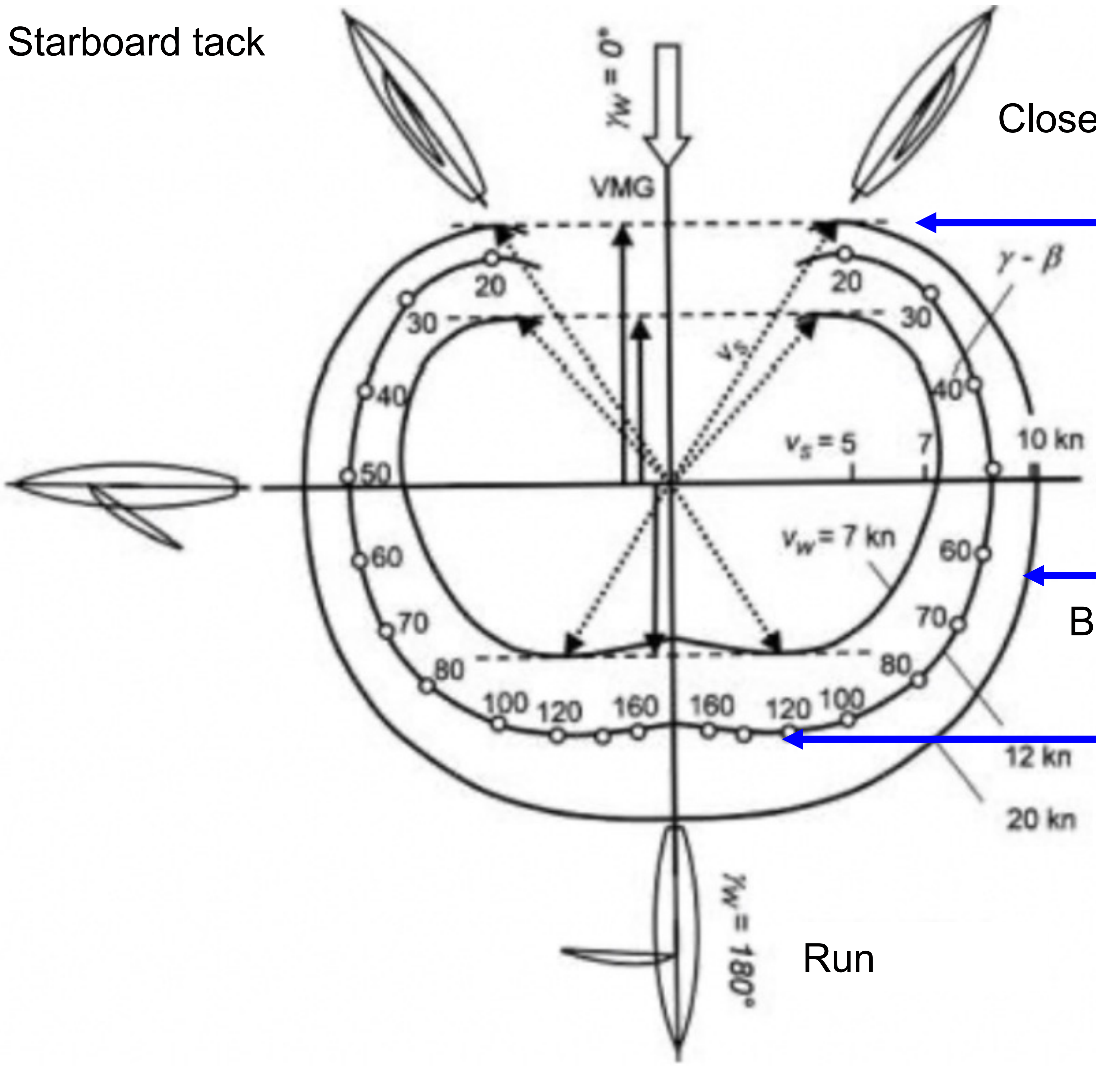
b) F_A determines the boat velocity v_s.

c) v_s and the hull angle β determine the lift L_H in water. Which must be equal to the sideways force S_A in air.



Drag-Lift polar diagram in water

The Velocity Polar Diagram



Common boats:

Peak $VMG \approx \frac{v_W}{2}$

at $\gamma_W \approx 45°$

at $\gamma \approx 30°$

High v_W ➔ v_S limited independent of angle

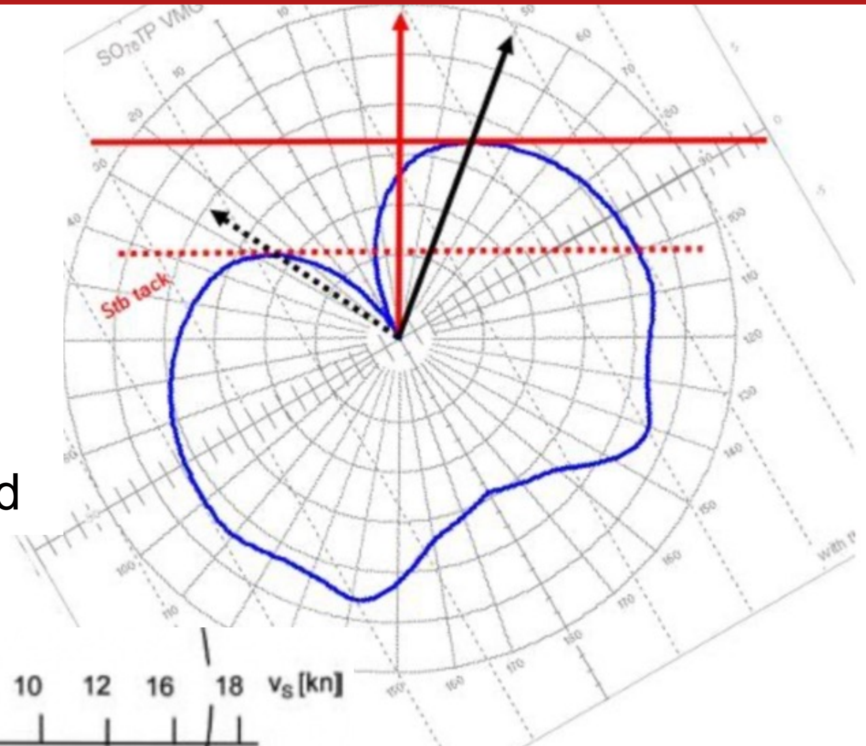
Peak downwind VMG

at $\gamma_W \approx 20°$

The VPD of a certain boat type is often hard get, because it depends on weight of the crew, type of sails, type of propeller, condition of the hull and sails, …

The velocity polar diagram shows the peak velocity that can be achieved for each angle to the true wind γ_W.

Use of the Velocity Polar Diagram

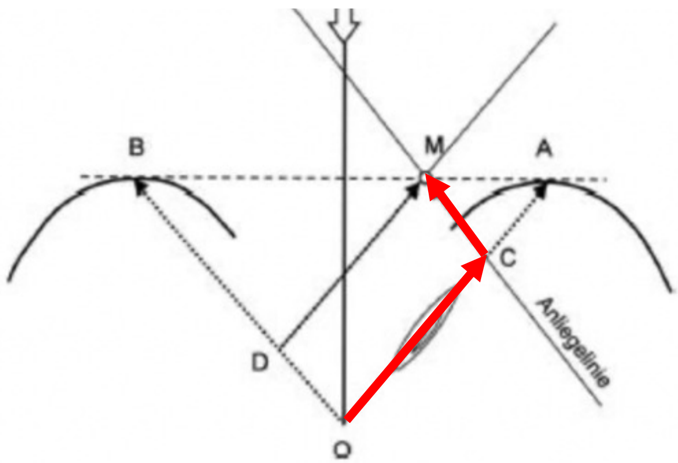


1) Determine the angle with the best velocity component in the desired direction.

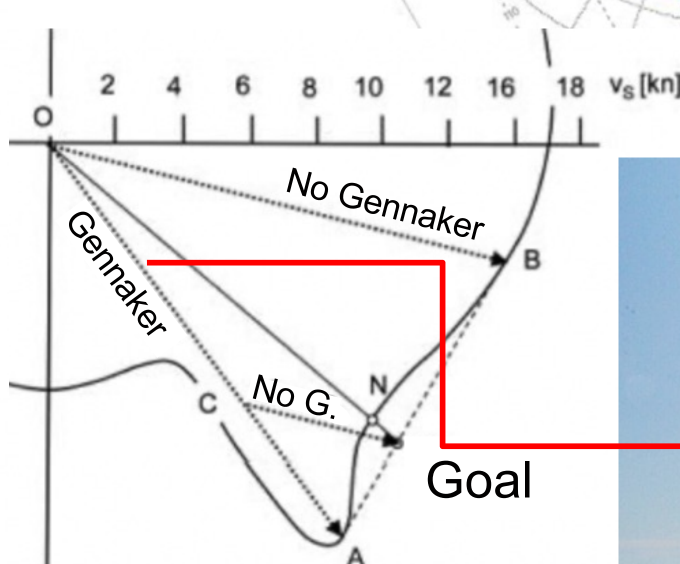
2) Determine how to divide a trajectory to increase the effective speed

a) Close reached

b) Broad reached



Fastest approach to the forbidden region. (Long Leg Leading, in case wind changes)



Possibly faster options

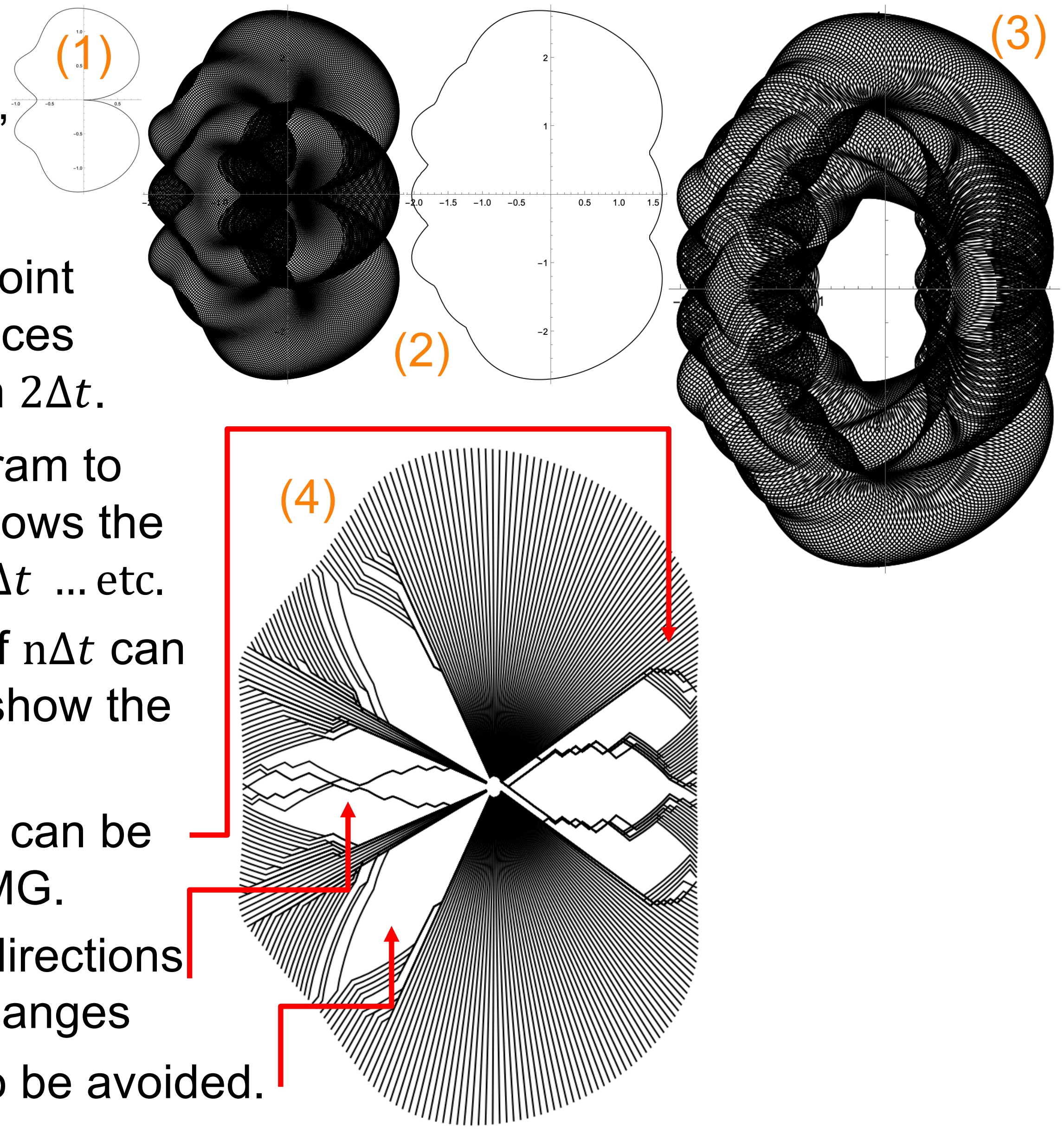
Determining the Best Path

1) The velocity polar diagram, times a small time traveled Δt, indicates the area one can reach in that time.
2) Adding this diagram to each point one can reach within Δt produces the envelope one can reach in $2\Delta t$.
3) Adding the velocity polar diagram to each point on the envelope shows the area that can be reached in $3\Delta t$ ... etc.
4) Every point on the envelope of $n\Delta t$ can be traced back to the start to show the fastest path for any angle.

➔ Destinations in the no-go zone can be reached by tacking with the VMG.

➔ Destinations in lower-velocity directions can be reached by direction changes

➔ Lower velocity directions are to be avoided.



Best Path for Variable Winds

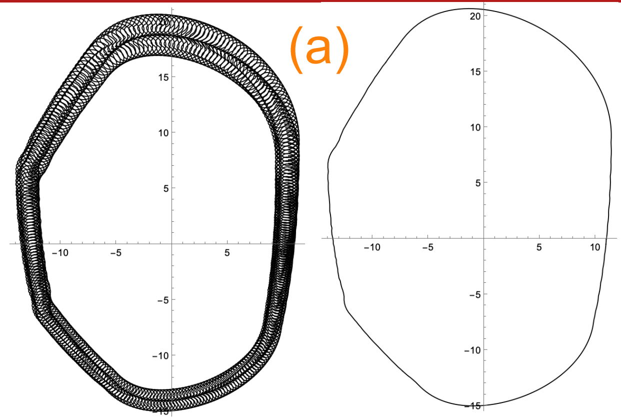
The same procedure of

a) Creating the envelopes that can be reached.

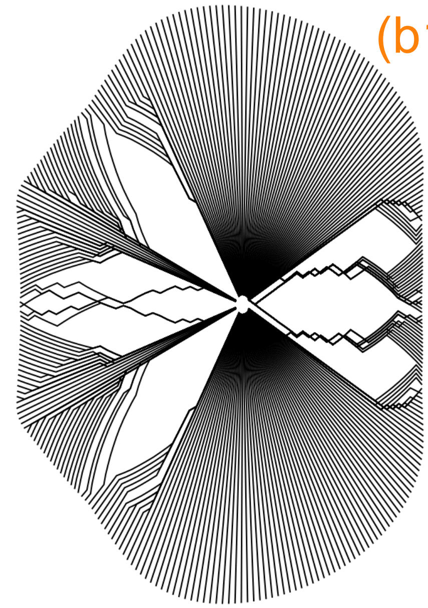
b) Tracking back the trajectories that lead to the envelop fastest. (b1) uniform wind, (b2) varying wind.

can be used when the wind depends on the position of the boat, and on the time when it is there.

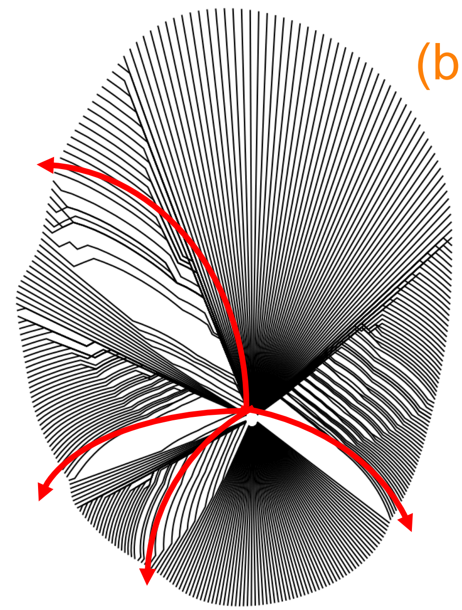
(a)



(b1)



(b2)



➔ Trajectories tend to bulge into strong-wind regions.

➔ When direction changes are needed to avoid lower-velocity directions, chose these to lead through stronger-wind areas.

➔ Lower velocity directions are to be avoided.